Circumcentre
Guided Investigation

Teacher Notes Answers

7 8 9 **10 11** 12

TI-Nspire™ Investigation Student 50 min

Introduction

A circumcircle passes through each vertex of a polygon. This investigation focuses on the specific case of the circumcircle as it applies to a triangle. The concept of a circumcircle and other similar geometric problems underpin concepts associated with electronic navigation. The circumcentre is the centre of the circumcircle and therefore, a point that is equidistant from each of the vertices.

Scan the QR code or use the link to watch a video to help set up the diagram on your calculator and to help check your answers.

https://bit.ly/Circumcentre

Geometry

Open a New TI-Nspire Document and insert a **Graphs Application**.

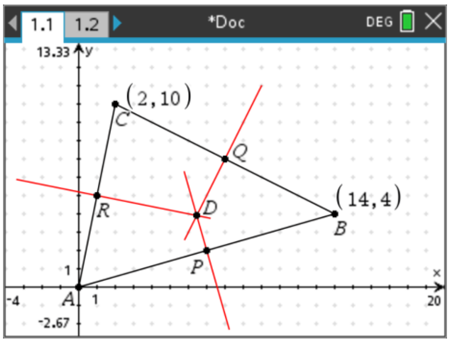
Draw a triangle with vertices:

A:(0, 0) B:(14, 4) C:(2, 10)

Construct perpendicular bisectors to sides: AB, BC and AC.

The aim is to determine the point where all three perpendicular bisectors intersect, the circumcentre of the triangle.

Note: Points P, Q & R have been labelled for reference only.



Question: 1.

Determine the gradient of side AB and hence the gradient of the perpendicular bisector (PD)

Answer: $\frac{Rise}{Run} = \frac{4-0}{14-0} = \frac{2}{7}$ Perpendicular bisector gradient: $\frac{2}{7} \times m_2 = -1$ therefore $m_2 = -\frac{7}{2}$

Question: 2.

Determine the coordinates of the midpoint on side AB. (Point P)

Answer: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (7, 2)$

Question: 3.

Determine the equation of the perpendicular bisector to side AB. (Line PD)

Answer: $y = -\frac{7}{2}(x-7) + 2$ which simplifies to: $y = -3.5x + 26.5$

Tip!

The translational form of a straight line is useful when the gradient and a point are known:

$y = m(x-h) + k$ is a straight line with gradient m passing through the point (h, k).

Remember to use your calculator to check your answers.

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Question: 4.

Determine the gradient of side AC and hence the gradient of the perpendicular bisector (RD)

Answer: $\frac{Rise}{Run} = \frac{10-0}{2-0} = 5$ Perpendicular bisector gradient: $5 \times m_2 = -1$ therefore $m_2 = -\frac{1}{5}$

Question: 5.

Determine the coordinates of the midpoint on side AC. (Point R)

Answer: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (1,5)$

Question: 6.

Determine the equation of the perpendicular bisector to side AC. (Line RD)

Answer: $y = -\frac{1}{5}(x-1)+5$ which simplifies to: $y = -0.2x+5.2$

Question: 7.

Determine the gradient of side BC and hence the gradient of the perpendicular bisector (QD)

Answer: $\frac{Rise}{Run} = \frac{10-4}{2-14} = -\frac{1}{2}$ Perpendicular bisector gradient: $-\frac{1}{2} \times m_2 = 1$ therefore $m_2 = 2$

Question: 8.

Determine the coordinates of the midpoint on side BC. (Point R)

Answer: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (8,7)$

Question: 9.

Determine the equation of the perpendicular bisector to side BC. (Line QD)

Answer: $y = 2(x-8)+7$ which simplifies to: $y = 2x-9$

Question: 10.

Use simultaneous equations to determine the point of intersection for the perpendicular bisectors: QD and RD.

Answer: Equations: $y = -0.2x+5.2$ & $y = 2x-9$

$$\begin{aligned} -0.2x+5.2 &= 2x-9 \\ 14.2 &= 2.2x \\ x &= \frac{71}{11} \end{aligned} \qquad \begin{aligned} y &= -0.2\left(\frac{71}{11}\right)+5.2 \\ y &= \frac{43}{11} \end{aligned}$$

Question: 11.

Verify the point of intersection (circumcentre) using the perpendicular bisectors QD and PD.

Answer: Equations: $y = -3.5x+26.5$ & $y = 2x-9$

$$\begin{aligned} -3.5x+26.5 &= 2x-9 \\ 35.5 &= 5.5x \\ x &= \frac{71}{11} \end{aligned} \qquad \begin{aligned} y &= 2\left(\frac{71}{11}\right)-9 \\ y &= \frac{43}{11} \end{aligned}$$

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Question: 12.

Verify that distances AD, BD and CD are equal.

Answer: Distance AD: $\sqrt{\left(\frac{71}{11}-0\right)^2+\left(\frac{43}{11}-0\right)^2}=\frac{\sqrt{6890}}{11}\approx 7.546$

Distance BD: $\sqrt{\left(\frac{71}{11}-14\right)^2+\left(\frac{43}{11}-4\right)^2}=\frac{\sqrt{6890}}{11}\approx 7.546$

Distance CD: $\sqrt{\left(\frac{71}{11}-2\right)^2+\left(\frac{43}{11}-10\right)^2}=\frac{\sqrt{6890}}{11}\approx 7.546$

Question: 13.

Determine the equation to the circle that passes through the three vertices.

Centre: $\left(\frac{71}{11},\frac{43}{11}\right)$ Radius = $\frac{\sqrt{6890}}{11}$

Equation: $\left(x-\frac{71}{11}\right)^2+\left(y-\frac{43}{11}\right)^2=\frac{6890}{121}$

Teacher Notes:

Students can verify their calculations for each question using the diagram they produce on their calculator. For students using a TI-Nspire CX (non-CAS), requesting 'exact' answers is a way of ensuring students have performed the calculations rather than relying purely on the calculator.

This activity is part of a 'series' that includes the Orthocentre, Centroid and the Euler line!

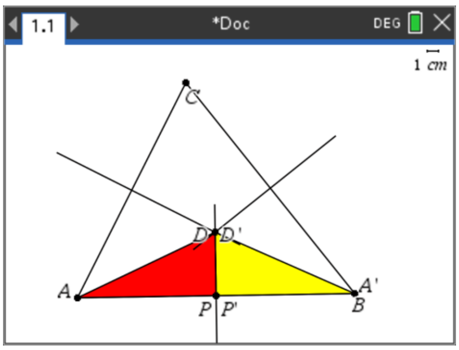
The Orthocentre and Centroid activities do not include a video, so it is recommended that this activity (circumcentre) be completed first. The geometric constructions and calculations are similar for the Orthocentre and Centroid, therefore students should be able to rely on their knowledge from this activity.

The Euler line is a straight line that passes through the circumcentre, orthocentre and centroid and provides a wonderful conclusion to the series!

Teachers are also encouraged to show the geometric proof of the circumcentre of a triangle.

Sample:

Triangle APD is reflected in the perpendicular bisector PD. This can be done using the reflection tool. Now students can be led through the proof:



- AP = PB (bisection);
- $\angle$ APD = $\angle$ BPD (perpendicular)
- DP is common
- $\therefore$ Δ APD $\equiv$ Δ BPD
- $\therefore$ AD = BD

This can be repeated on each side of the triangle leading to AD = BD = CD, thus D is equidistant from each vertex.

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