

Features Used

Differential Equations
Graphing Mode
NewProb

Setup

1
NewFold trannum

Transient Circuit Analysis: Numeric

This chapter describes how the differential equation plotter is used to solve the second-order circuit presented in Chapter 2 (Topic 8) by expressing it as a system of first-order differential equations.

Topic 9: RLC Circuit: Direction Field

Find $i(t)$ for $t > 0$ s for the circuit in Figure 1 in which the current at time $t = 0$ s is 10 A, that is, $i(0) = 10$ A, and the voltage across the capacitor at $t = 0$ s is 0, that is, $v_c(0) = 0$ V.

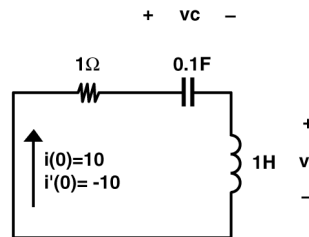


Figure 1. An RLC circuit

Kirchhoff's voltage law states that the sum of the voltages around every closed loop is zero. The voltage drop across a capacitor in the direction of current flow is

$$v_c = \frac{1}{C} \int i dt \quad (1)$$

and the voltage drop across an inductor in the direction of current flow is

$$v_l = L \frac{di}{dt}$$

The **DIFF EQUATIONS** plot requires that the equations be expressed in terms of $v_c'(t)$ and $i'(t)$. To do this, write equation (1) as

$$i(t) = C \frac{dv_c(t)}{dt} \text{ or } i(t) = C * v_c'(t)$$

Solve for $vc'(t)$ as

$$vc'(t) = \frac{i(t)}{C}$$

and for $i'(t)$ as

$$i'(t) = \frac{vl(t)}{L}$$

All equations must be entered as functions $y1(x)$, $y2(x)$, and so on. Therefore, let

$$vc = y1 \text{ and } i = y2$$

vl must be expressed in terms of $y1$ and $y2$. To do this, sum the voltages around the loop to get

$$i(t)R + vc(t) + vl(t) = 0$$

which is solved for $vl(t)$

$$vl(t) = -i(t)R - vc(t)$$

Substitution of $y1$ and $y2$ gives

$$vl(t) = -y2 * R - y1$$

or

$$i'(t) = y2' = \frac{vl(t)}{L} = \frac{-y2 * R - y1}{L}$$

As a result, these differential equations are

$$y1' = \frac{y2}{C} \text{ and } y2' = \frac{-y2 * r - y1}{ell}$$

The corresponding initial conditions are

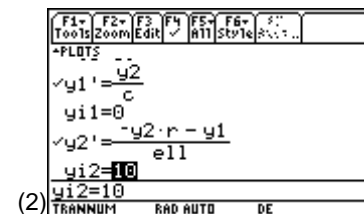
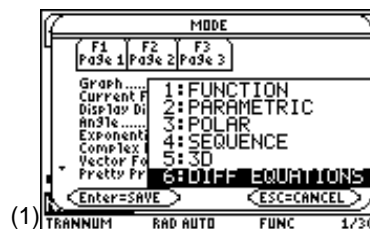
$$y1(0) = 0 \text{ and } y2(0) = 10$$

Note: "ell" is used to avoid confusion between "l" and "1."

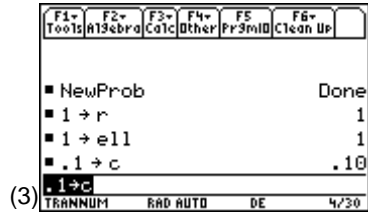
To enter these equations, follow these steps.

1. Clear the TI-89 by pressing $\boxed{2nd} \boxed{[F6]} \boxed{2:NewProb} \boxed{ENTER}$.
2. Press $\boxed{MODE} \rightarrow \boxed{6:DIFF EQUATIONS} \boxed{ENTER}$ to select the differential equation graphing mode as shown in screen 1.
3. Press $\boxed{\downarrow} \boxed{[Y=]}$ and enter the equations as shown in screen 2.

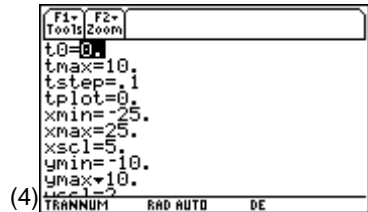
The $y1=0$ entry sets $vc(0)=0$ (remember $y1$ is $vc(t)$). The $y2=10$ entry sets $i(0)=10$.



