Nuclear Decay and Chain Reactions – ID: 9522

Time required
45 minutes

Topic: Nuclear Physics

- *Identify and write equations for the three forms of nuclear decay.*
- *Predict decay products.*
- *Perform half-life and decay constant calculations.*
- *Describe nuclear reactions and the conditions necessary for a chain reaction.*

Activity Overview

The activity begins with animations of alpha, beta, and gamma emissions. In studying the animations, students discover properties of the emitted particles. The next part of the activity has students make graphs of the counts of a Geiger counter vs. time for the decay of a short-lived nucleus, Po-218. They determine the decay constant and half-life. The final part of the activity looks at nuclear fission and simulates a chain reaction.

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Notes, Data & Statistics

Teacher Preparation

Review with students the relationships between the numbers of protons and neutrons and atomic number and mass number. Go over nuclear symbols, such as He-4 and $^{4}_{2}He$ *. Discuss the relationship between the two exponential function forms:* $f(x) = a \cdot b^{x}$ *and* $a \cdot e^{-c \cdot x}$*.*

- *The screenshots on pages 2–10 demonstrate expected student results. Refer to the screenshots on pages 11–14 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "9522" in the search box.***

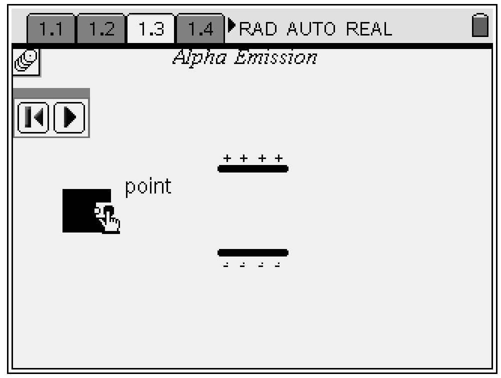
Classroom Management

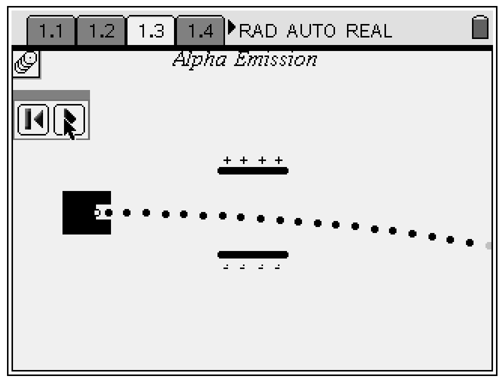
- *This activity is designed to be **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

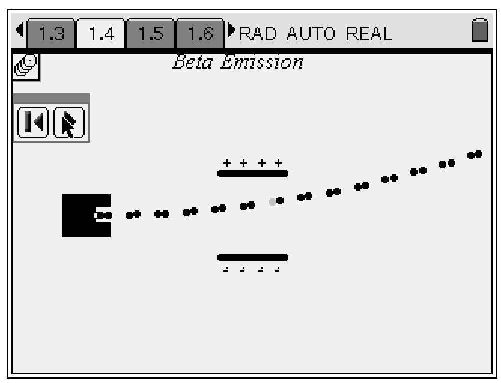
The following questions will guide student exploration in this activity:

- In what ways do radioactive nuclei decay?
- How are the rates of nuclear reactions characterized by a decay constant and a half-life?
- Why does nuclear fission lead to chain reactions?

Problem 1 – Nuclear decay processes







Step 1: Students should open the file **PhyAct_9522_NucDecay.tns** and read the first two pages. Pages 1.3, 1.4, and 1.5 show animations of alpha, beta, and gamma emissions, respectively. Each page shows a radiation source (black box) and two electrically charged plates (solid lines). On each page, students should put a **Geometry Trace** (**Menu > Trace > Geometry Trace**) on the solid circle just inside the radiation source. Then, they should run the animation by pressing the play button. Note: Students should not press esc after putting the **Geometry Trace** on the point. Students should observe the path traced out by each type of radiation. Then, they should answer questions 1–4.

Q1. Based on your observations, what is the sign of the charge on the alpha particle? Explain your answer.

