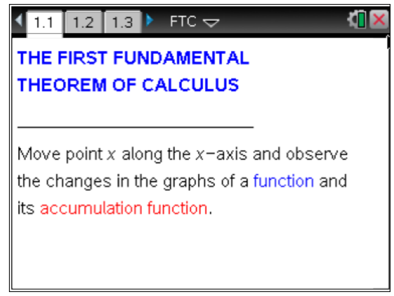
The First Fundamental Theorem of Calculus



Math Objectives

- Students will be able to identify the graphical connections between a function and its accumulation function.
- Students will be able to describe the relationship between a function and its accumulation function.
- Students will be able to apply and explain the first Fundamental Theorem of Calculus.
- Use appropriate tools strategically. (CCSS Mathematical Practice)
- Look for and express regularity in repeated reasoning. (CCSS Mathematical Practice)

Vocabulary

- signed area
- local maximum
- local minimum
- inflection point
- accumulation function

About the Lesson

- The intent of this lesson is to help students make visual connections between a function and its definite integral.
- Given a graph of a function and a graph of the function's accumulation function, students observe the coincidence of the accumulation function's extrema with the original function's zeros, and of the accumulation function's inflection point with the original function's extremum.
- A graph of the accumulation function and its derivative solidifies students' understanding of the relationship between the derivative of the accumulation function and the original function.
- The lesson concludes with students explicitly stating the first Fundamental Theorem of Calculus.

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- Use Class Capture to demonstrate that students can grab and drag the point *x* properly.
- Use Quick Poll to assess student understanding during the activity.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- FTC_Student.pdf
- FTC_Student.doc

TI-Nspire document

- FTC.tns

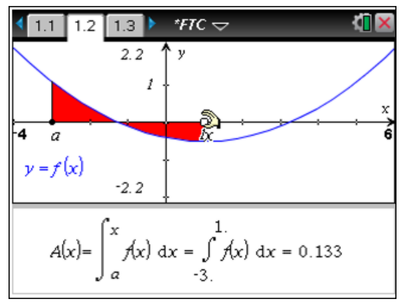
Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (☝) getting ready to grab the point. Then press ctrl 🖱 to grab the point and close the hand (✊).

Move to page 1.2.

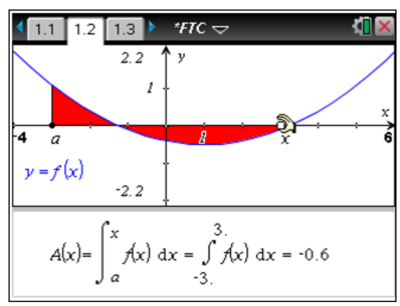
1. The graph shown is of the function $y = f(x)$. The **accumulation function** of $f(t)$ from a to x is given by $A(x) = \int_a^x f(x)dx$. The accumulation function measures the definite integral of f from a to x. For example, if you set a to –3, $A(2) = \int_{-3}^{2} f(x)dx$, you get the value of the definite integral of f from –3 to 2.

   Drag the point x along the x-axis to determine the values of the accumulation functions below:



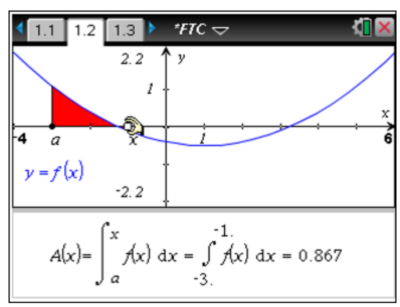
   a. $A(3) = \int_{-3}^{3} f(x)dx =$ ________________

   Answer: –0.6



   b. $A(0) = \int_{-3}^{0} f(x)dx =$ ________________

   Answer: 0.6



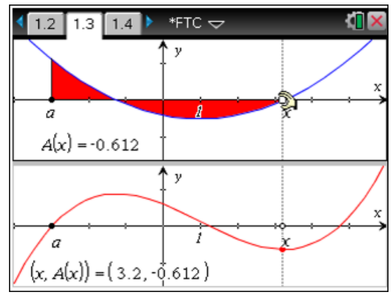
   c. $A(-1) = \int_{\square}^{\square} f(x)dx =$ ________________

   Answer: $\int_{-3}^{-1} f(x)dx = 0.867$

Move to page 1.3.

The top graph shows the original function, $y = f(x)$, and the shaded region between the graph of the function and the x-axis as the point x is dragged along the x-axis. The bottom graph shows the value of the definite integral for each upper limit x, with lower limit $a = –3$. Drag point x along the x-axis in the top graph to observe the relationship between the two graphs.

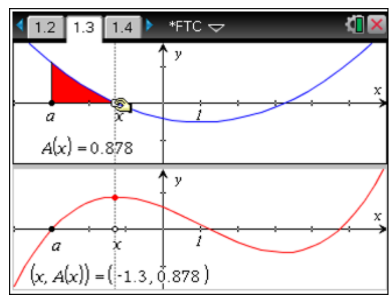
2. a. At what value(s) of x does the accumulation function, $A(x)$, have a local maximum? A local minimum? Explain how you know.



