

# The Cross Country Race – Exploring Midpoint & Gradient

Teacher Answers



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7 8 9 10 11 12



TI-Nspire CAS



Investigation



Student



50 min

## Aim

The aim of this investigation is to use various methods to explore, develop and apply the concepts of midpoint and gradient with respect to the Cartesian plane and linear functions.

## Equipment

For this activity you will need:


- TI-Nspire CAS
- TI-Nspire CAS file: Midpoint & Gradient

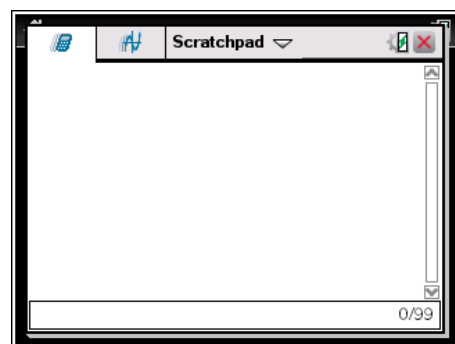
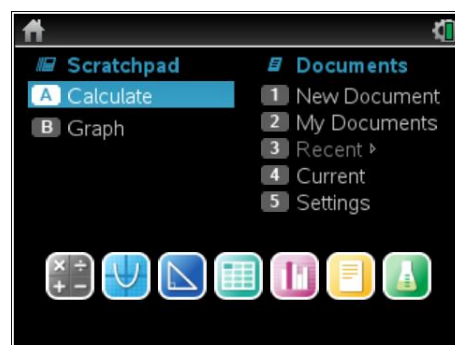
## Problem Description

We have looked at the idea of plotting points on a Cartesian plane. This activity will look at the meaning of the midpoint and gradient of a straight line. We will discover what the midpoint and gradient are and how they can be calculated.

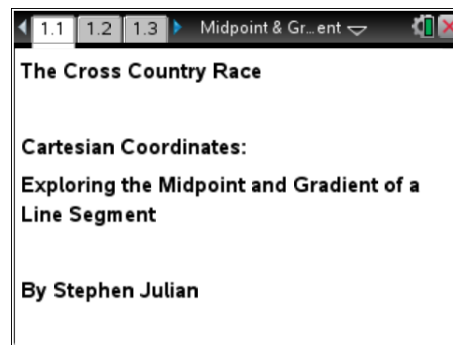
The activity requires access to the “Midpoint & Gradient” TI-Nspire document. This document should be loaded on your device before proceeding.



Part of in this investigation requires calculations to be performed. The Scratchpad is a place where calculations can be computed and then discarded. To access the Scratchpad press **home** and select **Scratchpad** (or press **A**). Alternatively, press the  key (this key is not available on a Clickpad).

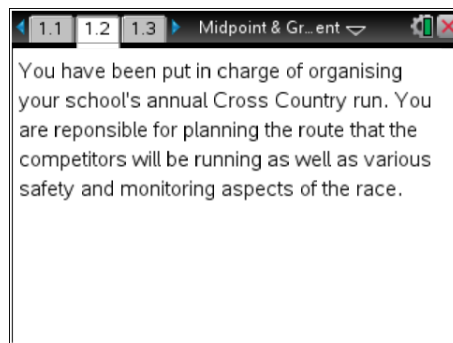


Open the “Midpoint & Gradient” file. Page 1.1 is the introductory page.



### Explanation of the Problem

Read pages 1.2 to 1.5 to outline the problem you are about to explore.



### Calculating the Midpoint

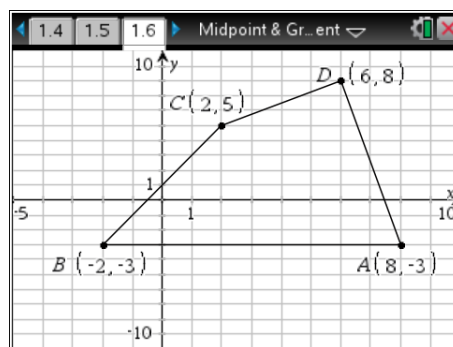
Navigate to page 1.6.

Measure the distance of the first leg (line segment AB). To do this press **menu > Geometry > Measurement > Length** then click on the line segment. The distance for the first leg should now be displayed. You may wish to drag this value to a better position on the screen.

