1.

(a) Find the Taylor polynomial of order 5 at 0 for $f(x) = e^{-2x}$. (4 marks)

(b) Use this Taylor polynomial to approximate $f(.5)$ to four decimal places. (3 marks)

Mark scheme:

(a) $f'(x) = -2e^{-2x}$

$f''(x) = 4e^{-2x}$

$f'''(x) = -8e^{-2x}$

$f^{IV}(x) = 16e^{-2x}$

$f^{V}(x) = -32e^{-2x}$ (M1)(A1)

At $a = 0$, the coefficients: $-2, 4, -8, 16, -32$ (A1)

$e^{-2x} \approx 1 - 2x + \dfrac{4}{2!}x^2 - \dfrac{8}{3!}x^3 + \dfrac{16}{4!}x^4 - \dfrac{32}{5!}x^5$ (A1)

(b) $e^{-2(.5)} \approx 1 - 2(.5) + \dfrac{4}{2!}(.5)^2 - \dfrac{8}{3}(.5)^3 + \dfrac{16}{4!}(.5)^4 - \dfrac{32}{5!}(.5)^5$ (M1)(A1)

≈ 0.3667 (A1)