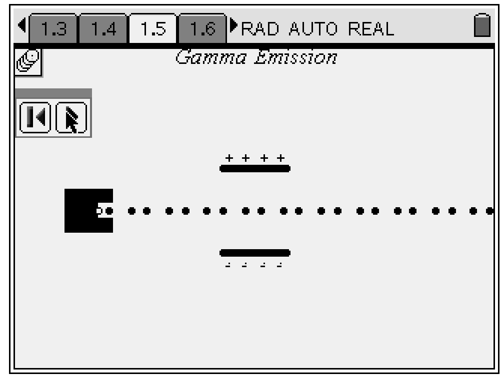
A. *The beam of alpha particles bends toward the negatively charged plate. Therefore, the alpha particles must be positively charged.*

Q2. Based on your observations, what is the sign of the charge on the beta particle? Explain your answer.

A. *The beam of beta particles bends toward the positively charged plate. Therefore, beta particles must be negatively charged.*

Q3. Based on your observations, what is the sign of the charge on the gamma particle? Explain your answer.

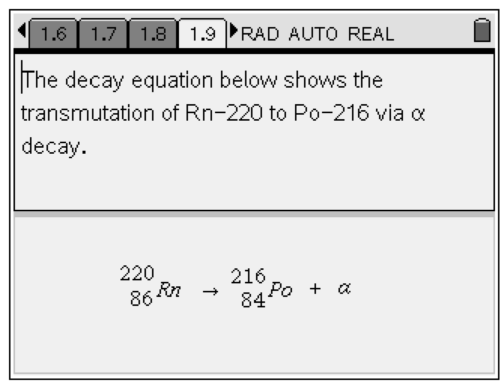
A. *The beam of gamma particles travels straight through the charged plates with no deflection. Therefore, gamma particles must have no electric charge.*



Q4. The magnitude of the charge on an alpha particle is twice the magnitude of the charge on a beta particle. However, notice that there is less curvature of the pathway of the alpha particles in an electric field. What can you conclude about the relative masses of alpha and beta particles? Explain your answer.

A. *The electric field exerts a stronger force on the alpha particle than on the beta particle. However, the alpha particle shows less deflection (acceleration) due to this larger force. Therefore, based on the equation F = ma, the alpha particle must have a greater mass than the beta particle.*

Step 2: Next, students should read page 1.9, which shows the nuclear decay equation for the decay of Rn-220 by alpha decay. Students should examine the decay equation and then answer questions 5–8. Note: You may need to review with students the conventions for writing nuclides (i.e., how to interpret a symbol such as $^{4}_{2}He$).



Q5. How many protons and how many neutrons are in the Rn-220 nucleus?

A. *86 protons and 220 – 86 = 134 neutrons*

Q6. How many protons and how many neutrons are in the Po-216 nucleus?

A. *84 protons and 216 – 84 = 132 neutrons*

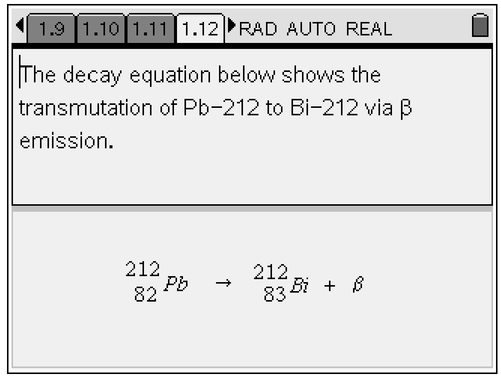
Q7. How many protons and how many neutrons does the α particle contain?

A. *2 protons and 2 neutrons*

Q8. The charge of an α particle is +2. Identify the α particle.

A. *An α particle is a He-4 nucleus.*

Step 3: Next, students will examine the nuclear decay equation on page 1.12, which shows how Pb-212 decays to Bi-212 via beta emission. After students have examined the decay equation, they should answer questions 9–16.



Q9. How many protons and how many neutrons are in a Pb-212 nucleus?

A. *82 protons and 212 – 82 = 130 neutrons*

Q10. How many protons and how many neutrons are in a Bi-212 nucleus?

A. *83 protons and 212 – 83 = 129 neutrons*

Q11. What happens to a neutron in the nucleus during the emission of a β particle?

A. *The neutron is transformed into a proton.*

Q12. The β particle is an electron. Is charge conserved in β emission? Explain your answer.

A. *Yes, charge is conserved. The positive charge of the proton formed from the neutron is balanced by the negative charge of the emitted β particle.*

Q13. The “particle” emitted in γ emission is a photon of very high-energy electromagnetic radiation. If a Dy-152 nucleus undergoes gamma emission, what will the resulting nuclide be? Explain your answer.

A. *Gamma rays have no mass or charge; they are pure energy. Therefore, when a nuclide decays through gamma emission, the mass and nuclear makeup of the nuclide will not change. When Dy-152 undergoes gamma emission, the resulting nuclide will be Dy-152.*

Q14. Po-218 decays by α emission. What is the new nuclide that is formed?