Copy your answer onto the diagram in the exercise section of this handout for **Question 1**.

Repeat this process for each of the four legs.

Answer **Question 2** in the exercise section of this handout.

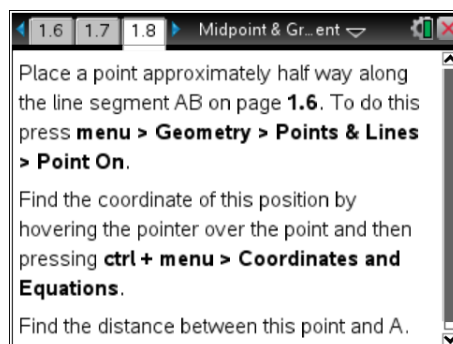


Pages 1.7 to 1.11 explain the process of finding the midpoint.

Place a point approximately half way along the line segment AB on page 1.6. To do this, press **menu > Geometry, Points & Lines > Points On**.

Find the coordinate of this position by hovering the pointer over the point and then pressing **ctrl + menu > Coordinates and Equations**.

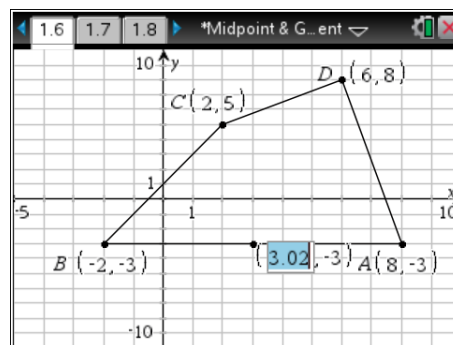
Find the distance between this point and A using the same steps as above.



Drag this point along the line segment AB until it is exactly half way. If you cannot get it exactly in the middle move the pointer onto the x-coordinate of the point and click twice so that the x-coordinate value is selected and input the closest whole number value to the one you current have.

Copy your answer onto the diagram in the exercise section of this handout for **Question 1**.

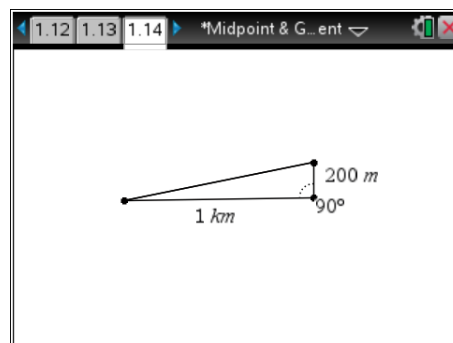
Repeat this process for the other three legs of the race and copy your answers onto the diagram in the exercise section of this handout for **Question 1**.



### Calculating the Gradient

Read pages 1.12 to 1.15 to view an outline of the next steps in the problem.

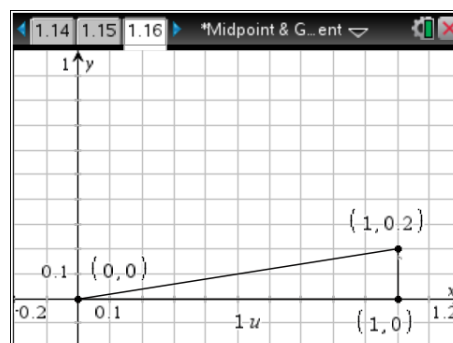
Now that you have worked out the course of the Cross Country and the positions of the teachers you now need to check an important aspect – safety regulations. The principal has told you that no one section of the run can have a hill that has a slope greater than a particular value. This is because of health concerns for the less fitter students who might struggle.



On page 1.17 you are asked to find the slope (gradient) of the hill. To do this return to page 1.16. Press **menu > Geometry > Measurement > Slope**. Select the segment that represents the hill, that is the line that joins (0, 0) and (1, 0.2).

Record your result in **Question 6** of this handout.

Measure the height and base of the triangle using the steps from earlier. Record your answers in this handout at **Question 6** and complete the requested calculations.

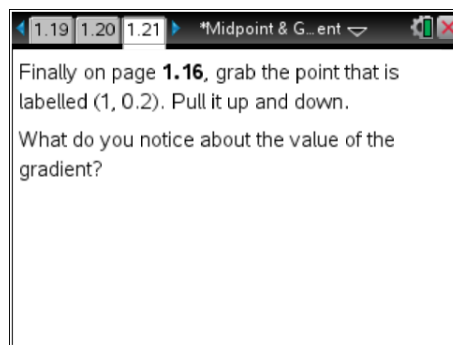


Now complete the calculations on this handout for **Question 7** related to finding the gradient using the three given points.

