Form a Line

Topic Areas

- Introduction to Linear Functions
- Slope
- Y-Intercept
- Linear Equations
- Introduction to Linear Fit

Objectives

- To encounter a linear function as an input-output rule both “pointwise” for particular input values and “in the aggregate”
- To gain a qualitative, experience-based sense of slope and y-intercept as attributes of the rules that correspond to linear functions
- To investigate slope as a relationship between the coordinates of two points that obey a linear rule
- To gain familiarity and fluency with the linear functions and their equations, by doing manual linear fits of class-created data

Introduction

In this lesson concept we use the Activity Center to build some basics in students’ understanding of linear functions. The linear relationship is clearly one of the “big ideas” of the algebra curriculum, and it paves the way for a deeper and more rounded understanding of the concept of “function” in general.

We approach the concept of a function as a “rule” by which any given “input” value can be converted to a corresponding “output” value. Students control points in the plane, moving them so that their x- and y-values fulfill the pointwise rule of the function they are investigating. As the whole class does this at the same time, the visual pattern of the function’s graph emerges in the public space of the Activity Center’s **Graph** tab. In this way, these activities provide students with an intuitive, visual introduction to the study of functions and their graphs.

Students can benefit from the transition from a pointwise understanding of a function to a more “holistic” and “global” one, throughout their study of mathematics. Thus, it can be useful to return to this kind of activity periodically throughout the year. For example, students can benefit from “building up” new functions from points, as they begin their study of quadratics, of exponential and logarithmic functions or, with a twist, of functions and their inverses.

In this particular case of introducing linear functions, be sure to make room for your students to describe the emerging patterns they are creating in their own language. Encourage them to articulate their emerging understandings of the patterns that they are creating together in the shared space. Solicit their explanations for the different attributes of graphs that they are seeing – aspects that they will later know as “slope” “y-intercept” and so on. Do not rush them to adopt those more formal terms, even though you may want to lead them toward noticing the attributes themselves.

Prerequisites and Materials

Prerequisites: Fundamental arithmetic and number skills.
Materials: The Activity Settings files, `FormALine1.act`, `FormALine2.act`, `FormALine3.act`, and `FormALine4.act` located on the CD.

Setting Up the Activity

From the TI-Navigator™ teacher computer home screen, open the Activity Center, . Then choose **File>Load>Load Activity Settings** to load the activity settings file, `FormALine1.act`. You will see that the window settings of the Activity Center’s **Graph** window are such that each pixel on the graphing calculator will correspond to one unit in the public graph space. The Contribute Points activity is also configured so that students cannot mark points beyond the one defined by their cursor.

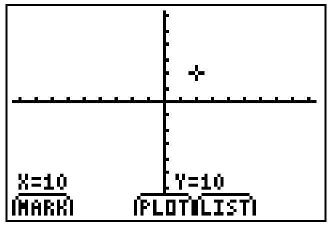
To see all of the settings that are contained in this file, simply click the **Configure** button.

In preparation for this activity you may also want to choose **View>Individualize Student Cursors**, so that the students’ cursors show up with distinctive color-shape combinations, making it easier for them to identify themselves.

Finally, you may wish to set up several equations. Do this by switching to the **Graph-Equation** tab, entering the equations into the **Y=** entry field. You can make them hidden for now by selecting them and clicking Hide. This will remove them from the visual field until you want to draw attention to them. You may also wish to mask their equations by choosing **View>Mask Teacher Input**. This way, you can avoid drawing attention to the symbolic representation of these functions at this time in your course, if you feel it is too early to do so.

Stage 1. Bootstrapping into Linear Functions

When you are ready to start, press the **Start Activity** button, . Ask your students to log in on their calculators and go to the Activity Center. After the system configures their calculators for the activity they will be able to move their points around in the Cartesian plane.



The students begin at the origin, (0, 0). As they begin to explore, you will see their cursors fan out over the plane. Ask your students to compare the display on their calculators with the public display in the activity center and see if they can identify which point is theirs.

When your students have gotten comfortable with moving their cursors, give them a simple rule that describes a linear function. For example,

Go to a place where your y-coordinate is twice your x-coordinate.

As they move their points, tell them that they should spread out, so that they are not on top of one another. When the points settle into the linear pattern, pause the activity, .

Ask the class to describe the pattern that they have made. Allow them to build their own language to describe the phenomenon that has emerged through their individual work.

If you like, you can switch to the **List-Graph** tab at this time. Sort the points in the table by the x-coordinate, in ascending order, telling the students what you are doing as you go. Identify the pairs of numbers in the table as another representation of the locations of their points.

Now, select the first row of the data table (the one with the smallest x-coordinate). Ask the students to notice which point has been highlighted. Now you can use the down-arrow key on your computer to move through the points from left to right. Have students call out the ordered pairs, confirming that each of the points is following the rule that the y-coordinate is two times the x-coordinate.

If you like, you can make the Y=2X line visible at this time, by switching momentarily to the **Graph-Equation** tab, selecting the equation and pressing **Show**.

