MATH AND SCIENCE @ WORK

AP* CALCULUS Educator Edition

TAKING A WALK IN THE NEUROSCIENCE LABORATORIES

Instructional Objectives

Students will

- analyze acceleration data and make predictions about velocity and
- use Riemann sums to find velocity and position.

Degree of Difficulty

For the average student in AP Calculus, this problem is at a moderate difficulty level.

Class Time Required

This problem requires 55–70 minutes.

- Introduction: 5–15 minutes
  - Read and discuss the background section with the class before students work on the problem.
  - Watch the video, *Vestibular Research in the Neuroscience Laboratory* (6:34), at http://youtu.be/oAoHYFZz5U4. (Optional)
- Student Work Time: 45 minutes
- Post Discussion: 5–10 minutes

Background

This problem is part of a series of problems that apply Math and Science @ Work in NASA's research facilities.

Over the duration of their mission, astronauts can go through many physical, psychological, and neurological changes. Exposure to reduced gravity during spaceflight can lead to adaptive changes in all systems of the human body, including the human nervous system.

The Neuroscience Laboratories at NASA Johnson Space Center investigate the effects of spaceflight on the human nervous system, with particular emphasis on posture and gait (manner of walking), eye-head coordination, perception, space motion sickness, and the involuntary sensory function of the inner ear.

The central focus for these laboratories is the development of countermeasures (ways to lessen or prevent spaceflight-related changes)

Grade Level
11–12

Key Topics
Relation between first and second derivatives, Riemann sums, position, velocity, acceleration

Degree of Difficulty
Moderate

Teacher Prep Time
15 minutes

Class Time Required
55–70 minutes

Technology
- TI-Nspire™ Learning Handhelds
- TI-Nspire document: *TakingAWalk.tns*

AP Course Topics
Derivatives:
- Derivative as a function
- Applications of derivatives

Integrals:
- Numerical approximations to definite integrals

NCTM Principles and Standards
Process:
- Problem solving

in the nervous system, particularly the function associated with adapting to the reduced gravity environment of space, and then re-adapting to gravitational environments.

Within the Neuroscience Laboratories, many different functions are tested. For example, researchers in the Motion Laboratory focus on the post-flight disturbances in balance and gait control—areas with which many astronauts struggle. This laboratory develops training programs that will facilitate recovery of normal mobility after long-duration spaceflight.

Figure 1: An instrumented test subject walks on a treadmill while viewing a rotating visual flow

Figure 2: Test subject on the motion base treadmill

Through a series of flight and ground-based studies, the Motion Laboratory is developing countermeasures built around treadmill exercise activities. The sensory conditions astronauts experience in space are simulated on Earth by varying the visual flow patterns, body load, and walking surface of the treadmill. When walking on the treadmill, a test subject's body weight can be "unloaded" by physically raising him or her off the treadmill surface using a body harness. The stability of the walking surface can also be manipulated in any combination of six degrees of freedom by changing the orientation of the motion base platform on which the treadmill is mounted (see Figure 2). Research shows that this training regimen promotes adaptive change in walking performance, therefore improving the ability of the astronaut to adapt to a new gravity environment.

AP Course Topics

Derivatives

- Derivative as a function
  - Corresponding characteristics of graphs of *f* and *f '*
  - Relationship between the increasing and decreasing behavior of *f* and the sign of *f '*
- Applications of derivatives
  - Interpretation of the derivative as a rate of change in varied applied contexts including velocity, speed, and acceleration

Integrals

- Numerical approximations to definite integrals
  - Use of Riemann sums to approximate definite integrals of functions represented by tables of values

NCTM Principles and Standards

Process

- Problem solving
  - Solve problems that arise in mathematics and in other contexts

Problem and Solution Key (One Approach)

Students are given the following problem information within the TI-Nspire document, TakingAWalk.tns, which should be distributed to their TI-Nspire handhelds. The data in Table 1 are also provided for the students on TI-Nspire page 1.3.

The data in Table 1 comes from a test subject who walked on a treadmill in one of the Neuroscience Laboratories. A measurement sensor was attached to the test subject's torso near the neck. Table 1 shows part of the data collected from the sensor. The first column is time (sec) and the second column is vertical acceleration (m/sec^2).

Teacher note: In the TI-Nspire document, the second column is labeled as accel_z because z is the vertical axis and the data is only vertical acceleration. Discuss this with students.