On page **1.16**, grab the point that is labelled (1, 0.2). Pull it up and down.

Answer **Question 9** in this handout.

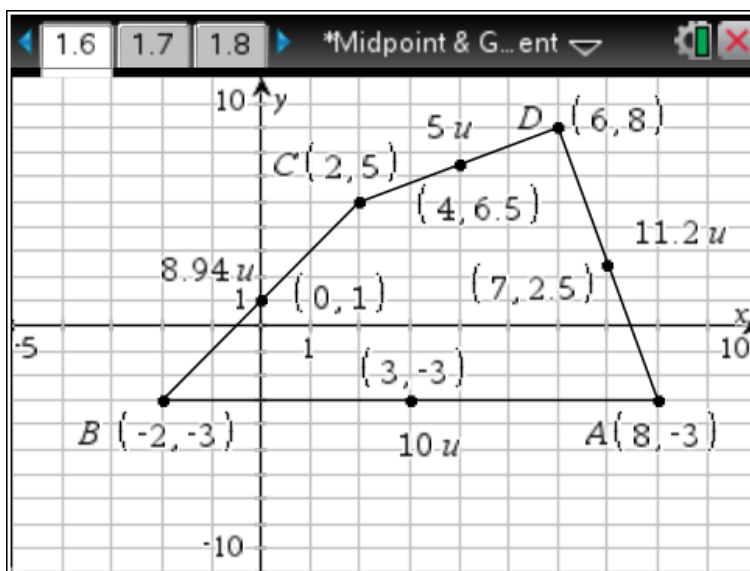
Finally attempt the extension questions in **Question 10**.



## Questions

- Record your results from page **1.7** to **1.11** on your TI-Nspire on the diagram below. Include the distance of each leg of the race and the positions and coordinates of each of the midpoints.

Distances, midpoints and their coordinates do not appear on the Student Activity handout.



2. Using the ratio 1 u : 0.2 km, what is the total length of this Cross Country course? Show your working out.

$$11.2 + 10 + 8.94 + 5 = 35.14 \text{ u}$$

$$35.14 \times 0.2 = 7.028 \text{ km}$$

3. Looking at the coordinates of each of the midpoints, try to come up with a quicker way of finding the midpoint for each of the line segments. Explain your process, using the coordinates for A and B as an example to show your method.

Add the x-coordinates of the end points of the segment and then do the same for the y-coordinates.

Divide each answer by 2 to get an x-coordinate and y-coordinate for the midpoint.

$$\text{i.e. } \frac{8 + ^{-}2}{2} = 3$$

$$\frac{^{-}3 + ^{-}3}{2} = ^{-}3$$

$$\therefore \text{Midpoint} = (3, ^{-}3)$$

4. Using your method from Question 3, find the midpoints of the other three line segments. Show working to justify your answers.

