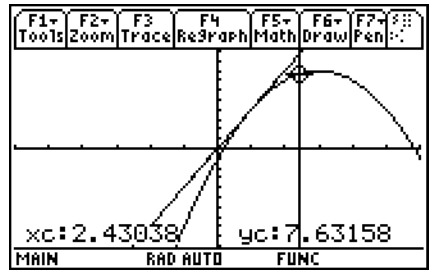
Linear Approximation

Name ________________________

Class ________________________

Introduction

Linear approximation uses a tangent line to estimate values of a function near the point of tangency. For this reason, *linear approximation* is also referred to as *tangent line approximation*.



On the graph to the right, let the graph let *a* be the point where the tangent touches the graph, *L*(x) be the tangent and *f*(*x*) be the function.

On the picture, the point *x* is the *x* coordinate of the vertical line.

Draw a vertical line from *a* to the *x*-axis.

Draw horizontal lines from *a*, f(*x*), and the intersection of the vertical line with the tangent line.

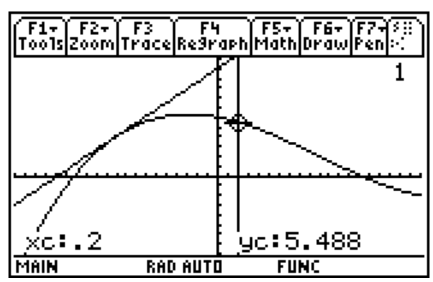
At this stage, you should have three points on the *y*–axis: *f(a), f(x), L(x).* Label them.

- Which of these points can you use to represent the estimate, or linear approximation, of *f(x)* near *a*?

- How can you use these labels to represent the *error* associated with this estimate?

- Is this estimate an *overestimate* or an *underestimate*? Explain.

Investigating linear approximation



On the graph at the right, $\mathbf{f1}(x)=x^3-3x^2-2x+6$ is shown. The tangent line at $a = -1$ is $L(x)=7x+11$. The trace object *q* is at the point (0.2, 5.488).

If you draw a vertical line from the *x*–axis through this point, you will get the point *p* = (0.212658, 12.4886).

The distance *pq*=7.039997 or 7.04.

- What numerical value represents the linear approximation of **f1**(*q*) near a = –1?

- What numerical value gives the error associated with this linear approximation?

- What is the true value of **f1(**q**)?**

- Is this an underestimation or an overestimation?

Repeat the process above and complete the table below for the *x*-values given.

	p	**distance *pq***	**linear approx. of f1(q)**	**real value of f1(q)**	**error**	**underestimation/ overestimation**
x = –0.2						
x = –0.5						
x = –0.6						
x = –1.2						

- At what x-value(s) is the error less than 0.5?

- What do you notice about graph of the function and the graph of the tangent line as you get close to the point of tangency?

- Based on your observations, explain why the relationship between a tangent and a graph at the point of tangency is often referred to as *local linearization*.

Typically, you will have a function but not a graph to find the linear approximation.

- Find the derivative of **f1**(x)= $x^3 - 3x^2 - 2x + 6$. Evaluate it at $x = -1$. This is the slope of the line. Use the slope and the point (–1, 4) to get the equation of the line.

- The tangent line $L(x)$ =

- What is $L(-1.03)$? What does this value represent?

- Calculate the error with this estimate.

Underestimates versus overestimates

Graph the function f1(x) = $x^3 - 3x^2 - 2x + 6$ and place a tangent line a = 1.

- If you were to draw a point p on the graph to the left of a = 1, is the approximation an *overestimate* or an *underestimate*?
- If you draw a point p on the graph to the right of a = 1, is the approximation an *overestimate* or an *underestimate*?
- What is the significance of the point of tangency?
- Generalize your findings about when a linear approximation produces an overestimate and when it produces an underestimate.

Finding intervals of accuracy

Suppose we have the following question.

How close to –1 must x be for the linear approximation of f1(x) = $x^3 - 3x^2 - 2x + 6$ at a=–1 to be within 0.2 units of the true value of **f1(*x*)**?

Graph **f2 = f1 + 0.2** and **f3 = f1 – 0.2** with f1 and the tangent line.

- How would you use the graphs to answer the question posed in this problem?
- How close to –1 must x be for the linear approximation of f1(x) = $x^3 - 3x^2 - 2x + 6$ at a = –1 to be within 0.2 units of the *true* value of **f1**(x)?

Now we want to ask the same questions when the point of tangency is at a = 1.

- How does this situation differ from the one we just had?
- Use graphical or algebraic methods to find an interval that ensures the linear approximation at a = 1 is accurate to within 0.2 units of **f1**(x).