A. *Pb-214*

Q15. Write the nuclear decay equation for the decay of U-235 by α emission.

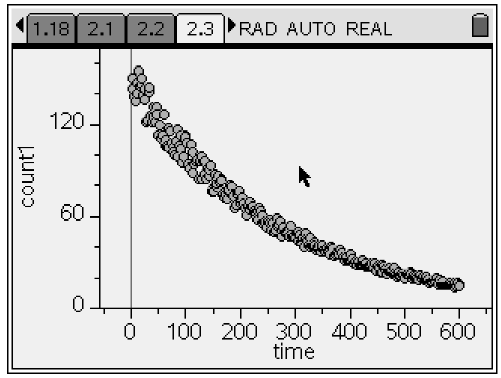
A. ${}^{235}_{92}U \rightarrow {}^{4}_{2}He + {}^{231}_{90}Th$

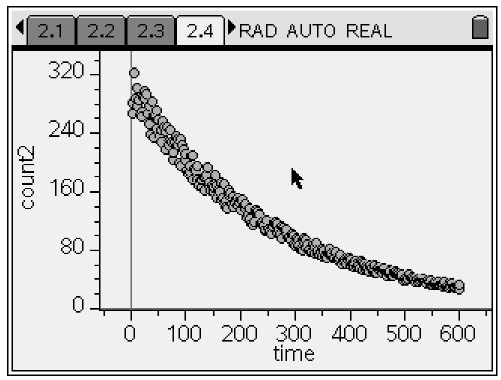
Q16. Write the nuclear decay equation for the decay of Pt-197 by β emission.

A. ${}^{197}_{78}Pt \rightarrow {}^{0}_{-1}e + {}^{197}_{79}Au$

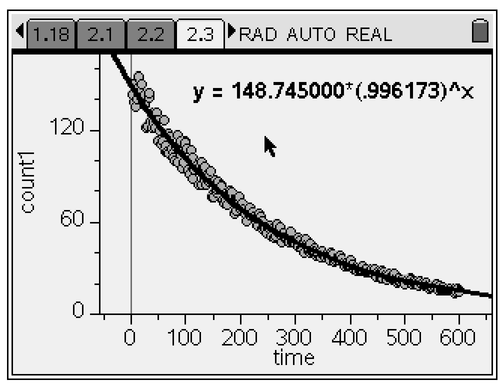
Problem 2 – Rates of nuclear decay

Step 1: Next, students should read the text on page 2.1 and then move to page 2.2. The *Lists & Spreadsheet* application on page 2.2 contains data collected with a Geiger counter that was used to measure the number of α particles emitted from two samples of Po-218. The mass of the first sample was half that of the second sample. In the experiment, the counts were recorded at 2 sec intervals for 10 minutes. Students should make a plot of **count1** vs. **time** in the *Data & Statistics* application on page 2.3. Then, they should make a plot of **count2** vs. **time** in the *Data & Statistics* application on page 2.4.





Step 2: Next, students should add an exponential regression line (**Menu > Actions > Regression > Show Exponential**) to each plot. Then, they should answer questions 17 and 18.

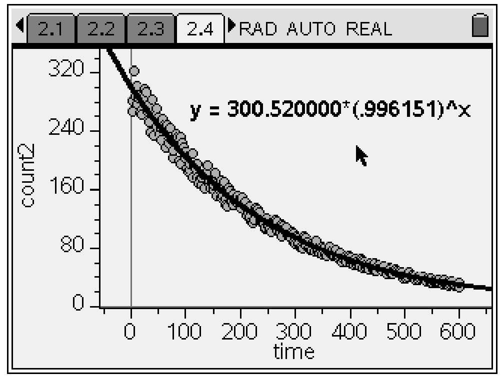


Q17. How do the two regression equations compare?

A. *The equation for* ***count1*** *vs.* ***time*** *has a smaller value of* a *than does the equation for* ***count2*** *vs.* ***time****, but they have very similar values for* b; *apparently, parameter* a *depends on the size of the sample, but parameter* b *does not.*

Q18. How do the counts compare for the two experiments as the samples decay?