<u>Answer:</u> Local maximum: around $x = –1.2$ or $x = –1.3$
Local minimum: around $x = 3.2$

The local maximum is where the largest value of $A(x)$ occurs, and the minimum is where the smallest value of $A(x)$ occurs.

b. Drag point x to the x-value at which $A(x)$ has a local maximum. What do you notice about the value of the original function, $f(x)$, at that point?

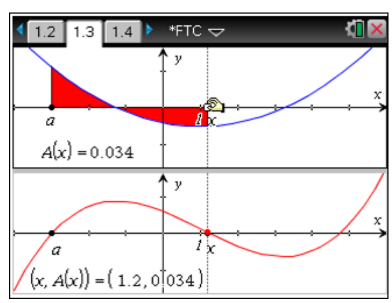


<u>Answer:</u> $f(x)$ is zero where $A(x)$ has a local maximum.

c. Drag point x to the x-value at which $A(x)$ has a local minimum. What do you notice about the value of the original function, $f(x)$, at that point?

<u>Answer:</u> $f(x)$ is zero where $A(x)$ has a local minimum.

d. At what value of x does the accumulation function, $A(x)$, have an inflection point? Explain how you know.



<u>Answer:</u> $A(x)$ has an inflection point at a value of x of about 1.2. The inflection point is where $A(x)$ changes from concave down to concave up.

e. Drag point x to the inflection point of $A(x)$. What do you observe about the original function, $f(x)$, at that point?

Answer: This appears to be the same x-value where $f(x)$ has a minimum.

> **Teacher Tip:** Because the x-axis is scaled in increments of 0.1, students may not be able to move the point x until it is at the location of the extrema or the inflection points. However, it is not important that students attain these values exactly, but that they understand the connections between the extrema and inflection point of A(x) and the zeros and minimum of $f(x)$.

3. a. Over what interval(s) is $A(x)$ increasing? Decreasing?

Answer: $A(x)$ is increasing over $(–4, –1.2) \cup (3.2, 6)$ and decreasing over $(–1.2, 3.2)$.

b. What do you observe about $f(x)$ over the interval(s) where $A(x)$ is increasing? Over the interval(s) where $A(x)$ is decreasing?

Answer: When $A(x)$ is increasing, $f(x)$ is positive. When $A(x)$ is decreasing, $f(x)$ is negative.

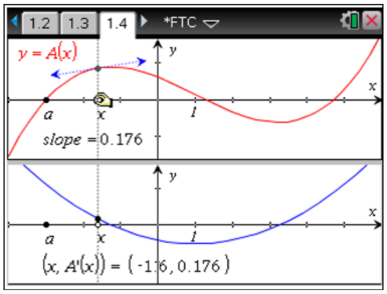
4. Based on your observations in questions 2 and 3, what do you believe the relationship between the functions $f(x)$ and $A(x)$ to be? Explain your reasoning.

Answer: $f(x)$ is the derivative of $A(x)$.

> **CCSS Mathematical Practice:** Use appropriate tools strategically. Mathematically proficient students will use technology in questions 2 and 3 to explore the connections between $f(x)$ and $A(x)$ and conclude that $f(x)$ is the derivative of $A(x)$ based on their observations.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of the lesson.

Move to page 1.4.



5. The top graph on page 1.4 is the graph of the accumulation function, $A(x)$, for the function $f(x)$ from previous pages.

   a. Drag point x and observe the changes in both graphs. What is the graph on the bottom of the page measuring? How do you know?

   Answer: The graph on the bottom gives the slopes, or the derivative, of the accumulation function. You can tell because its points are of the form $(x, A'(x))$ and because the bottom graph corresponds to the slopes of the top graph.

   b. What is the relationship between the bottom graph on page 1.4 and the original function, $f(x)$?

   Answer: They are the same function.

   c. Based on your observations, what is the relationship between the functions $f(x)$ and $A(x)$? How do you know? How does this compare to your answer to question 4?

   Answer: $f(x)$ is the derivative of $A(x)$. It is the same as the graph of the derivative given on page 1.4. This is the same hypothesis from question 4.

6. Complete the following: $\frac{d}{dx}A(x) = \frac{d}{dx}\int_a^x f(t)dt$ = _______________. Explain your reasoning.

   Answer: $f(x)$. You have determined in the preceding questions that the derivative of the accumulation function is the original function.

> **CCSS Mathematical Practice:** Look for and express regularity in repeated reasoning.
> Mathematically proficient students will notice that the derivative of the accumulation function is the original function and generalize the First Fundamental Theorem of Calculus.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of the lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The first Fundamental Theorem of Calculus.

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Note 1

Question 4: *Quick Poll*

As students complete item 4, send a Quick Poll asking: Using the same function, what would have to change in the top graph in order for the coordinates of the critical points to change in the bottom graph?

Answer: The value of a would have to change.

Note 2

Question 6: *Quick Poll (Multiple Choice)*

After students complete the activity worksheet, send a Quick Poll asking: If the point $(x, A'(x))$ is the point (2, 0), what will be true about the corresponding point on the top graph?

Answer: The point on the graph of $y = A(x)$ will be a relative maximum or relative minimum point.