Slope Fields Forever

ID: 12320

Time Required
20 minutes

Activity Overview

In the first part of this activity, students will explore particular solutions to a differential equation for different initial conditions. Slope fields are investigated, and students are asked to describe their observations when initial conditions are changed. In the second part of the activity, students match differential equations to their slope fields. Students will check their solutions by running a script file on their handheld devices.

Topic: Slope fields

- *Graphical solutions to differential equations.*
- *Identify the differential equation for a certain slope field.*

Teacher Preparation and Notes

- *Before beginning this activity, send the defield.89t and dematch.89t files to students. Students will need these files to successfully complete this activity.*
- *Students should know how to execute each command line of the text file (press* F4*).*
- *The script file dematch.89t can be used to check students' solutions to the matching section in the second part of the activity.*
- ***To download the TI-89 documents (.89t files) and student worksheet, go to education.ti.com/exchange and enter "12320" in the keyword search box.***

Associated Materials

- *SlopeFieldsForever_Student.doc*
- *defield.89t*
- *dematch.89t*

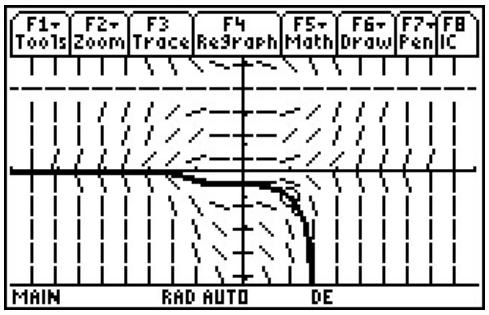
Suggested Related Activities

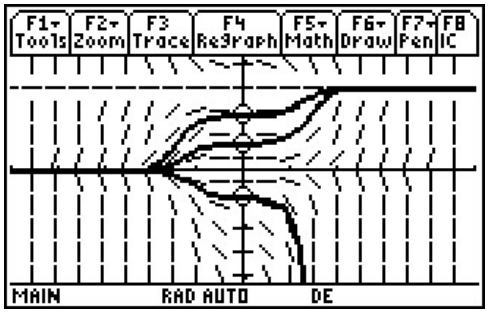
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Logistic Growth, Differential Equations, Slope Fields (TI-89 Titanium) — 5514*
- *Differential Equations and Slope Fields – Activity 7 (TI-89 Titanium) — 4279*
- *Introduction to Slope Fields (TI-84 Plus) — 4369*
- *Using Slope Fields (TI-84 Plus) — 4370*

Part 1 – Introducing Slope Fields

The activity begins with students plotting a slope field for the differential equation $\frac{dy}{dt} = \frac{1}{2}t^2\left(3y - y^2\right)$.





Students will learn that slope fields graphically give a particular solution to a differential equation when given an initial condition. Students enter different initial conditions and observe the particular solutions that are formed in Questions 1 and 2.

Discussion Questions

- *Looking at the differential equation, what values give a slope of zero? Can you see that this is true on the slope field?* (Answer: $t = 0$, $y = 0$, $y = 3$)
- *What benefit might there be in using a slope field to graphically find a solution?* (Answer: If students have already been introduced to separable differential equations, they may think that all DEs can be solved to find the general and particular solution. However, many differential equations cannot be solved algebraically. Numerically, the slope at each point on the graph can be determined and a graphical solution can be found.)

Questions 3–6 ask students to see why the slopes are what they are for several representative points. Question 7 has students consider what makes a differential equation zero.

Question 8 leads students in beginning the thought process for matching a differential equation to a slope field. See Part 2 for more details.

Student Solutions

1. Students may observe that the slope is different at different points. It is not the same all along a horizontal or vertical line. This shows that the DE depends on both *t* and *y*. As the initial condition is changed, different curves are formed.

2. The solution approaches $y = 0$ and/or $y = 3$ and levels out when $t = 0$.

3. $m = -2$ for $(-1, -1)$

4. $m = 0$ for $(0, -3)$

5. $m = 1$ for $(1, 1)$

6. $m = -2$ for $(1, -1)$

7. The slope is zero when $t = -1$.

8. The slope only depends on *y*, since it has the same slope along horizontal lines. $\frac{dy}{dt} = \frac{1}{2}y$. (It cannot be the first choice because the slopes are not always positive.)

Part 2 – DE matching

The following are the strategies mentioned on the student worksheet. This is what students can think about in order to identify the DE that corresponds to the slope field. The last of these 5 will often be the most important in determining the match. It is recommended that the teacher pause and allow students to give the answer to the problems listed below.

i) If the DE only depends on *t*, then the slopes are the same along _____

(Answer: vertical lines).

ii) If the DE only depends on *y*, then the slopes are the same along _____

(Answer: horizontal lines).

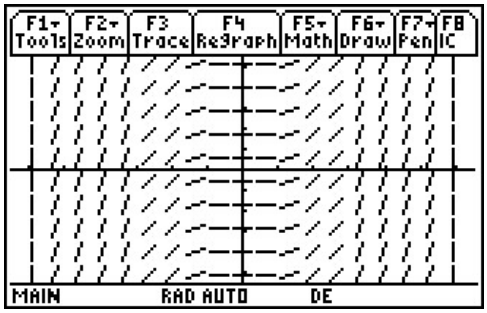
iii) If the DE is something like $\frac{dy}{dt} = t + y$, then the slopes are the same along _____

(Answer: oblique lines).

iv) Does the slope field look sinusoidal? There will be a sine or cosine function in the DE.

v) Consider a specific point. What values make the slope zero? What values of *t* and *y* make the slope 1?

Students should complete the matching activity using the strategies discussed above. Once finished, students can check their answers by running the script file *dematch.89t*. This script file will set up and plot the slope fields for each differential equation from the matching activity in the order they are presented.



Student Solutions

1. E
2. C
3. A
4. G
5. B
6. D
7. I
8. H
9. F
10. K
11. J