Ask the students to describe the relationship between that line and the points that they have entered. Pick any two of the class's points that are adjacent to one another. Are there points between these two points that follow the "y is twice the x" rule? How does the line show this?

Hide the line Y=2X again, and switch back to the **List-Graph** tab.

Tell the class that you will next ask them to try following a different rule: this time, they're going to move to a point where their y-coordinate is equal to their x-coordinate. Furthermore, tell them you want them to stay with the same x-coordinate that they have now, so that they will only be changing their y-coordinate – moving the point up or down.

Give them some time to think about what they are going to do to make their y-coordinate equal to their x-coordinate.

Ask the class who is going to move down. Who is going to move up? What do the people who are going to move down have in common? How about the ones that are going to move up? If there is someone at the origin, ask whether there is anyone in the class who will not have to move at all and highlight the student at the origin.

Optional Poll Idea
Instead of asking for a show of hands, you could ask a Quick Poll question here, . For example, you could send a **Multiple Choice A-C** poll, and ask students to answer "A" if they are going to increase their y-coordinates and move up; "B" if they are going to stay where they are; and "C" if they are going to decrease their y-coordinates and move down. Students may have a hard time interpreting the bar graph that results, as a new visual representation of the situation. We suggest that you not force a correct explanation to emerge – simply note that it seems some people will be moving upwards and some downwards, while one will stay the same. You can return to the poll's bar graph later in the activity, when students will have already seen the relationship between Y=2X and Y=X and will be more confident in making interpretations of that visual.

Discussion Note: Some students will perceive that the points have gotten *closer* to each other (or "more crowded"). This is true, of course, though it doesn't directly illuminate the discussion at hand. You might suggest that students take this observation down as an "open question" for further reflection. It is a rich concept and can be related to slope (through "change in Y") but it may be better tabled for now.

Resume the activity, , letting the students move to make their y-coordinate equal to their x-coordinate. Pause the activity, . Discuss the results with the class. If you decided to give the optional poll described above, you can return to discuss what kinds of points would have moved up, what kinds would have moved down; what kind would have stayed the same?

Ask the students how this pattern relates to the pattern of "y twice your x". At this point, you might choose to make both the Y=2X and Y=X equations visible for discussion, by switching to the **Graph-Equation** tab, selecting both, and pressing **Show**.

Now ask what would happen if you followed the rule,

Make your y-coordinate the opposite of your x-coordinate.

What would the graph of this rule look like? Resume the activity, , and find out. Pause the activity, , to discuss; then once you are ready to move on, stop the activity, .

Stage 2. Getting Back into Line – Slope and Variation

To set up this stage of the activity, load the settings file, `FormALine2.act`. Switch to the **Graph-Equation** tab and make just the Y=2X equation visible again. Start the activity, ▶: your students will now have the graph of that equation as the background on their screens. Ask them to move to a point on the line so that they are not on top of one another.

Pause the activity, ⏸, to get the class's attention. Tell your students you want them to experiment with the kinds of movements they can do to leave the line and come back. How many steps to right or left, followed by how many clicks up or down? Ask them to take out a sheet of paper (or use the worksheet below) to record <u>three</u> combinations that they discover. When they have found three, they should exchange papers with a partner to see three more. Resume the activity, ⏸, and let them go to work. Ask them to finish their work and return to the line.

Option: You can turn on grid lines by selecting **Grid** on the **Graph** tab's toolbar. Since the gridlines will be at a larger scale than most students' experiments, they can also help to "up level" the pattern discussion.

Pause the activity, ⏸, to regain the class's attention, asking for volunteers to describe what they and their partner have found. Is there a pattern? The students, of course, are discovering the concept of slope as a relationship between "change in X" and "change in Y" (or "run" and "rise"). Take some time to explore these relationships and generalize them, looking at connections between the beginning and ending coordinate pairs.

Next, ask students to write their coordinate pairs down, and then turn to another neighbor and compare their points. How could one of you move to get to the other? What steps would you have to take to get to your partner? How do those steps compare to the steps your partner would have to take to get to you?

Stop the activity, ■. Repeat it with Y=(2/3)X, by switching to the **Graph-Equation** tab, entering the equation, and then making it the only visible equation.

After your students have explored this new function, ask them to propose a connection between the coefficient of X and the horizontal and vertical movements that relate any two points on the line?

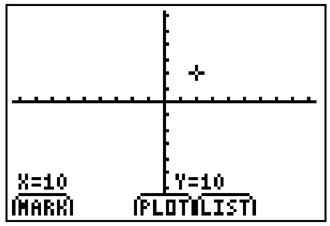
Stop the activity, ■. Next, try it again with the line Y=(2/3)X + 5. Keep Y=2/3 X visible on the screen, as well. But ask students to gather on the top graph. Choose another neighbor and talk about needed horizontal and vertical movements.

Stage 3. Found in Translation – Getting to Y-Intercept

Stop the activity, ■, and hide any visible equations. Load the activity settings file, `FormALine3.act`, and set students up to contribute points. Start the activity, ▶, and ask students to move to a point where their y-coordinates are equal to their x-coordinates. Pause the activity, ⏸, and graph the equation

Y=X by switching momentarily to the **Graph-Equation** tab and entering the equation there.

