A Little Light Work – ID: 8547

By Russell Brown

Time required
45 minutes

Activity Overview

In this activity, students will study graphs of light intensity at various distances from a light sensor. They will use power and linear regressions to determine the relationship between light intensity and distance from the light source. They will also develop their own models for the relationship. This activity will help reinforce the idea that light intensity decreases with distance according to an inverse-square law.

Concepts

- *Inverse-square relationships*
- *Propagation of light from a point source*
- *Mathematical models of physical phenomena*

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *light point source (e.g., small lightbulb on a base)*
- *meter stick*
- *Vernier Light Sensor*
- *Vernier EasyLink™or Go!®Link interface*
- *copy of the student worksheet*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Calculator, Graphs & Geometry, Lists & Spreadsheet, Data & Statistics

Teacher Preparation

Review the concept of an inverse-square relationship with students before they carry out this activity.

- *You may wish to dim the classroom lights during the data collection procedure.*
- *You should experiment with the light sources students will be using to determine the appropriate range setting for the switch on the Light Sensor. Give students this information before they begin data collection.*
- *The screenshots on pages 2–7 demonstrate expected student results. Refer to the screenshots on page 8 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file and student worksheet, go to education.ti.com/exchange and enter "8547" in the search box.***

Classroom Management

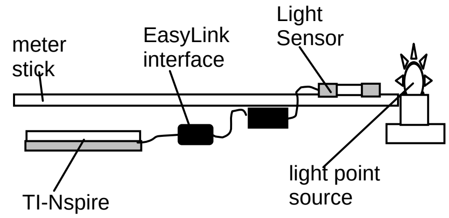
- *This activity is designed to be **student-centered**, with the teacher acting as a facilitator while students work cooperatively. The student worksheet guides students through the main ideas of the activity and provides questions to guide their exploration. Students should answer the questions on blank paper.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration in this activity:

- What is the relationship between the intensity of light and the distance from its source?
- How can we create mathematical models based on quantitative data for this relationship?

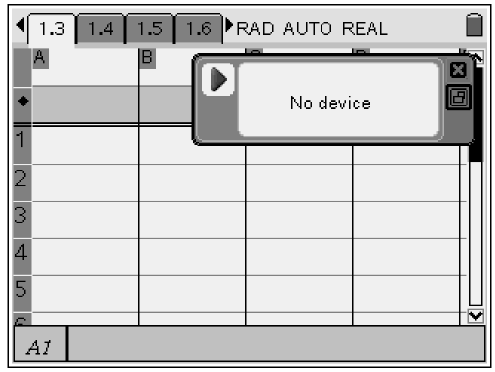
Students will first collect data on intensity of light as a function of distance from its source. They will use three different methods to develop equations relating intensity and distance. Before beginning the activity, briefly discuss what students have observed about the relationship between light intensity and distance. Remind them that light travels outward in all directions from a point source, and that the wave fronts form spheres of increasing radius.

Part 1 – Using a regression to quantify the intensity-distance relationship



Step 1: Students should open the file **PhyAct02_light_intensity_EN.tns** and read the first two pages. They should then place a meter stick on the floor or on another flat, level surface. They should place the light source at the zero end of the meter stick. They should then connect a Vernier Light Sensor to an EasyLink interface (if using a handheld) or a Go!Link interface (if using a computer). Note: For students to observe an inverse-square relationship between distance and intensity, the Light Sensor must be at the same vertical position (height) as the center of the light source. A small LED or flashlight bulb mounted on a base and connected to a battery would be an appropriate point source. *Do not use a flashlight*. A flashlight contains a parabolic reflector that changes the direction of some of the light emitted by the bulb. This makes the light “appear” to be brighter than it actually is—that is, it will not show an inverse-square relationship between distance and intensity. If a small bulb mounted on a base is not available, you can also use a bare light bulb on a desk lamp. In this case, you may wish to cut a small hole in a piece of cardboard and hold it in front of the bulb so that only a thin beam of light comes out. This will more closely approximate a point source.

Step 2: Students should move to page 1.3, which contains an empty *Lists & Spreadsheet* application, and insert a data collection box. They should then connect the EasyLink or Go!Link to their handheld or computer. Tell students which setting (0-600, 0-6000, or 0-150,000 lux) on the Light Sensor to use.



