Teacher Notes

Activity 3

To Infinity and Beyond!

Objectives

- Develop an understanding of what it means to take a limit "at" infinity
- Develop an understanding of behavior that prevents limits from occurring by means of chaos or oscillation
- Estimate limits from graphs and tables of values
- Connect the ideas of end behavior, horizontal asymptotes, and limits at infinity

Materials

- TI-84 Plus / TI-83 Plus

Teaching Time

- 55 minutes

Abstract

This activity will examine limits as the input moves toward infinity. It will also summarize the idea of limits and examine limits that do not exist for chaotic reasons. The problems at the end of this activity make use of the ideas from prior limit activities.

Management Tips and Hints

Prerequisites

Students should:

- have an understanding of the imprecise nature of electronic utilities and what happens when the precision limits are reached.
- be able to manipulate graphs and tables of values manually and with the graphing handheld.
- have a basic understanding of function language.
- be able to identify rational, exponential, and trigonometric functions.
- be able to use the **VARS Menu** to produce function values.
- be able to store output values in lists.

Student Engagement

Working in pairs or small groups is recommended for maximum student engagement.

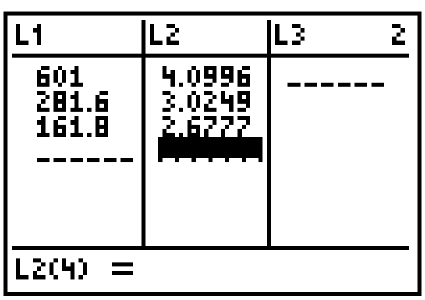
Evidence of Learning

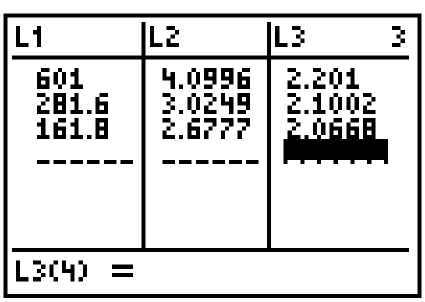
- Given a function, students will be able to state and explain the limit at infinity or explain why it fails to exist.
- Given a graph, students will be able to state and explain the limit at infinity or explain why it fails to exist.

Common Student Errors/Misconceptions

- Students often misinterpret infinity as an actual value to be substituted in a function.
- Students may incorrectly estimate infinity by pushing the graphing handheld beyond its precision limits and misinterpreting the result.
- Students may fail to investigate thoroughly the *chaotic* behavior and incorrectly conclude that a limit exists when it does not.

Activity Solutions

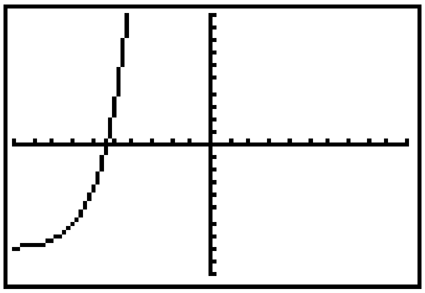
1. n/a
2. {601, 281.6, 161.8}
3. n/a
4. 

5. 

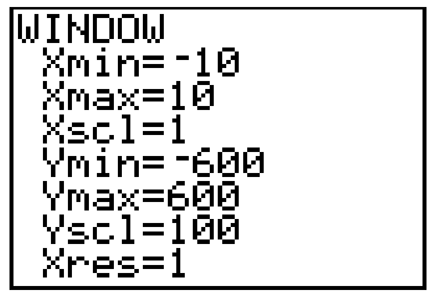
6. Answers will vary, but the data seems to be getting close to a value of 2.
7. {2.00000002, 2, 2}, according to the graphing handheld.

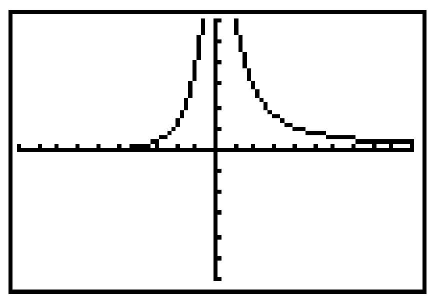
8. Answers will vary. The question is leading students to say that it appears so; however, the function never actually reaches the value of 2. Care should be taken to distinguish between the value of $f(x)$ actually being 2 and the graphing handheld producing a value of 2 because of its precision capabilities. This is a good place for discussion of *equals* versus *approaches very closely.*

9.

