The Geometric Mean

ID: 9466

Time required
30 minutes

Activity Overview

In this activity, students will establish that several triangles are similar and then determine that the altitude to the hypotenuse of a right triangle is the geometric mean between the segments into which it divides the hypotenuse.

Topic: Ratio, Proportion & Similarity

- *Prove and apply the Mean Proportional Theorem for triangles.*

Teacher Preparation and Notes

- *In a right triangle, the altitude from the right angle to the hypotenuse will be the geometric mean between the segments of the hypotenuse.*
- ***X** is the geometric mean between **A** and **B** if* $\frac{A}{X} = \frac{X}{B}$*; this can also be expressed as* $X^2 = AB$*, or* $X = \sqrt{AB}$
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "9466" in the keyword search box.***

Associated Materials

- *GeometricMean_Student.doc*
- *GeometricMean.tns*

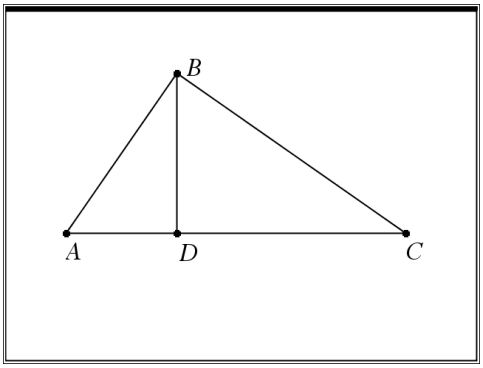
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Geometry: Concurrent Lines, Medians, and Altitudes with Cabri Jr. (TI-84 Plus) — 7288*
- *Geometric Mean Investigation (TI-Nspire technology) — 9944*
- *Geometric Mean with TI-Nspire (TI-Nspire technology) — 9655*

Introduction

Triangle *ABC* is a right triangle with right angle $\angle ABC$ and an altitude from the right angle to the hypotenuse. Therefore, $m\angle ADB = 90°$ and $m\angle CDB = 90°$. Using knowledge of similar triangles, we could conclude that $\triangle ABC$ is similar to $\triangle ADB$ and is similar to $\triangle BDC$. When we create proportions from the two smaller triangles, we get: $\frac{AD}{BD} = \frac{BD}{DC}$.



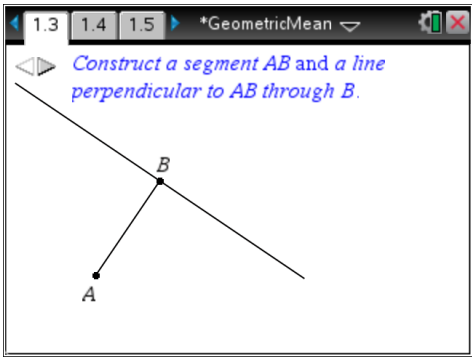
Simplifying this we have $BD^2 = AD \times DC$.

The line segment $\overline{BD}$ is called the **Geometric Mean** between $\overline{AD}$ and $\overline{DC}$.

Problem 1 – The Geometric Mean

On page 1.4, students are to construct $\overline{AB}$ and a perpendicular line to $\overline{AB}$ through *B*.

Note: If the points were not labeled as they were created, students can click on a point and press ctrl menu to select **Label** after they exit the **Perpendicular** tool.

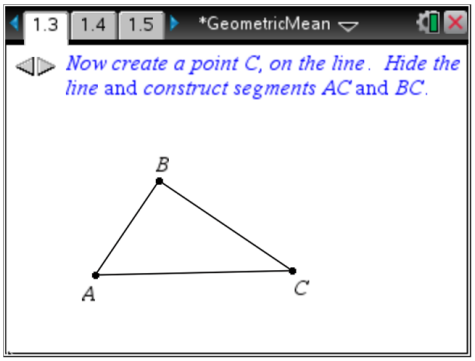


TI-Nspire™ Navigator™ Opportunity: *Live Presenter*

See Note 1 at the end of this lesson.

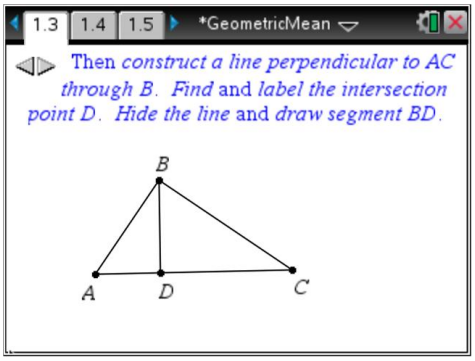
Students are to use the **Point On** tool to create a point *C* on the perpendicular line (menu **> Points & Lines > Point On**). If needed, drag to extend the perpendicular line.

Then, they can hide the perpendicular line with the **Hide/Show** tool and construct line segments connecting *A* to *C* and *B* to *C*.



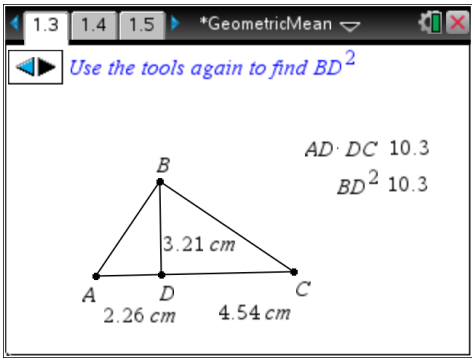
Students will construct a line that is perpendicular to $\overline{AC}$ through point *B*. They can then construct the point of intersection of this perpendicular line and $\overline{AC}$, labeling it *D*.

They should hide the perpendicular line and construct a line segment connecting *B* to *D*.