A. *At any given time, there are twice as many counts recorded for the second sample as for the first sample.*



Step 3: Next, students should use the regression equation they found for **count1** vs. **time** to determine the decay constant, λ, for Po-218. To do this, students will need to understand the relationship between the exponential equation the TI-Nspire generates, which is of the form $y = ab^x$, and the more traditional exponential equation that describes radioactive decay, which has the form
$y = ae^{bx}$. Explain to students that the rate of radioactive decay can be represented by the equation below, in which t is time, N is the number of radioactive nuclei present, and λ is the decay constant:

$$decay\ \ rate = \frac{\Delta N}{\Delta t} = \lambda N$$

This equation can be represented by a differential equation. When the differential equation is solved, the following equation relating N, N_0 (the number of radioactive nuclei present at t = 0), t, and λ is obtained:

$$N = N_0 e^{-\lambda t}$$

A Geiger counter, of course, does not count the number of radioactive nuclei present at a given time; it counts the number of decays that occur in a given amount of time. The number of decays (C) that occur in the sample and the number of decays that occurred at t = 0 (C_0) are related to N by the following equations:

$C = \lambda N$

$C_0 = \lambda N_0$

Therefore, we can derive an equation relating *C, C_0*, *t*, and *λ*, as shown below:

$$N = N_0 e^{-\lambda t}$$

$$\lambda N = \lambda N_0 e^{-\lambda t}$$

$$C = C_0 e^{-\lambda t}$$

$$C = C_0 (e^{-\lambda})^t$$

This equation has the form $y = ab^x$, where $y = C$, $a = C_0$, $b = e^{-\lambda}$, and $x = t$. Once students understand this derivation, they should answer questions 19 and 20.

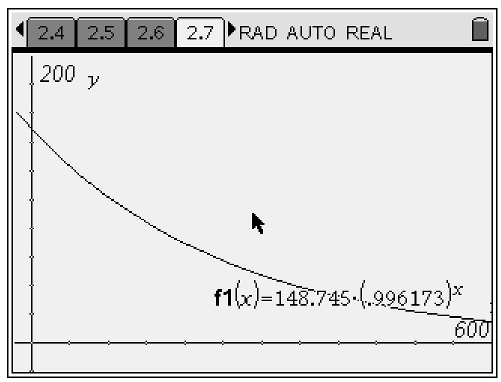
Q19. According to the regression equation for **count1** vs. **time**, what is the value of $e^{-\lambda}$ for Po-218?

A. *0.996173*

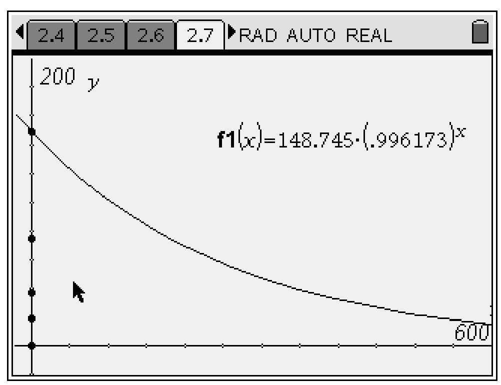
Q20. What is the value of the decay constant *λ*?

A. *0.003834 sec^{-1}; if students struggle with this calculation, remind them that if $e^x = y$, then $x = ln(y)$.*

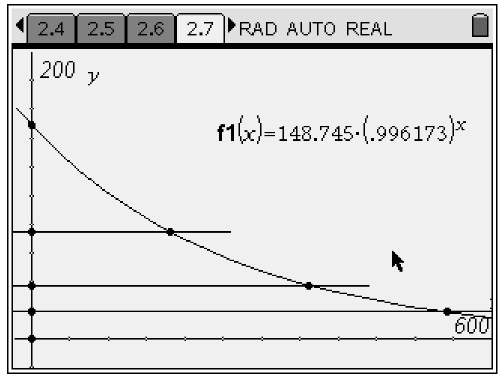
Step 4: Next, students should move to page 2.7, which contains an empty *Graphs & Geometry* application. Students should plot the exponential regression equation they found for **count1** vs. **time** as **f1** on this graph. After students have graphed **f1**, they should hide the entry line on the screen by pressing ctrl G.



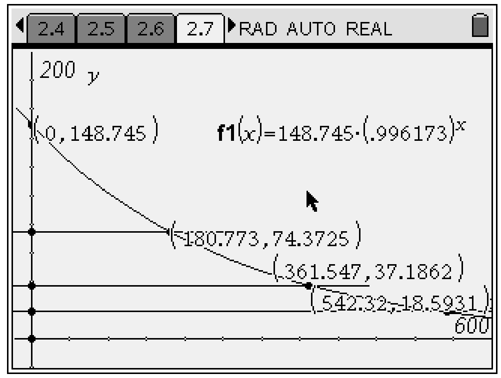
Step 5: Next, students should use the **Intersection Point(s)** tool (**Menu > Points & Lines > Intersection Point(s)**) to mark the intersection between **f1** and the *y*-axis and to mark the origin (the intersection between the *x*-axis and the *y*-axis). Then, they should use the **Midpoint** tool (**Menu > Construction > Midpoint**) to construct the midpoint between the origin and the *y*-intercept of **f1**. They should then construct the midpoint of this point and the origin, and the midpoint between that point and the origin, as shown to the right.



