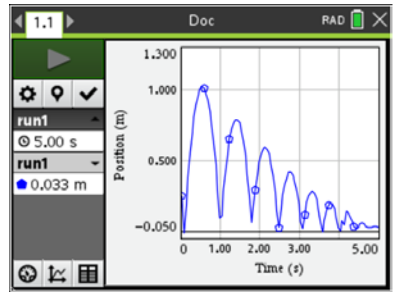
About the Lesson

This activity examines the motion of a ball as it falls under the influence of gravity. The parameters in the vertex form of the quadratic equation $y = a(x – h)^2 + k$ are determined to describe the behavior of a ball bounce. As a result, students will:

- Collect motion data and graph scatter plots.
- Determine the quadratic equation for a ball bounce.
- Determine the value of the coefficient *a* in the vertex form of a quadratic equation.
- Explore the effect of *a* on the shape of the graph.
- Interpret the meaning of the value of *a* in the context of the problem situation as one half of the acceleration due to gravity.



Vocabulary

- Vertex
- Vertex form of a quadratic function
- Vertical reflection, vertical stretch, and vertical compression

Teacher Preparation and Notes

- Students should have worked with translations, reflections, and vertical stretches and compressions of functions.
- This activity provides an opportunity for math-science connections.
- This activity is best performed with at least three students: one to hold the CBR™ 2 and press the trigger, one to release the ball, and one to run the calculator.

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire™ CX II. It is also appropriate for use with the rest of the TI-Nspire CX family. Slight variations to these directions may be required if using other handheld models.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Activity Materials

- CBR™ 2 motion sensor, USB CBR 2 to handheld cable, and TI-Nspire CX II
- Bouncing ball (Avoid using a soft or felt-covered ball such as a tennis ball.)
- Recommended: TI-Nspire™ CX Premium Teacher Software or TI-Nspire™ CX CAS Premium Teacher Software

This activity could be demonstrated using the TI-Nspire™ CX Premium Teacher Software so the entire class can see the process. If you only have one CBR 2 motion sensor, send the data to each student's handheld after collecting the data. If you have enough CBR 2 devices, have students work in small groups and collect data.

Setup

See directions in the Student Activity.

Data Collection

See directions in the Student Activity.

Data Collection Questions

6. What quantity is represented along the horizontal axis? **Answer:** time

   What are its units? **Answer:** seconds

7. What quantity is represented along the vertical axis? **Answer:** position or height

   What are its units? **Answer:** meters or feet

8. What does the highest point on the plot represent? **Answer:** the maximum height of the ball

   What does the lowest point represent? **Answer:** the floor

Select a Region of the Graph

See directions in the Student Activity.

Data Analysis

Method 1 – Vertex Form of a Quadratic Function; Determining the Value of *a*

See directions in the Student Activity.

Data Analysis Method 1 Questions

2. For any one bounce, a plot of distance vs. time has a parabolic shape. One form of the equation that describes this motion is $y = a(x - h)^2 + k$ where (h, k) is the vertex of the parabola and a is the vertical stretch or compression factor of the graph. This equation is called the *vertex form*.

   Record the x- and y-coordinates of the vertex as h and k here:

   h = ______ k = ______

   Sample Answer: Answers will vary.

4. Before storing a value for $\boldsymbol{a}$, predict what the graph would look like if $\boldsymbol{a} = 1$.

   Answer: The graph will be concave up. It needs to be vertically reflected and vertically stretched to match the bounce.

5. Before storing a value for $\boldsymbol{a}$, predict what the graph would look like if $\boldsymbol{a} = 0$.

   Answer: The graph will be the horizontal line $y = \boldsymbol{k}$.

8. Record the value of a that works best: $\boldsymbol{a}$ =_________

   Answer: Answers will vary but the value of $\boldsymbol{a}$ should be approximately one-half the acceleration due to gravity, –4.9 meters/second^2 or –16 feet/second^2.

9. Using this value of $\boldsymbol{a}$ and the $\boldsymbol{h}$ and $\boldsymbol{k}$ values you reported in Step 2, write the vertex form of the quadratic equation. y =________________

   Answer: Answers will vary but should be of the form $y = a(x - h)^2 + k$.

10. What effect does each have on the graph of the parabola?

    a. The sign (positive or negative) of ***a***?

    Answer: Positive ***a*** value, the graph opens up; negative **a** value, the graph opens down. If ***a*** < 0, the graph is concave down; if ***a*** > 0, the graph is concave up.

    b. |***a***| > 1?

    Answer: If |***a***| > 1, the graph is a vertical stretch of $y = x^2$.

    c. |***a***| < 1?

    Answer: If |***a***| < 1, the graph is a vertical compression of $y = x^2$.

11. For the same bounce, press **Menu > Analyze > Graph Trace**, and trace along the plot to identify the x- and y-coordinates of a point that is not the vertex. Record the coordinates here.

    x = ________ y = ________ (to 2 decimal places)

    Answer: Answers will vary.

12. Substitute the coordinates of the vertex (from Step 2) and the coordinates of a point on the plot of the parabola ((x, y) from Step 11) into the vertex form of a parabola, $y = a(x - h)^2 + k$, to solve for the value of a.

    a = ____________________________

    Record the equation of the parabola in vertex form.

    y = ____________________________

    Answers: Answers will vary but the value of a should be approximately one-half the acceleration due to gravity, –4.9 meters/second^2 or –16 feet/second^2.

14. How does the value of ***a*** from Question 8 compare to the value of ***a*** from Question 12? Which graph (f1(x) or f2(x)) provides a better fit for the bounce? Explain.

    Sample Answer: Answers will vary.

Method 2 – Quadratic Regression

Quadratic Regression Questions

2. What is your regression equation? ________________________

Answer: Answers will vary but the value of ***a*** should be approximately one-half the acceleration due to gravity, –4.9 meters/second^2 or –16 feet/second^2.

3. How does the value of ***a*** in the regression equation compare to the values of ***a*** you determined in Method 1?

Answer: Answers will vary.

Extension:

1. Would you expect your classmates to have the same value of ***a*** for their trials, or do you think the ***a*** value would vary? Explain your answer.

Sample Answer: Same; student explanations will vary depending on their previous experiences.

2. Determine the values of ***a*** found by other groups of students in your class. How do these values compare to your value of ***a***?

Sample Answer: Same or very close.

3. If a ball that was more or less bouncy was used, would it affect the value of ***a*** in the equation? Explain your response.

Answer: No, it would only affect the vertices of the bounces.

Lead students into a discussion about the acceleration of falling objects due to gravity.

- Remember, the ball was not thrown; it was dropped. After an object is released, it is acted upon by gravity (neglecting air resistance). So ***a*** depends on the acceleration due to gravity, –9.8 meters/second^2 or –32 feet/second^2.
- The negative sign indicates that the acceleration is downward.

Note: Depending on students' knowledge of physics, decide how far to take this discussion.