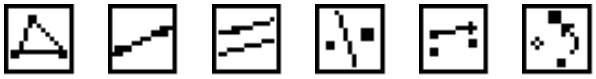
EXPLORATIONS ™

Activity 9

Objective

- To investigate the relationships among the different types of rigid transformations.

Cabri® Jr. Tools



Connecting Translations, Reflection, and Rotations

Introduction

Now that you have investigated the three types of rigid transformations — translations, rotations, and reflections — you will investigate the relationships among these transformations in this activity.

Part I: Reflections in Parallel lines

Construction

Construct a triangle and reflect it across two parallel lines.

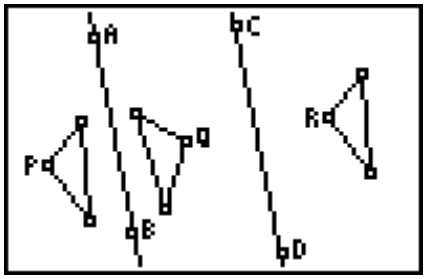
Draw a small scalene triangle in the lower left corner of the screen. Label one of the vertices *P*.

Draw $\overleftrightarrow{AB}$ to the right of your triangle.

Construct $\overleftrightarrow{CD}$ parallel to $\overleftrightarrow{AB}$.

Reflect the triangle across $\overleftrightarrow{AB}$. Label the image of *P* as point *Q*.

Reflect the triangle containing *Q* across $\overleftrightarrow{CD}$. Label the image of *Q* as point *R*.



Note: *If necessary, adjust the lines so that all three triangles are visible on the screen.*

Exploration

Using different transformation tools, explore ways to make the triangle containing *P* map onto the triangle containing *R*. Be sure to drag vertex *P* and the pre-image triangle to observe this transformation for different pre-images.

Questions and Conjectures

Describe a transformation that is equivalent to a composition of reflections across parallel lines. Describe the properties of this transformation as they relate to the reflecting lines.

Part II: Reflections across two intersecting lines

Construction

Construct a triangle and reflect it across two intersecting lines.

Clear the previous construction.

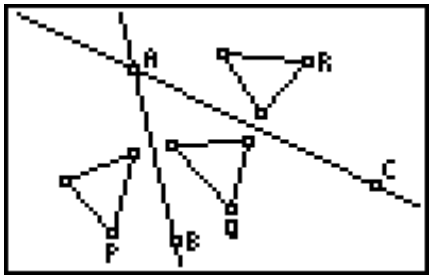
Draw a small scalene triangle in the lower left corner of the screen. Label one of the vertices *P*.

Draw $\overleftrightarrow{AB}$ to the right of your triangle.

Construct $\overleftrightarrow{AC}$ as shown.

Reflect the triangle across $\overleftrightarrow{AB}$. Label the image of *P* as point *Q*.

Reflect the triangle containing point *Q* across $\overleftrightarrow{AC}$. Label the image of *Q* as point *R*.



Note: *If necessary, adjust the lines so that all three triangles are visible on the screen.*

Exploration

Using different transformation tools, explore ways to make the triangle containing *P* map onto the triangle containing *R*. Be sure to drag vertex *P* and the pre-image triangle to observe this transformation for different pre-images.

Questions and Conjectures

1. Describe a transformation that is equivalent to a composition of reflections across intersecting lines. Describe the properties of this transformation as they relate to the reflecting lines.
2. Does your answer to Question 1 change if you reflect across $\overleftrightarrow{AC}$ first and then across $\overleftrightarrow{AB}$? Explain your reasoning.
3. If the Cabri® Jr. application could have only one rigid Transformation tool, which tool would you recommend and why?

Teacher Notes

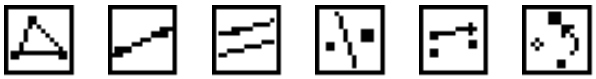
Activity 9

Connecting Translations, Reflections, and Rotations

Objective

- To investigate the relationships among the different types of rigid transformations

Cabri® Jr. Tools



Additional Information

It is recommended that you complete Activities 6, 7, and 8 prior to doing this activity.