Step 6: Next, students should construct lines parallel to the *x*-axis through the points they constructed in step 5. They should use the **Parallel** tool (**Menu > Construction > Parallel**) to construct these lines. Then, they should use the **Intersection Point(s)** tool to mark the intersections between these lines and **f1**.



Step 7: Next, students should use the **Coordinates and Equations** tool (**Menu > Actions > Coordinates and Equations**) to mark the coordinates of these intersection points, as well as the intersection between **f1** and the *y*-axis. Then, they should answer questions 21–26.



Q21. What are the *y*-coordinates of the intersection points?

A. *148.7, 74.4, 37.2, and 18.6*

Q22. What is the relationship between successive *y*-coordinates of the intersection points?

A. *Each value is one-half the previous value.*

Q23. What are the *x*-coordinates of the intersection points?

A. *0, 180.8, 361.5, and 542.3*

Q24. What is the relationship between successive *x*-coordinates of the intersection points?

A. *The difference between successive values is constant, about 181 sec.*

Q25. A radioisotope's half-life is the amount of time required for one-half of a given sample to decay. Each radioisotope has a unique half-life. Based on the graph, what is the half-life of Po-218?

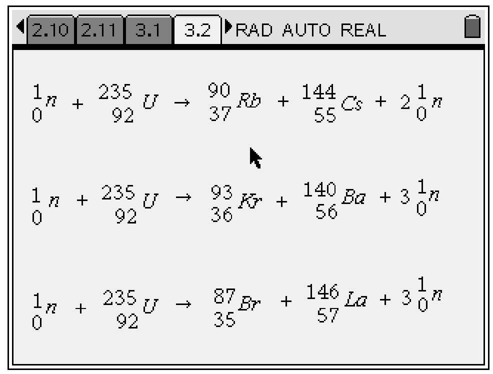
A. *about 181 sec*

Q26. A radioisotope's decay constant and its half-life ($t_{1/2}$) are related by the equation $t_{1/2} = \frac{0.693}{\lambda}$. Use this equation to calculate the half-life of Po-218, and compare your results to those you obtained from the graph.

A. $t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.003834} = 180.8$ *; the calculated value is very close to the value obtained from the graph.*

Problem 3 – Fission and chain reactions

Step 1: Next, students should read page 3.1, which describes nuclear fission. They should then examine the three nuclear decay equations on page 3.2, which show three fission reactions that a U-235 nucleus can undergo. After studying the equations, students should answer questions 27–29.



Q27. How many neutrons does each reaction consume?

A. *one neutron each*

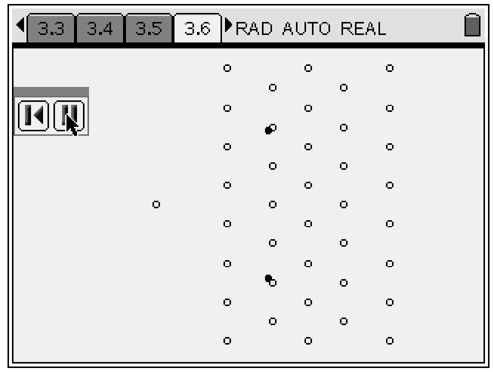
Q28. How many neutrons does each reaction produce?

A. *two or three*

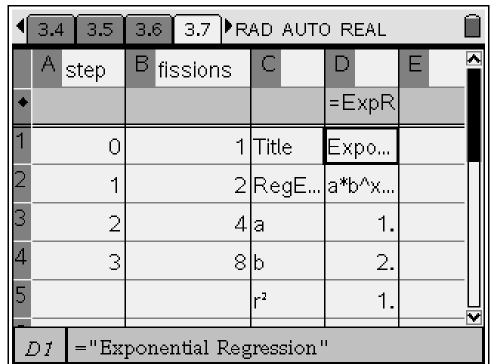
Q29. How could these reactions produce an escalating chain reaction?