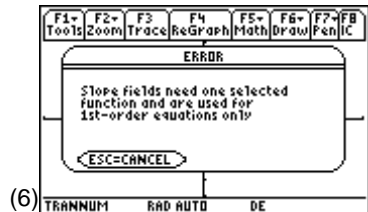
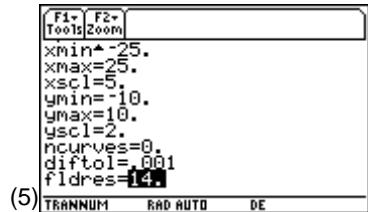
4. Press **[HOME]** and enter values for **r**, **e11**, and **c** as shown in screen 3.



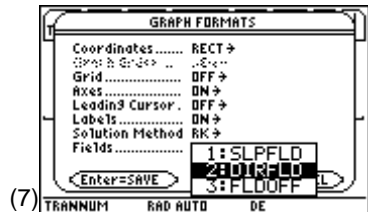
5. Press **[WINDOW]** and set the window variables to the values as shown in screens 4 and 5.



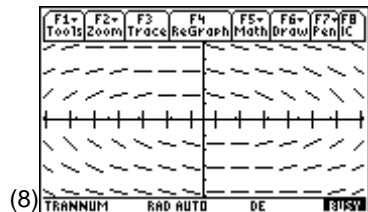
6. Press **[GRAPH]** to graph the results. An error message appears first (screen 6). Slope fields are used for 1st-order equations only.



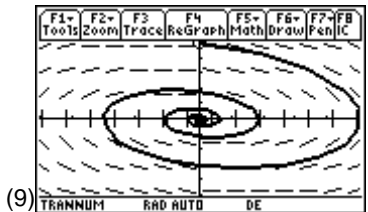
7. Press **[ESC]** **[F1]** to set the proper graph format. Move the cursor down to **Fields** at the bottom and select **2:DIRFLD** as shown in screen 7.



8. Press **[ENTER]**. The graph first shows a direction field (screen 8). The short lines indicate the direction in which **i** and **vc** change with respect to each other at each point in the solution space. This plot has **vc** as **y1** on the x-axis and **i** as **y2** on the y-axis. Initial values were entered in the Y= Editor.



After a short time, the trajectory of the solution for the given initial conditions appears (screen 9).



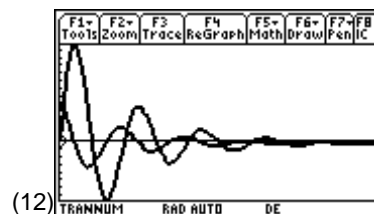
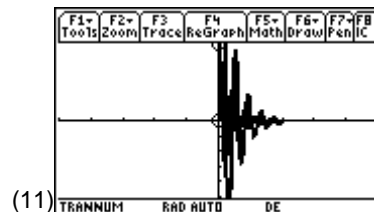
It takes a few seconds to complete this plot. The time can be reduced by pressing \square [WINDOW] and setting **tmax** to a smaller value or setting **tstep** to a larger one. The smaller **tmax** causes the plotting to stop sooner. The larger **tstep** results in a less smooth plot.

The thicker solution line starts at $v_c = 0$ and $i = 10$ (screen 9). It then circles down and to the right, spiraling clockwise to 0,0. This shows how the energy moves back and forth between the current in the inductor $E_L = \frac{Li^2}{2}$ and the voltage on the capacitor $E_C = \frac{Cvc^2}{2}$. The resistor is converting some of the energy to heat; so after a long time all the energy is lost as heat and $i(t)$ and $v_c(t)$ are 0.

Topic 10: RLC Circuit: Time Domain

A more traditional way of viewing the voltage across the capacitor (**y1**) and current through the inductor (**y2**) is to plot them versus time.

1. You can do this easily from the Graph screen by pressing \square [1] and selecting **Fields 3:FLDOFF** (screen 10).
2. Press \square [ENTER] to save the change, and screen 11 appears.
3. Press \square [WINDOW] and set **xmin** to 0, **xmax** to 10, **ymin** to -15, and **ymax** to 25.
4. Press \square [GRAPH] to display a clearer plot, as shown in screen 12.



Note: Pressing \square [F2] **A:ZoomFit** will produce a graph with **ymin** and **ymax** automatically set to display the whole function.

Topic 11: RLC Circuit: Multiple Initial Conditions

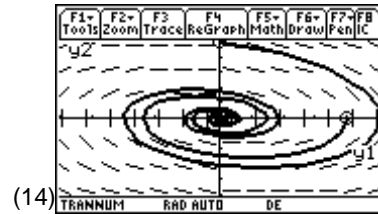
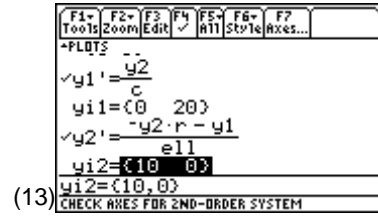
The Y= Editor can be used to set multiple initial conditions. The first example (Topic 9) initially set the current, i (**y2**), to 10 A and the voltage v_c (**y1**) to 0 V. The plot for the current $i = 0$ A and the voltage $v_c = 20$ V is added to the original plot by entering lists of initial conditions in the Y= Editor.