Table 1: Vertical acceleration of a test subject who walked at a normal pace on a treadmill in the Neuroscience Laboratories

Time (sec)	Acceleration (m/sec^2)
0.00	-4.305020916
0.02	-6.364482106
0.04	-6.32596376
0.06	-6.500910727
0.08	-1.501415973
0.10	23.10789083
0.12	10.02004670
0.14	2.945086430
0.16	12.91133145
0.18	8.133487355
0.20	9.127715358
0.22	8.955759236
0.24	6.259131273
0.26	4.717210286
0.28	3.708324781
0.30	2.046347978
0.32	-0.624171481
0.34	-2.6311823
0.36	-4.385754433
0.38	-5.664348618
0.40	-7.524490023
0.42	-8.825604387
0.44	-7.051679576
0.46	-1.02600842
0.48	6.884695354
0.50	7.485673859
0.52	3.843485811
0.54	9.729917039
0.56	8.676480809
0.58	6.835736946
0.60	6.038568746
0.62	4.208792058
0.64	4.80875954
0.66	2.128342368

Time (sec)	Acceleration (m/sec^2)
0.68	-0.002515208
0.70	-1.599918134
0.72	-3.619421353
0.74	-5.961877413
0.76	-7.679560356
0.78	-9.401999874
0.80	-10.96935964
0.82	-10.05849293
0.84	-7.956654692
0.86	-0.261320539
0.88	15.57593726
0.90	0.999822617
0.92	6.126813152
0.94	8.368961568
0.96	5.977433249
0.98	6.842496904
1.00	4.309617979
1.02	4.404795652
1.04	1.679738457
1.06	-0.074290732
1.08	-1.194951937
1.10	-3.896116275
1.12	-6.193762972
1.14	-7.776284353
1.16	-9.032151946
1.18	-10.04421953
1.20	-9.990987964
1.22	-9.9416564
1.24	-7.876564619
1.26	1.726172226
1.28	10.15765401
1.30	3.604958809
1.32	8.662195794
1.34	9.380231592

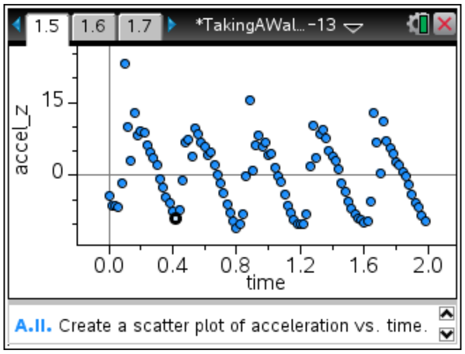
Time (sec)	Acceleration (m/sec^2)
1.36	7.559980507
1.38	5.093448023
1.40	3.895199996
1.42	2.97402008
1.44	1.397794943
1.46	-1.546336189
1.48	-3.487565684
1.50	-5.29653065
1.52	-6.844008628
1.54	-8.010422071
1.56	-8.730919794
1.58	-9.22676161
1.60	-9.780502436
1.62	-9.557058181
1.64	-5.497351597
1.66	12.91056347
1.68	6.884465739
1.70	0.410255819
1.72	11.02674725
1.74	7.200456072
1.76	5.726827097
1.78	4.589098954
1.80	2.604763805
1.82	2.090026913
1.84	0.649630821
1.86	-0.199668585
1.88	-1.865004983
1.90	-3.914575793
1.92	-5.613636372
1.94	-6.429382243
1.96	-8.388814882
1.98	-9.363141599

A. Analyze the vertical acceleration data of the test subject found in Table 1.

I. What patterns do you see in the data?

The data appears periodic, with negative (and then positive) values repeating.