A. *Each reaction produces more neutrons than it consumes. Therefore, after each reaction occurs, there are "extra" neutrons in the system. These neutrons can collide with other uranium nuclei and cause more reactions. Each of these single reactions could cause two or three further reactions, so the number of reactions would increase over time.*

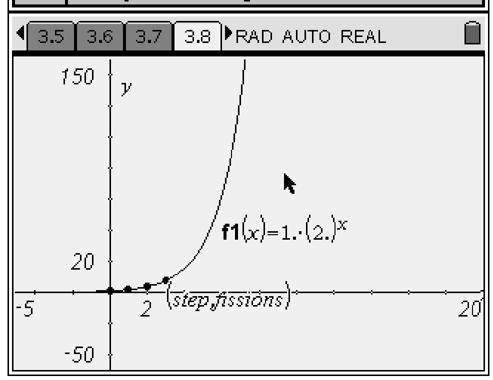
Step 2: Next, students should read the text on page 3.5 and then move to page 3.6. Page 3.6 contains an animated simulation of the early stages of a nuclear chain reaction. Students should play the animation and observe the effects. In particular, students should pay attention to the number of fissions that occur at each "step" (i.e., time interval). For the purposes of this simulation, students can equate the number of neutrons present at each step with the number of fissions that occur at that step.



Step 3: Next, students enter the number of fissions that occur at each "step" in the *Lists & Spreadsheet* application on page 3.7. Then, they should calculate an exponential regression (**Menu > Statistics > Stat Calculations > Exponential Regression**) for **fissions** vs. **step**. They should use **fissions** for their Y list and **step** for their X list in the regression, and they should store the best-fit equation in **f1**.



Step 4: Next, students should move to page 3.8, which contains an empty *Graphs & Geometry* application. They should plot **f1** in the application. Then, they should change the graph to a scatter plot and plot **fissions** vs. **step**. Finally, they should answer questions 30 and 31.



Q30. List the number of fissions that occur at steps 5, 10, and 15.

A. *32; 1,025; and 32,768; if students struggle with this, remind them that the number of fissions that occur at step* x *is equal to* 2^x.

Q31. Describe the shape of the curve of **fissions** vs. **time**.

A. *The curve is exponential.*

Nuclear Decay and Chain Reactions – ID: 9522

(Student)TI-Nspire File: *PhyAct_9522_NucDecay.tns*

1.1 1.2 1.3 1.4 RAD AUTO REAL

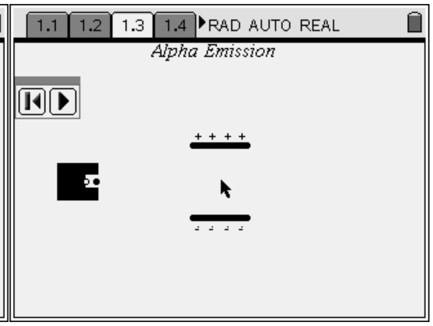
NUCLEAR DECAY AND CHAIN REACTIONS

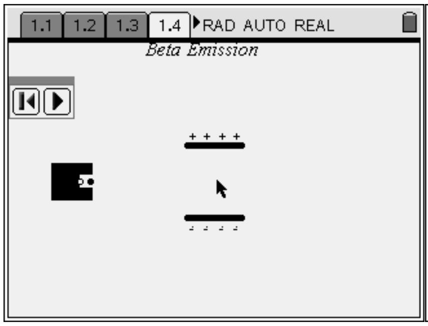
Physics

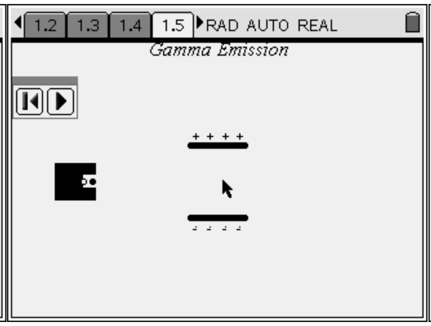
Nuclear Physics

1.1 1.2 1.3 1.4 RAD AUTO REAL

On the next three pages you will see animations of α, β, and γ emissions from radioactive nuclei. The emitted particles pass through an electric field. Use a Geometry Trace to observe the path of each type of particle.