1. Press \blacktriangleright [WINDOW] to set **xmin** back to -25, **xmax** to 25, **ymin** to -10, and **ymax** to 10, and press \blacktriangleright [1] to set **Fields** back to **DIRFLD**. This will enable redrawing the direction field and **y1** versus **y2** solution.
2. The lists of initial conditions are added (in the Y= Editor) as shown in screen 13.

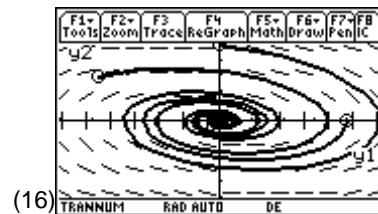
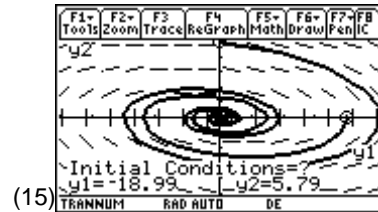
y1: [ENTER] [2nd] [t] [0] [.] [20] [2nd] [)] [ENTER]
 y2: [ENTER] [2nd] [t] [10] [.] [0] [2nd] [)] [ENTER],

3. Pressing \blacktriangleright [GRAPH] shows both solutions (screen 14). Notice the second set of initial conditions produces a second curve which begins on the positive x-axis.
4. Initial conditions can also be added graphically. Press [2nd] [F8] and use the cursor keys to move to the desired initial conditions in the **y1-y2** plane (screen 15).

5. Press [ENTER] to add the graph of this new set of initial conditions to the display (screen 16).



Note: Pressing \blacktriangleright [1] and selecting **Labels 2:ON** activates the **y1** and **y2** axes labels.

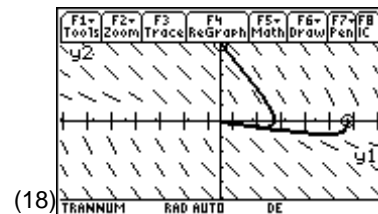
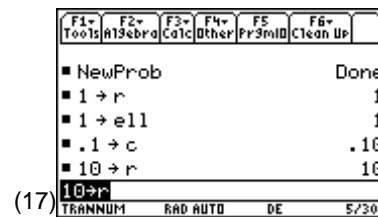


Topic 12: RLC Circuit: Adjusting the Circuit Parameters

If the resistor, R, is increased, the energy in the circuit should be dissipated in the resistor more rapidly. This change can be investigated by simply changing r.

1. Press [HOME] and increase r to 10 by entering **10** [STO] r as shown in screen 17.
2. Press \blacktriangleright [GRAPH] to display the solutions to the previous sets of initial conditions as shown in screen 18.

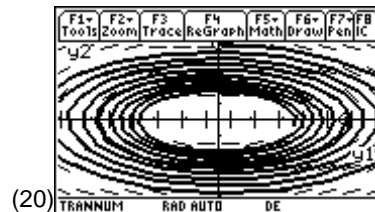
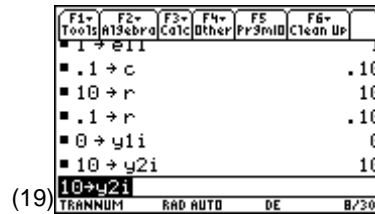
Notice that the direction field has changed dramatically, and now the energy quickly goes to zero from both sets of initial conditions.



3. Making r smaller will cause the current and voltage to die out more slowly. Return to the Home screen, and try $r = 0.1$. Also, reset the initial conditions to the original single set, that is, $y_1=0$ and $y_2=10$ (screen 19) to keep the graph from becoming too cluttered.

4. Press \square [GRAPH] to see the result (screen 20).

What would happen if r were set to 0 or to a negative value? Try it!



Tips and Generalizations

The examples presented here show how to plot the numeric solution to a second-order differential equation by expressing it as a system of two first-order differential equations. The numeric differential equation solver is not limited to two equations, so higher-order differential equations also can be solved.

The response of most circuits is composed of two parts, transient and steady-state. Chapters 2 and 3 have considered transient responses.

Often the steady-state response is of more interest. Chapter 4 shows how the TI-89 manipulates complex numbers (phasors) to find sinusoidal steady-state solutions.