Fly Straight

Unit 3 – Application – Teacher

7 8 9 10 11 12

TI-Nspire™ Coding Language Student 50 min

TI-Basic

Teacher Notes:

This task is available as another version that includes a loop structure. This lesson includes the IF <condition> Then statement covered in Unit 3 and uses the Spreadsheet application to cover the 'recursive' requirement for the sequence. This approach provides an opportunity to reinforce the purpose of Functions and Programs.

A Power-Point slide show is also available for this activity to help students understand the sequence.

Problem Statement

There are all sorts of patterns created by sequences such as the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13 …

In this coding exercise we look at a very usual sequence. The aim is to see if an observable pattern exists as the sequence progresses. The sequence starts with the first term defined as: $t_1 = 1$.

Term Number:	n	1	2
Term:	t_n	1	

The second term is calculated using the term number (2) and the previous term (1), highlighted below. To calculate the next term check to see if there are any common factors between the current term number and the previous term.

Term Number:	n	1	2
Term:	t_n	1	

IF the highest common factor is 1 then we add the two highlighted values (above), then add 1 to the result: 2 + 1 + 1 = 4.

Term Number:	n	1	2	3
Term:	t_n	1	4	

We're ready to calculate the next term. We apply the same criteria, **IF** the highest common factor between the two highlighted values is 1 **THEN** we add the highlighted values (above), then add 1 to the result: 3 + 4 + 1 = 8.

Term Number:	n	1	2	3	4
Term:	t_n	1	4	8	

We're ready to calculate the next term. Applying the same criteria, the highest common factor between the highlighted values (4 and 8) is 4. If the highest common factor is not equal to 1, the term (8) is divided by the highest common factor, in this case 4. Our new term becomes: 8 ÷ 4 = 2.

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We'll do two more terms for practice.

Term Number:	n	1	2	3	4	5
Term:	t_n	1	4	8	2	

The highest common factor of the two highlighted values is 1, therefore the next term is 5 + 2 + 1 = 8.

Term Number:	n	1	2	3	4	5	6
Term:	t_n	1	4	8	2	8	

For the second time we see that the highest common factor of the two highlighted values is not equal to 1. The highest common factors of 6 and 8 is 2, therefore the next term will be equal to 8 ÷ 2 = 4

Check the next two terms in the table below.

Term Number:	n	1	2	3	4	5	6	7	8
Term:	t_n	1	4	8	2	8	4	12	3

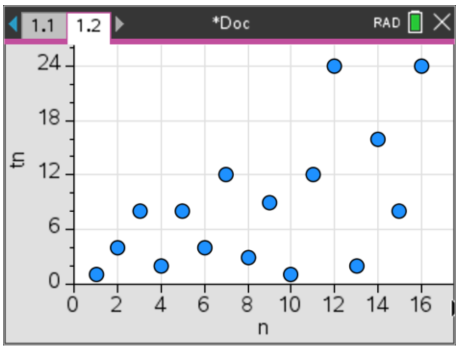
Question: 1.

Determine the missing terms in the table. (Check the answers with your teacher before continuing)

Term Number:	n	8	9	10	11	12	13	14	15	16
Term:	t_n	3	1	12	24	2	16	8	24	3

Question: 2.

Create a scatterplot for the data collected so far with the term number (n) on the x axis (independent variable) and the term (t_n) on the y axis (dependent variable). Include a copy of your scatterplot in your answers and identify any patterns if they exist.



Note: Students may draw the scatterplot by hand or populate a spreadsheet and use the calculator.

Answer: In this early scatterplot there are no obvious patterns. Students may observe that almost all the points lie under the line y = 2x + 1, or that prime numbers cause terms to increase in value.

Examples: $t_7 = 12$, $t_{11} = 24$ and $t_{13} = 16$.

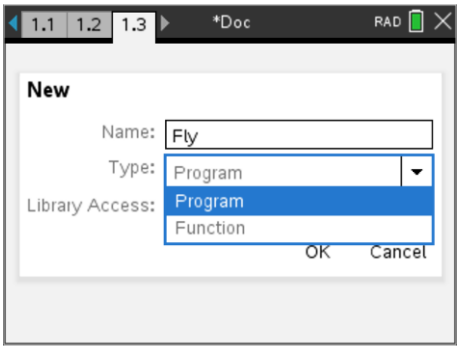
Students may also notice that the majority of the terms (at this stage) are even.

Coding the Sequence

Insert a new program/function.

Name: Fly

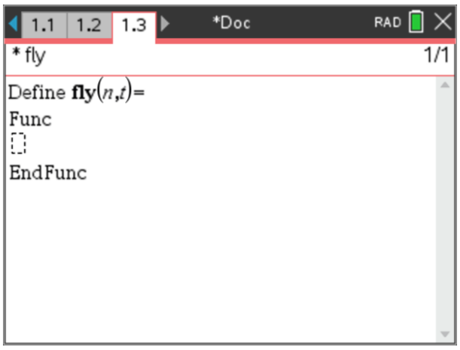
Type: Program or Function? Choose the most appropriate!
This tool requires spreadsheet (cell) access and only returns a single numerical value.



The code requires two input values, one for the term number (n) and one for the term (t_n). These values can be entered directly when the code is called from the corresponding application.

n = Term number

t = Term

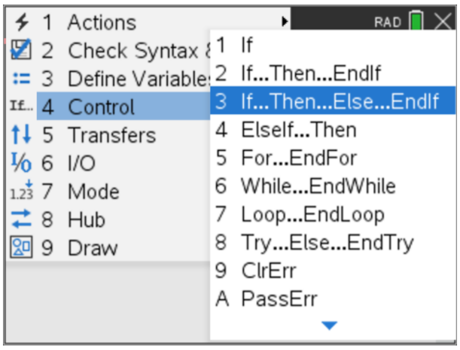


An **IF** … **THEN** … **ELSE** .. statement will do all the hard work.