1.3 1.4 1.5 1.6 RAD AUTO REAL

1. Based on your observations, what is the sign of the charge on the alpha particle? Explain your answer.

2. Based on your observations, what is the sign of the charge on the beta particle? Explain your answer.

1.4 1.5 1.6 1.7 RAD AUTO REAL

3. Based on your observations, what is the sign of the charge on the gamma particle? Explain your answer.

1.5 1.6 1.7 1.8 RAD AUTO REAL

4. The magnitude of the charge on an alpha particle is twice the magnitude of the charge on a beta particle. However, notice that there is less curvature of the pathway of the alpha particles in an electric field. What can you conclude about the relative masses of alpha and beta particles? Explain your answer.

1.6 1.7 1.8 1.9 RAD AUTO REAL

The decay equation below shows the transmutation of Rn−220 to Po−216 via α decay.

$$^{220}_{86}Rn \rightarrow \, ^{216}_{84}Po + \alpha$$

1.7 1.8 1.9 1.10 RAD AUTO REAL

5. How many protons and how many neutrons are in the Rn−220 nucleus?

6. How many protons and how many neutrons are in the Po−216 nucleus?

1.8 1.9 1.10 1.11 RAD AUTO REAL

7. How many protons and how many neutrons does the α particle contain?

8. The charge on an α particle is +2. Identify the α particle.

1.9 1.10 1.11 1.12 RAD AUTO REAL

The decay equation below shows the transmutation of Pb−212 to Bi−212 via β emission.

$$^{212}_{82}Pb \rightarrow \, ^{212}_{83}Bi + \beta$$

1.11 1.12 1.13 1.14 RAD AUTO REAL

9. How many protons and how many neutrons are in a Pb−212 nucleus?

10. How many protons and how many neutrons are in a Bi−212 nucleus?

1.11 1.12 1.13 1.14 RAD AUTO REAL

11. What happens to a neutron in the nucleus during the emission of a β particle?

12. The β particle is an electron. Is charge conserved in β emission? Explain your answer.

1.12 1.13 1.14 1.15 RAD AUTO REAL

13. The "particle" emitted in γ emission is a photon of very high−energy electromagnetic radiation. If a Dy−152 nucleus undergoes gamma emission, what will the resulting nuclide be? Explain your answer.

1.13 1.14 1.15 1.16 RAD AUTO REAL

14. Po−218 decays by α emission. What is the new nuclide that is formed?

1.14 1.15 1.16 1.17 RAD AUTO REAL

The symbols for α and β particles are shown below.

α: ${}^{4}_{2}He$ β: ${}^{0}_{-1}e$

1.15 1.16 1.17 1.18 RAD AUTO REAL

15. Write the nuclear decay equation for the decay of U−235 by α emission.

16. Write the nuclear decay equation for the decay of Pt−197 by β emission.

1.16 1.17 1.18 2.1 RAD AUTO REAL

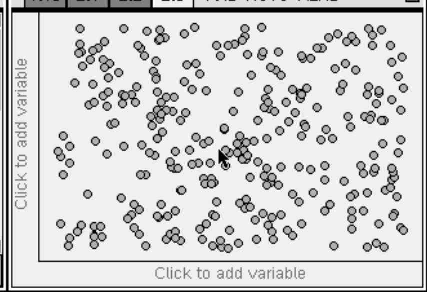
The next page shows the number of α particles emitted by two samples of Po−218 over the course of 10 minutes. The mass of the first sample was one−half that of the second sample. The data were collected using a Geiger counter. The data are shown in the spreadsheet on the next page.

1.17 1.18 2.1 2.2 RAD AUTO REAL

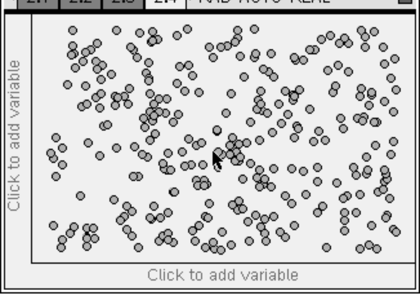
	A time	B count1	C count2	D
◆				
1	2	142.975	281.714	
2	4	150.008	267.228	
3	6	138.485	322.004	
4	8	135.319	287.751	
5	10	140.436	276.458	

A1 2

1.18 2.1 2.2 2.3 RAD AUTO REAL



2.1 2.2 2.3 2.4 RAD AUTO REAL



2.2 2.3 2.4 2.5 RAD AUTO REAL

17. How do the two regression equations compare?

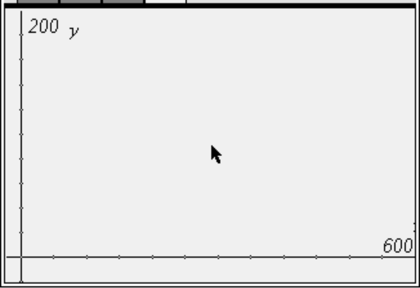
18. How do the counts compare for the two experiments as the samples decay?