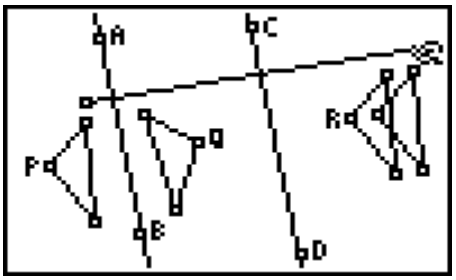
Orientation — refers to the rotation order of corresponding vertices. If the pre-image is represented by the vertices *A*, *B*, and *C* rotated in counter-clockwise order, then orientation will be preserved in the image when the vertices *A*1, *B*1, and *C*1 are in counter-clockwise order.

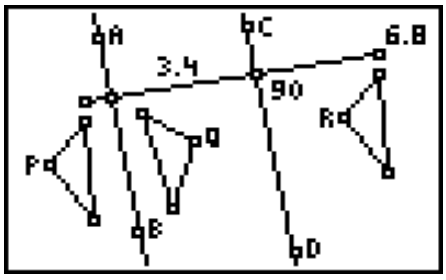
Part I: Reflections in Parallel lines

Answers to Questions and Conjectures

Describe a transformation that is equivalent to a composition of reflections across parallel lines. Describe the properties of this transformation as they relate to the reflecting lines.

A composition of reflections over parallel lines is equivalent to a translation. Use a segment to translate the triangle containing *P* onto the triangle containing *R*. As the endpoint of the segment is dragged, the image of the translation can be placed exactly on top of the triangle containing *R*.





The length of the segment that translates the pre-image onto the triangle containing *R* is twice as long as the distance between the two parallel lines of reflection. The translation segment is perpendicular to the reflecting lines and the orientation of the two triangles is identical. Since the translation is perpendicular to the lines of reflection, each set of corresponding points is equidistant from their corresponding reflection line.

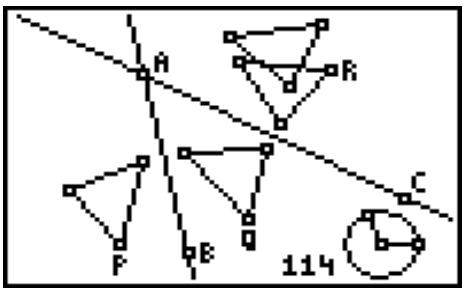
This creates two pairs of congruent segments with one from each pair making up the distance between the two reflecting lines. Hence, the distance between the reflecting lines is half that total length of the translation.

Part II: Reflections across Two intersecting Lines

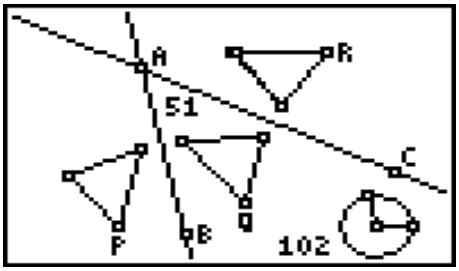
Answers to Questions and Conjectures

1. Describe a transformation that is equivalent to a composition of reflections across intersecting lines. Describe the properties of this transformation as they relate to the reflecting lines.

   The rotation of the pre-image triangle (containing point *P*) onto the final image containing point *R* can be accomplished by rotating the pre-image by an angle that is twice as large as the angle between the two reflecting lines.

   One way to explain why the angle of rotations must have this relationship is to think of the reflecting line as the angle bisector of the angle of rotation between any two corresponding points on the pre-image and the first image or the first image and the second image. This creates two pairs of equal angles with one angle from each pair between the reflecting lines, thus half the total angle of rotation.

2. Does your answer to Question 1 change if you reflect across $\overleftrightarrow{AC}$ first and then across $\overleftrightarrow{AB}$? Explain.

   The order of reflections will not change the relationship between the angle of rotation and the angle formed by the intersecting lines. However, the image and the direction of rotation will change except in the special case of the reflecting lines being perpendicular.

3. If the Cabri® Jr. application could have only one rigid transformation tool, which tool would you recommend and why?

   The **Reflection** tool can be used to create images that are equivalent to translations and rotations and should be the recommended tool. This exploration shows that a translation is a composition of two reflections over parallel lines and a rotation is the composition of two reflections over intersecting lines.