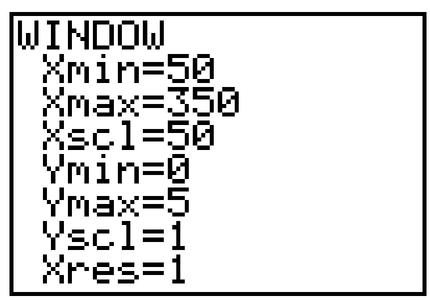


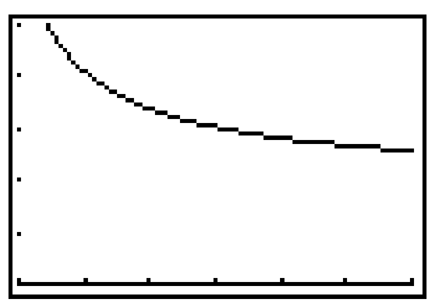
10. Answers will vary. Some students may keep trying to show the behavior in Quadrants II and III even though the question is leading them toward $x \to \infty$. Shown is one possible window and the resulting graph



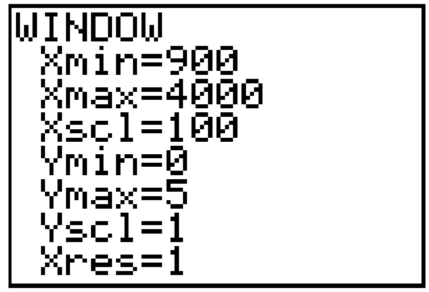


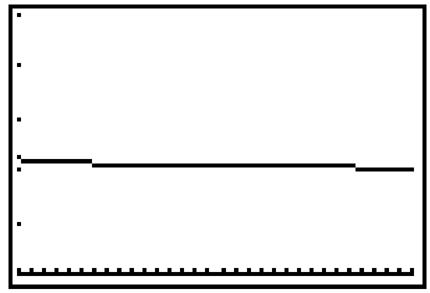
11. Answers will vary. Students will find it harder to include the behavior in Quadrants II and III. This would be a good place to remind students that the behavior sought is really "*x* is increasing toward infinity." It is also a good place to point out that if it is not always possible to get a good graph of the entire function, sometimes portions need to be looked at to get a true picture of the full behavior.





12. Answers will vary. One example is shown.





13. $\lim_{x \to \infty} \frac{2x^2 + 200x + 1000}{x^2 + 1} = 2$

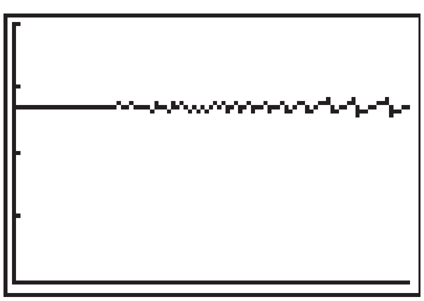
14. $\lim_{x \to -\infty} \frac{2x^2 + 200x + 1000}{x^2 + 1} = 2$

15.

X	L1	X	L2	X	L3
1	2	100	2.7048	1,000	2.7169
2	2.25	200	2.7115	2,000	2.7176
3	2.3704	300	2.7138	3,000	2.7178

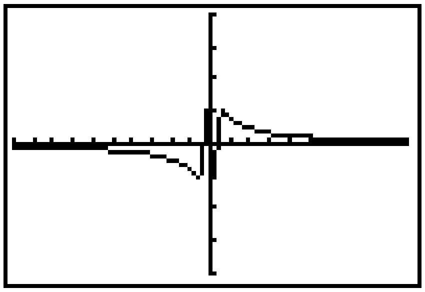
16. The graphing handheld produces values {2.71828182832, 1, 1}.

17.