2.3 2.4 2.5 2.6 RAD AUTO REAL

19. According to the regression equation for **count1** vs. **time**, what is the value of $e^{-\lambda}$ for Po−218?

20. What is the value of the decay constant λ?

2.4 2.5 2.6 2.7 RAD AUTO REAL



2.5 2.6 2.7 2.8 RAD AUTO REAL

21. What are the y−coordinates of the intersection points?

22. What is the relationship between successive y−coordinates of the intersection points?

2.6 2.7 2.8 2.9 RAD AUTO REAL

23. What are the x−coordinates of the intersection points?

24. What is the relationship between successive x−coordinates of the intersection points?

2.7 2.8 2.9 2.10 RAD AUTO REAL

25. A radioisotope's half−life is the amount of time required for one−half of a given sample to decay. Each radioisotope has a unique half−life. Based on the graph, what is the half−life of Po−218?

2.8 2.9 2.10 2.11 RAD AUTO REAL

26. A radioisotope's decay constant and its half−life ($t_{1/2}$) are related by the equation $t_{1/2}=\frac{0.693}{\lambda}$. Use this equation to calculate the half−life of Po−218, and compare your results to those you obtained from the graph.

2.9 2.10 2.11 3.1 RAD AUTO REAL

In the process of fission, a neutron strikes a heavy nucleus and splits the nucleus into two smaller nuclei. Three different fission reactions for U−235 are shown on the next page.

2.10 2.11 3.1 3.2 RAD AUTO REAL

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{90}_{37}Rb + {}^{144}_{55}Cs + 2\,{}^{1}_{0}n$$

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{93}_{36}Kr + {}^{140}_{56}Ba + 3\,{}^{1}_{0}n$$

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{87}_{35}Br + {}^{146}_{57}La + 3\,{}^{1}_{0}n$$

2.11 3.1 3.2 3.3 RAD AUTO REAL

27. How many neutrons does each reaction consume?

28. How many neutrons does each reaction produce?

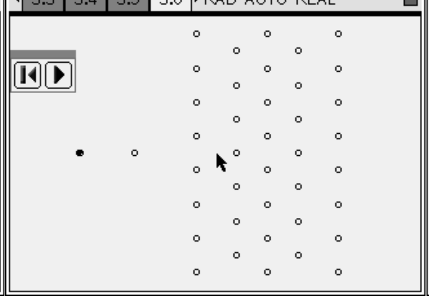
3.1 3.2 3.3 3.4 RAD AUTO REAL

29. How could these reactions produce an escalating chain reaction?

3.2 3.3 3.4 3.5 RAD AUTO REAL

The neutrons released in fission reactions can trigger a chain reaction. The simulation on the next page illustrates the early stages of a chain reaction. The white dots represent the U−235 nuclei, and the black dots represent neutrons.

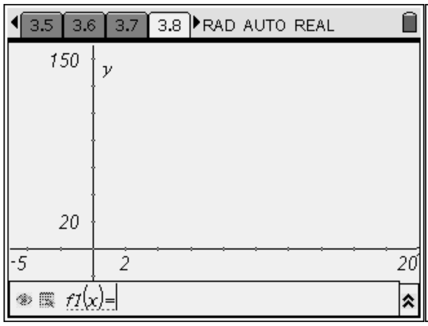
3.3 3.4 3.5 3.6 RAD AUTO REAL

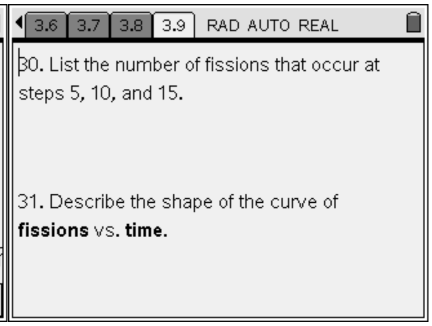


3.4 3.5 3.6 3.7 RAD AUTO REAL

	A step	B fissions	C	D	E
◆					
1	0	1			
2	1	2			
3	2				
4	3				
5					

A1 0


3.5 3.6 3.7 3.8 RAD AUTO REAL
150
y
20
-5
2
20
f1(x)=


3.6 3.7 3.8 3.9 RAD AUTO REAL
30. List the number of fissions that occur at steps 5, 10, and 15.
31. Describe the shape of the curve of **fissions** vs. **time.**