GCD (Greatest Common Divisor) is the same as HCF (Highest Common Factor) and will be used as the testing condition.

IF GCD(n,t) = 1 **THEN**

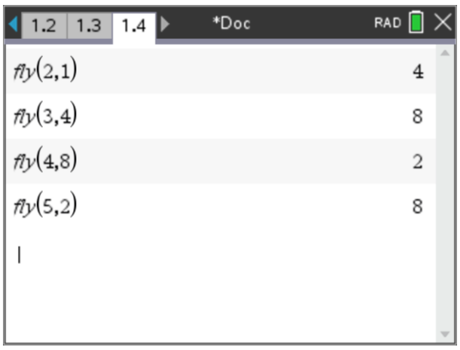
Insert the two sequence rules in the corresponding positions.



Save and run your code (Ctrl + R).

Test the values from the table, term number then term value.

Terms 2 to 5 are shown opposite.

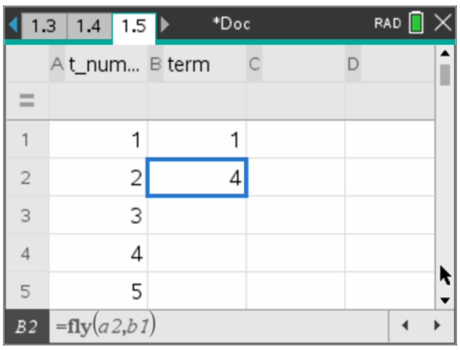


Insert a Spreadsheet application and create meaningful column names.

Column A will hold the term numbers. In cell A2 enter: = A1 + 1

Fill the formula down to cell A20 … menu > Data (3) > Fill (3)

In cell B2 use the fly code to generate the terms, fill this formula down to cell B20 and check the values against those computed in Question 1.



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Question: 3.

What is the 20th term in the sequence?

Answer: $t_{20} = 48$

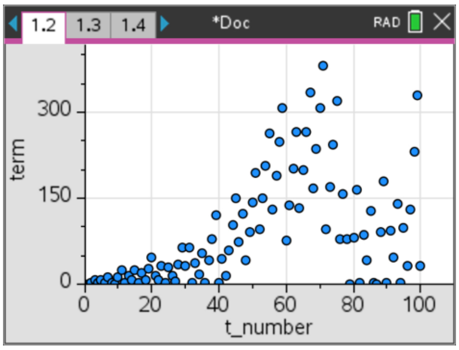
Question: 4.

Create a scatter plot for the first 20 terms of the sequence. Is there a clear pattern in the data?

Answer: No – there is no clear pattern in the data.

Question: 5.

Fill the terms in the spreadsheet down to cover the first 100 terms of the sequence. Return to the Data and Statistics application, change the window setting (zoom data) and study the scatterplot. Is there a clear pattern or does the data look 'chaotic'?



Answer: There is no clear pattern, however, it is interesting to see that the data is 'spreading out' and from term 76 to 95 there are several points that appear to form a 'straight' line.

Question: 6.

Continue your search, determine whether or not this sequence eventually generates a pattern.

Answer: Most students will likely give up before seeing the remarkable pattern. It is a lovely example of 'why' we need tools like algebra so we don't need to look 'forever'. Algebra allows us to prove for all numbers. In this example we eventually get to: $t_{641} = 2$. This is the first time an odd numbered term = 2, then ...see teacher notes!

Teacher Notes:

This proof is not required by middle school students, but could be asked of students in a higher level. Consider the following where an odd numbered term is equal to 2:

$t_{2n+1} = 2$

t_{2n+2} : The highest common factor of 2n + 2 and 2 will be 2, therefore $2 \div 2 = 1 \therefore t_{2n+2} = 1$

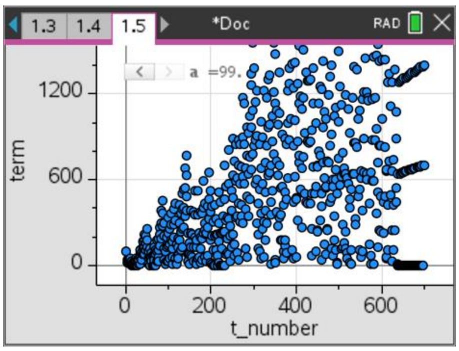
t_{2n+3} : The highest common factor of 2n + 3 and 1 will be 1, therefore $t_{2n+3} = 2n + 3 + t_{2n+2} + 1 = 2n + 5$

t_{2n+4} : The highest common factor of 2n + 4 and 2n + 5 is 1, therefore: $t_{2n+4} = 2n + 4 + 2n + 5 + 1 = 4n + 10$

t_{2n+5} : The highest common factor of 2n + 5 and 4n + 10 is 2n + 5 ... which means $t_{2n+5} = (4n + 10)/(2n+5) = 2$.

The cycle will now continue since 2n + 5 is an odd term number that is equal to 2.

We can easily test this "theory". Change the first term of the sequence to 2, ($t_1 = 2$) The sequence starts with an odd term number that is equal to 2, alas, that sequence is not particularly exciting! It does however pose the question: "What other starting terms might result in an interesting pattern / chaos?"



Such an investigation is easy to explore. Insert a slider into the Data and Statistics application, set cell A1 equal to the slider variable and start exploring!

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