II. Create a scatter plot of acceleration vs. time. What patterns do you see in the scatter plot?



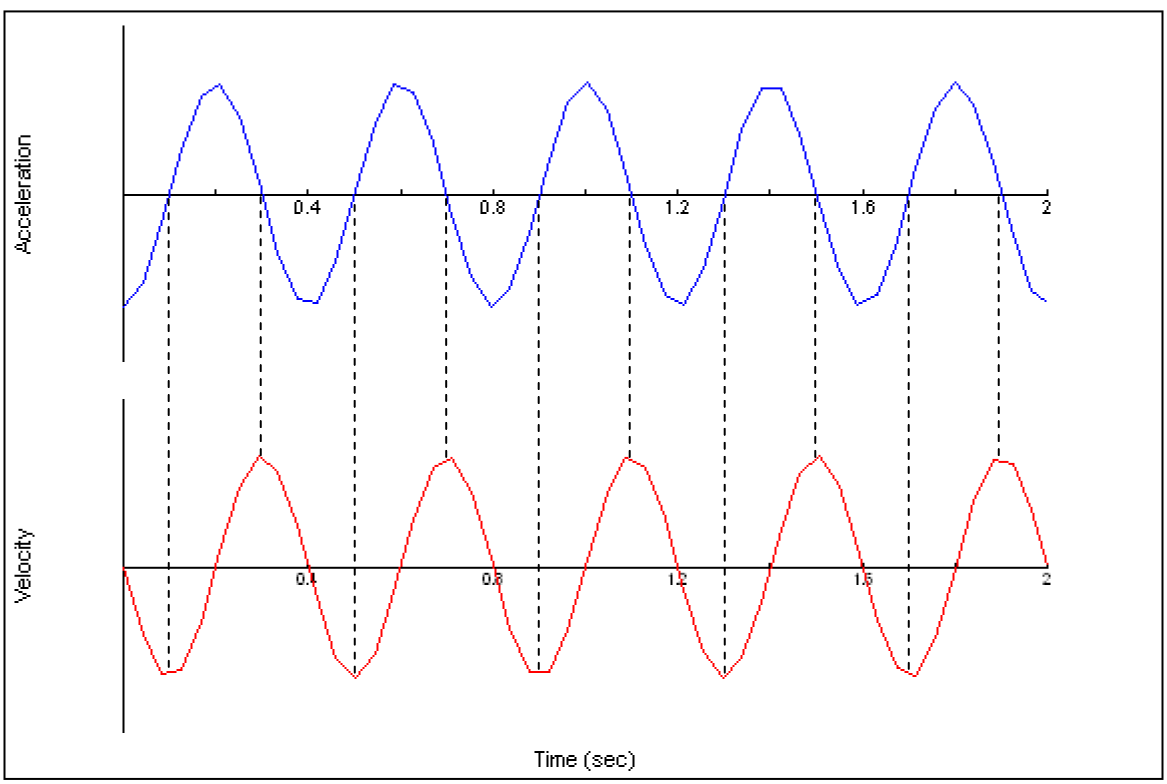
The data shows a periodic pattern.

III. How would you explain the pattern found in context of the given situation?

As the subject walks up and down, his/her vertical velocity is changing between positive and negative.

B. Analyze the velocity.

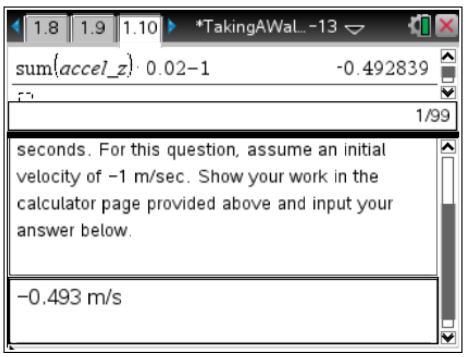
I. Sketch the graph of acceleration that you found in question A part III. Then sketch a prediction of the velocity and show how it corresponds to the acceleration graph.



The velocity graph will also be periodic. It will have positive slope where the graph of acceleration is positive and negative slope where the graph of acceleration is negative. The extrema of velocity should correspond to the roots of acceleration.

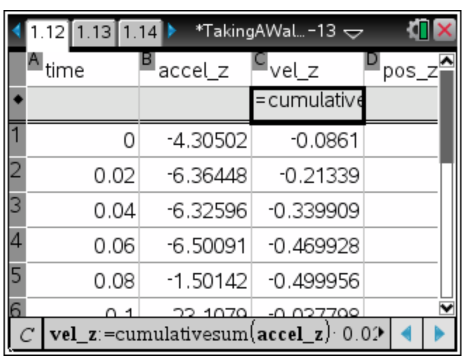
When an acceleration function is given for a situation, one can use definite integrals to find velocities at specified times. In this situation there is no acceleration function to integrate; however, numerical methods can be used to find velocities. For example, we could compute a left Riemann sum of acceleration directly from the data in the table.

II. The **sum** command can be used to find the sum of a list. Use this command to find the left Riemann sum that approximates the velocity at two seconds. For this question, assume an initial velocity of -1 m/sec. Show your work in the calculator page provided.



III. In the spreadsheet on page 1.12, enter the following command in the formula cell for the column labeled vel_z, then go on to page 1.13: **vel_z:=cumulativesum(accel_z) • 0.02+v0**.