Midpoint of BC =

$$\frac{^{-}2 + 2}{2} = 0$$

$$\frac{^{-}3 + 5}{2} = 1$$

$$\therefore \text{Midpoint} = (0, 1)$$

Midpoint of CD =

$$\frac{2 + 6}{2} = 4$$

$$\frac{5 + 8}{2} = 6.5$$

$$\therefore \text{Midpoint} = (4, 6.5)$$

Midpoint of DA =

$$\frac{6 + 8}{2} = 7$$

$$\frac{8 + ^{-}3}{2} = 2.5$$

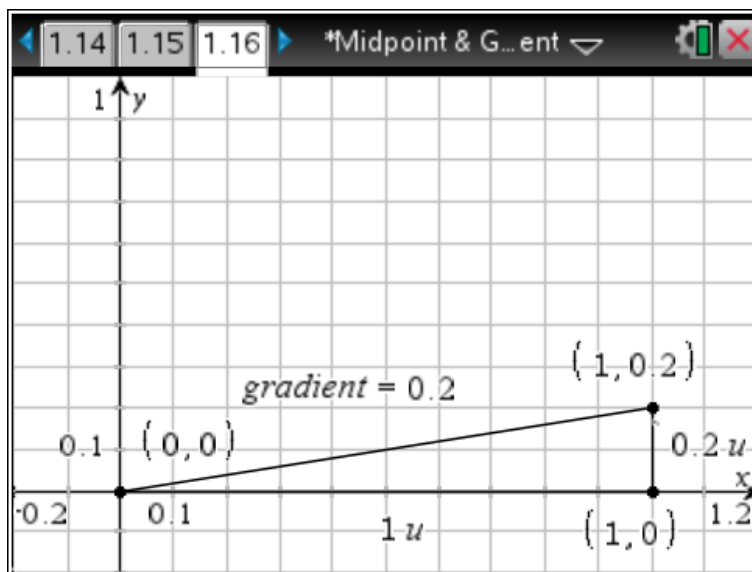
$$\therefore \text{Midpoint} = (7, 2.5)$$

5. What value did you get for the slope (gradient) of the hill using the method on page 1.17?

$$\text{Gradient} = 0.2$$

6. On the diagram below clearly label the height and base length of the triangle as found by using your TI-Nspire CAS.

Distances and slopes do not appear on the Student Activity Handout.



$$\begin{aligned} \text{Gradient} &= \frac{\text{Height}}{\text{Base Length}} \\ &= \frac{0.2}{1} \\ &= 0.2 \end{aligned}$$

7. Using the coordinates from the diagram on page 1.16 develop a method for finding the gradient without having to measure the height and base length of the triangle. Show your method below using the coordinates from this diagram as an example to illustrate your method.  
Hint: Use the concept of “rise” (the difference in height) divided by “run” (the difference in the base length).

$$\text{Height} = 0.2 - 0 = 0.2$$

$$\text{Base Length} = 1 - 0 = 1$$

$$\therefore \text{Gradient} = 0.2 \div 1 = 0.2$$

8. Based on the principal’s comment about the course having no slope (gradient) greater than 0.25, is the course safe?

The course is safe (greatest slope is 0.2)

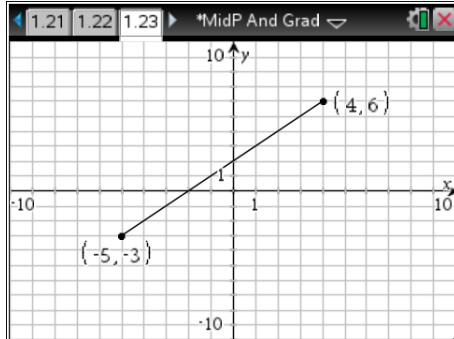
9. What do you notice happens to the gradient value when you drag the point (1, 0.2) up and down?

The gradient becomes a larger value as the line is made steeper and smaller when the line is flatter.

**Extension:**

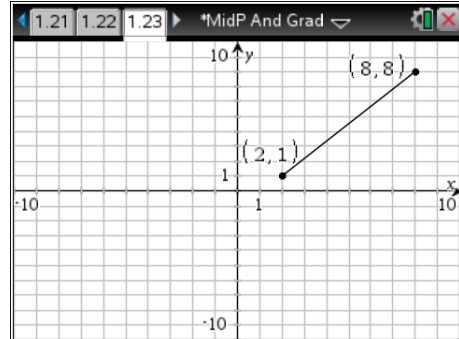
10. For each of the following diagrams below find:
- the coordinate of the midpoint
  - the gradient value (correct to two decimal places if necessary)

(i)



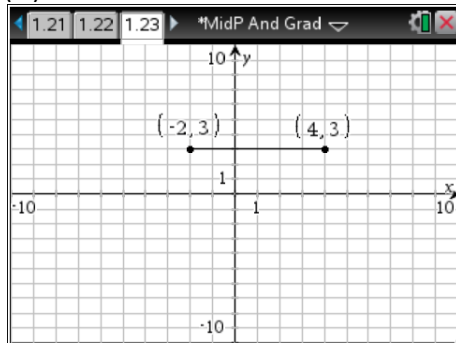
Midpoint =  $(-0.5, 1.5)$   
 Gradient = 1

(ii)



Midpoint =  $(5, 4.5)$   
 Gradient = 1.17 (to 2 d.p.)

(iii)



Midpoint =  $(1, 3)$   
 Gradient = 0