Direct students to measure segments $\overline{AD}$, $\overline{DC}$ and $\overline{BD}$.

Note: Since $\overline{AD}$ and $\overline{DC}$ were not constructed as segments, students will measure their lengths by clicking the two endpoints. For example, after selecting menu **> Measurement > Length**, click on point A and then point D to measure the length of segment $\overline{AD}$.



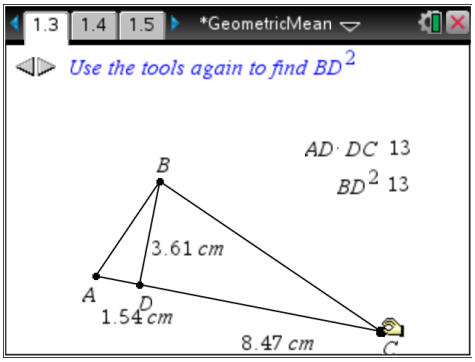
Students should select the **Text** tool and enter the formula *AD* × *DC* on the screen. Then, they can use the **Calculate** tool to find the product of the lengths of $\overline{AD}$ and $\overline{DC}$.

Note: After choosing the **Calculate** tool,, press (or enter) once on the text of the formula. Then, click additional times to select the value of each variable in the formula.

Students are to use the **Text** and **Calculate** tools again to find the square of the length of the altitude $\overline{BD}$. They should drag point *A* and point *C* so that *AD* = 3 and *DC* = 7. Students may need to increase the number of displayed digits. The length of the altitude, $\overline{BD}$, should be an estimate for the value of $\sqrt{21}$.

Note: Students may need to increase the number of displayed digits. To do this, place the cursor over a measurement, then press the plus key.

Students should manipulate their sketch again so that *AD* = 1 and *DC* = 3. They should see that the measurement of $\overline{BD}$ is the square root of ***AD·DC***. They are to consider how to use this figure to find other radical values and answer the questions on the worksheet.



TI-Nspire™ Navigator™ Opportunity: *Quick Poll (Open Response)*

See Note 2 at the end of this lesson.

Student Solutions

1. Sample table:

Length of $\overline{AD}$	Length of $\overline{DC}$	$AD \times DC$	Length of $\overline{BD}$	BD^2
3.199 cm	5.683 cm	18.18 cm^2	4.264 cm	18.18 cm^2
3.308 cm	4.404 cm	14.57 cm^2	3.817 cm	14.57 cm^2

2. $BD = \sqrt{3}$ cm ≈ 1.732 cm

3. *Sample answers:*

a. AD = 2 cm, DC = 5 cm

b. AD = 2 cm, DC = 3 cm

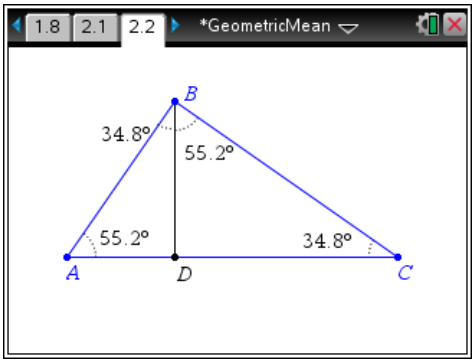
c. AD = 5 cm, DC = 10 cm

d. AD = 8 cm, DC = 9 cm

Problem 2 – Similar Triangles

On page 2.2, students will determine that the triangles are similar by measuring the angles in the figure.

On the worksheet, students are to write a similarity relationship for the three triangles in the figure. They are to determine what other proportions are true about the figure besides $\frac{AD}{BD} = \frac{BD}{DC}$ and identify other geometric means that occur when an altitude is constructed to the hypotenuse of a right triangle.



TI-Nspire™ Navigator™ Opportunity: *Class Capture*

See Note 3 at the end of this lesson.

Student Solutions

4. $\triangle ABC \sim \triangle ADB \sim \triangle BDC$

5. $\frac{AB}{BC} = \frac{BD}{DC} = \frac{AD}{BD}$, $\frac{AB}{AD} = \frac{BC}{BD} = \frac{AC}{AB}$

6. $\frac{AC}{AB} = \frac{AB}{AD}$

Exercises

The exercises in this section reinforce the skills learned in this activity.

Student Solutions

1. $BD = 9$ in.

2. $DC = 16$ in.

3. $BD = 3\sqrt{6}$ in.

4. $BD = 5\sqrt{2}$ cm, $AB = 5\sqrt{3}$ cm, and $BC = 5\sqrt{6}$ cm

5. $AD = \frac{1}{2}$ cm, $AB = \frac{\sqrt{17}}{2}$ cm, and $BC = 2\sqrt{17}$ cm

TI-Nspire™ Navigator™ Opportunity: *Class Capture or Quick Poll*

See Note 4 at the end of this lesson.

TI-Nspire™ Navigator™ Opportunities

Note 1
Problem 1, *Live Presenter*
To ensure students are following the steps correctly, you could have different students show different steps of the construction. Have one student show drawing the segment. Then have the next student show creating the perpendicular line, and so on.

Note 2
Problem 1, Question 2 *Quick Poll (Open Response)*
Use Quick Poll to see if students found the correct answer to Question 2.

Note 3
Problem 2, Question 5 *Class Capture*
As students to explain the different ratios they found. For each student, show their screen as they describe one of the ratios. Have them point out the line segments or parts of the triangles that have the indicated ratios.

Note 4
Apply the Math, Questions 1-5 *Class Capture or Quick Poll*
Gather the answers from students to ensure they are finding the correct lengths. You can use either Quick Poll to ask the whole class for their results to any of the questions or Class Capture and have one student explain how they found the unknown lengths.