Draw the students' attention to the **MARK** button on their calculators. Tell them that this will allow them to mark points on the screen.

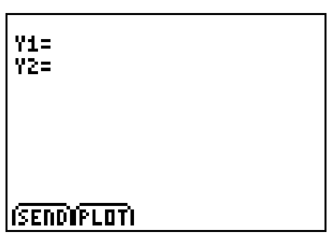


Now ask what would happen to the line if everyone added 2 to their y-coordinates. Let the students make predictions and discuss. Once they've grappled with this together for a little while, tell them that you'd like them to investigate the question as a class.

To begin, you'd like them to use the **MARK** button to create a point where they are right now, on the line Y=X. Next, on your count, you'd like them all to move their cursors up one unit and mark <u>those</u> points – points created by increasing the y-coordinates of the original points by one. Finally, again on your count, you'd like them to move up one more unit and then mark those points as well. The finished result will show the whole class marching up two steps from the line Y=X.

Resume the activity, , and do the experiment. Pause the activity, , and discuss.

Now stop the activity, . Switch to allow students to contribute equations by choosing **Equations** from the **Contribute** drop-down box. Switch to the **Graph-Equation** tab and adjust the division between the graph window and the Equation Gallery to optimize the visibility of the graph. Then start the activity, , and the students will receive the new configuration – an equation-entry screen with two entry fields.



Ask the class to enter two equations, one for each set of shifted points. The students' calculators will be configured to allow the students to preview the graphs of their equations before submitting (using the **PLOT** button), and they will see the points and the line on the **Graph** tab's contents (the points they have submitted) as a background for their preview. They will also be able to use trial and error, resubmitting their equation and changing their answers until you stop or pause the activity.

When the students have converged on two lines through the data points they have created, stop the activity, , and discuss. Then, if the group seems comfortable with the relationship between Y=X, Y=X+1, and Y=X+2, try "raising the bar" a little, as follows.

Clear the data of the Activity Center for the new run, by choosing **Edit>Clear Activity Data**. Switch back to having the students contribute points, and this time, ask them to move first to a point where their y-coordinate is half of their x-coordinate. Start the activity, .

Pause the activity, , when they have arrived, and drop in the line Y=.5X. Tell them than now you'd like them to march downward, and to go a bit further than one step. Tell them that when you resume the activity, you'd like them to follow your count, stepping downward. Now, ask them to take 3 steps upward on your count. Resume the activity, , and ask them to step 1, 2, 3 steps down, and then mark their place. Then 1, 2 more steps down and mark their place. Stop the activity, , and switch to contributing equations by choosing **Equations** from the **Contribute** drop-down box.

Before you begin, switch to the **Graph-Equation** tab and adjust the division between the graph window and the Equation Gallery to optimize the visibility of the graph. Start the activity, , and watch the equations come in. Pause the activity, , if they need some group discussion to grapple with Y=.5X-3 and Y=.5X-5. This will allow you to resume, , and let them continue to work. When they are done, stop the activity, .

Stage 4. Found in Translation – Getting to Y-Intercept

Load the activity settings file `FormALine4.act`, set up for students to contribute points by changing the **Contribute** drop-down box back to **Points** in the "configure" drop down, if necessary, and start the activity, . Ask students to "make a line" – any line they want. When they are all lined up, ask them to choose **MARK** to mark their point.

Stop the activity, , switch to let the class contribute equations using the **Contribute** drop-down box, and start the activity again, . Ask your students to experiment to find the equation of the line that they have made. You might have them work with a partner for the first time.

Repeat the activity any number of times. It is a fun exercise for students and provides excellent practice with the slope-intercept form of the line.

As the students become more proficient, consider putting constraints on the line that the group-makes. For example, *Make yourselves a line that has a negative slope and a positive y-intercept.*

In this case, students can discuss errors in the Equation Gallery in terms of their slopes and y-intercepts (e.g., “That one can’t be right because it has a negative y-intercept.”).

Ideas for Extending “Found in Translation”
You can “raise the bar” on this activity by removing the **Let students view graphs of equations** option and/or the **Let students resubmit equations** option in the **Configure Equations** dialog box, accessible by pressing the **Configure** button when **Equations** is selected in the **Contribute** dialog box. If you remove both of these options, students must make a single “best guess” about the equation of the line, based only on their observation of the public space and their knowledge of the point that they themselves contributed.

"Getting Back into Line" Worksheet Name ___________________________

	Your Starting Point		**Your** Moves		**Your** Ending Point	
	Starting X-Coordinate	Starting Y-Coordinate	Number and direction of Left-Right Movements	Number and direction of Up-Down Movements	Ending X-Coordinate	Ending Y-Coordinate
#1						
#2						
#3						

	Your Partner's Starting Point		**Your Partner's** Moves		**Your Partner's** Ending Point	
	Starting X-Coordinate	Starting Y-Coordinate	Number and direction of Left-Right Movements	Number and direction of Up-Down Movements	Ending X-Coordinate	Ending Y-Coordinate
#1						
#2						
#3						