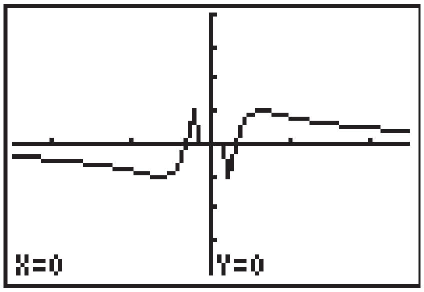


18. Although the value of the limit is e, it is not to be expected that students will recognize such a value. A reasonable answer is 2.718.

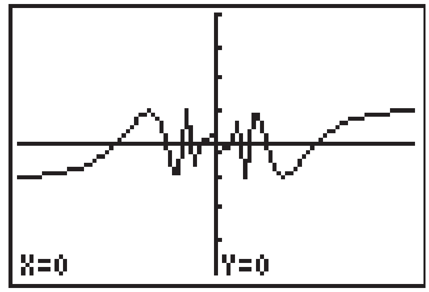
19.



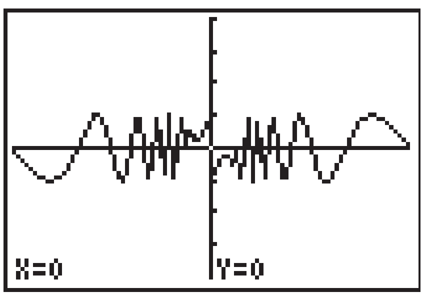
20. First Zoom



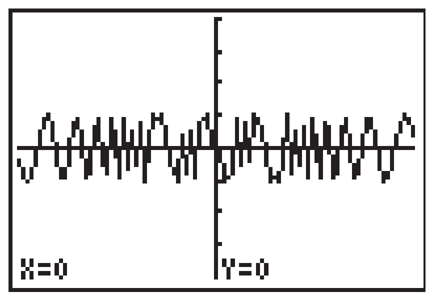
Second Zoom



Third Zoom

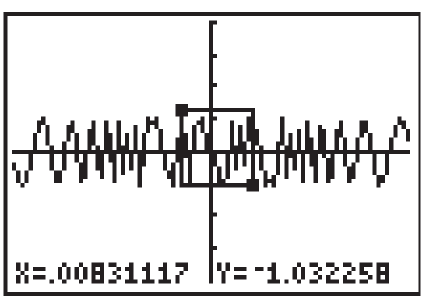


Fourth Zoom

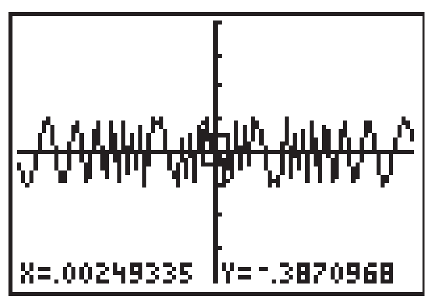


21. Answers will vary. Shown are two possibilities.

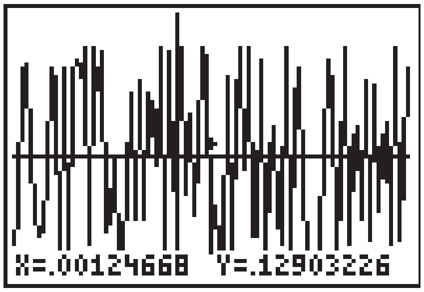
Zoom Box 1



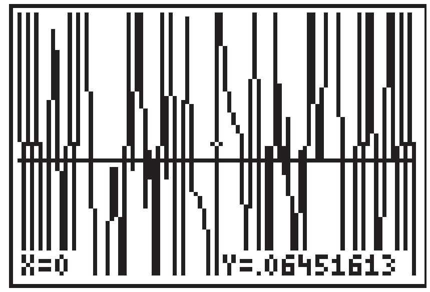
Zoom Box 2



Zoom Result 1



Zoom Result 2



22. $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist.

Although it does not move toward infinity or negative infinity, the function oscillates wildly at $x = 0$ and never gets near a single value.

23. 1

24. $\frac{1}{2}$

25. The limit does not exist. (There are two different limits as 0 is approached from different sides.)

26. $\frac{5}{4}$

27. $^{-}4$

28. 3

29. $\frac{1}{4}$