Note: v0 represents the initial velocity of the test subject.

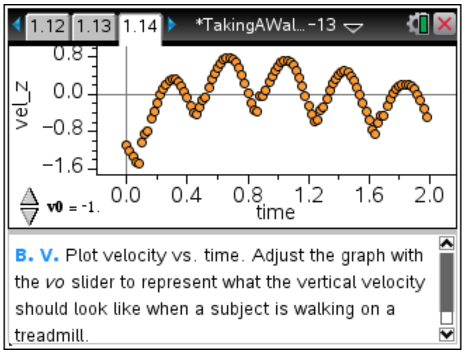


IV. Why did the command: **vel_z:=cumulativesum(accel_z) • 0.02+v0** calculate velocities?

This command could be thought of as a rectangular approximation method. Each rectangle corresponds to the change in velocity for a 0.02 second interval. This change in velocity is added to the previous velocity to obtain a new velocity.

V. Plot velocity vs. time. Adjust the graph with the *v0* slider to represent what the vertical velocity should look like when a subject is walking on a treadmill. Explain your selection of *v0*.

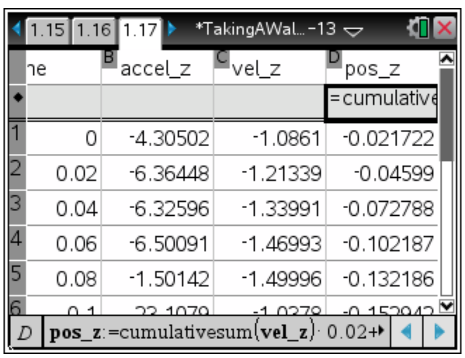
Answers may vary, but about half the velocity graph should be positive and half should be negative to reflect the change in direction each time the subject is moving up (+) or down (-).



C. Analyze the position.

I. Use numerical methods to find positions. On page 1.17, enter the following command in the formula cell for the column labeled pos_z, then go on to page 1.18: **pos_z:=cumulativesum(vel_z) • 0.02+z0**.

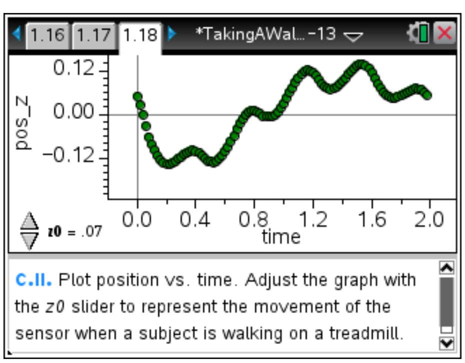
Note: z0 represents the initial position of the test subject.



	ne	B accel_z	C vel_z	D pos_z
◆				=cumulative
1	0	-4.30502	-1.0861	-0.021722
2	0.02	-6.36448	-1.21339	-0.04599
3	0.04	-6.32596	-1.33991	-0.072788
4	0.06	-6.50091	-1.46993	-0.102187
5	0.08	-1.50142	-1.49996	-0.132186

II. Plot position vs. time. Adjust the graph with the *z0* slider to represent the movement of the sensor when a subject is walking on a treadmill. Explain your selection of *z0*.

Answers may vary depending on the student's choice of a reference frame for the vertical position of the sensor on the subject's torso. All answers should be either the graph shown or a vertical translation of the graph shown. While discussing the solutions, students should be able to explain their reference frame for the value of z0 they chose. If they choose the position of the treadmill to be zero, the slider should be placed at a higher value because the sensor is placed on the subject's torso.



D. The reduced gravity experienced during spaceflight can lead to a loss of balance when an astronaut first returns to Earth. How might the graphs change if the test subject has just returned from an extended stay on the International Space Station?

Rather than being regular and periodic, he/she might show erratic spikes and uneven behavior as the subject loses balance and adjusts stride rate and stance.

Scoring Guide

Suggested 9 points total to be given.

Question		**Distribution of points**
A	*3 points*	1 point for noticing data is periodic (part I) 1 point for correct graph (part II) 1 point for explaining why the data is periodic (part III)
B	*4 points*	1 point for correct prediction of velocity graph including connections to the acceleration graph (part I) 1 point for correct left hand Riemann sum (part II) 1 point for correct explanation of the command (part IV) 1 point for correct graph including vertical translation (part V)
C	*1 point*	1 point for correct graph and explanation of *v0* and *z0*
D	*1 point*	1 point for correct prediction of irregularity

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school AP Calculus instructors.

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