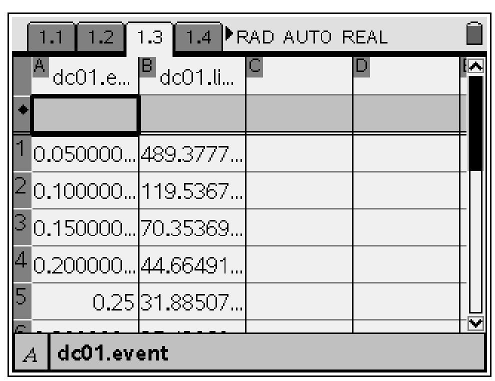
Step 3: Students should place the sensor at the 5 cm mark on the meter stick. With the light off, they should zero the sensor (**Menu > Sensors > Zero**).

Step 4: Next, students set up the data collection software to **Events with Entry** mode.

Step 5: Next, students turn the light on and record the light intensity at the 5 cm mark. Note: If necessary, students can alter the locations at which they collect data. You may wish to experiment with the flashlights students will use to determine the distances that yield the best results. (For example, if students hold the sensor too close to the flashlight initially, the data may not follow an inverse-square curve. Similarly, if students hold the sensor too far away from the light, the intensity may be so low that the sensor cannot detect it precisely.)

Step 6: Students move the sensor to the next mark and repeat the data collection.

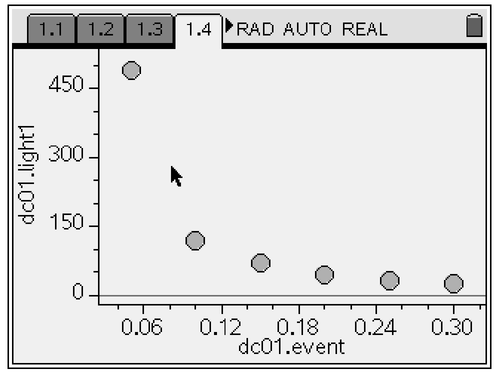
Step 7: Students repeat the data collection four more times. Once they have collected six data points, they can close the data collection box and disconnect the sensor.



Q1. Briefly describe how intensity changes as distance increases. Does the change appear to be linear (that is, do uniform increases in distance produce uniform change in intensity)? Justify your answer.

A. *Intensity decreases as distance increases, but the relationship does not appear to be linear. Each 0.02-unit increase in distance does not produce a uniform decrease in intensity.*

Step 8: Next, students create a plot of intensity vs. distance in the *Data & Statistics* application on page 1.4.



Q2. Describe the shape of the graph of intensity vs. distance.

A. *The data appear to lie on a curved line.*

Q3. What types of mathematical functions produce graphs with shapes like this?

A. *Graphs of functions of the type* $y = \frac{1}{x^2}$, *also known as inverse-square functions, have shapes similar to that of this graph.*

Step 9: Next, students use a power regression to find the best-fit equation for the data.

Q4. Write the equation that best fits the data on your answer sheet. Round the calculated values to three significant figures.

A. *The equation will vary depending on the light source used. For the sample dataset shown, the equation is the following (*I *is intensity and* d *is distance):*

$$I = (3.0)d^{-2}$$

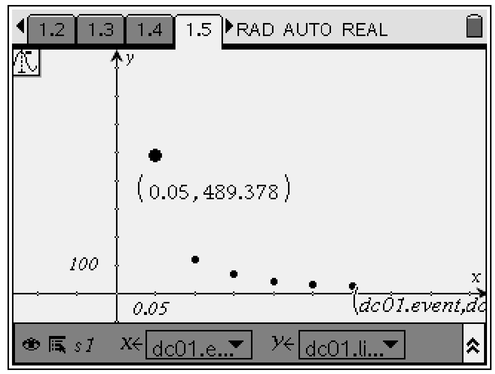
Q5. Why do you think relationships like these are called "inverse-square laws"?

A. *Relationships like these are called "inverse-square laws" because one of the variables is equal to a constant times the inverse of the square of the other variable.*

Q6. Does the graph of the regression equation appear to fit the data well?

A. *Answers will vary. If the collected data are sufficiently precise, the curve should fit them quite well.*

Part 2 – Building your own model for the relationship



Step 1: In this part of the activity, students attempt to fit an inverse-square curve to the data. They are first asked to plot the data on a scatter plot and use the **Trace** function to identify the coordinates of the leftmost point, as shown.

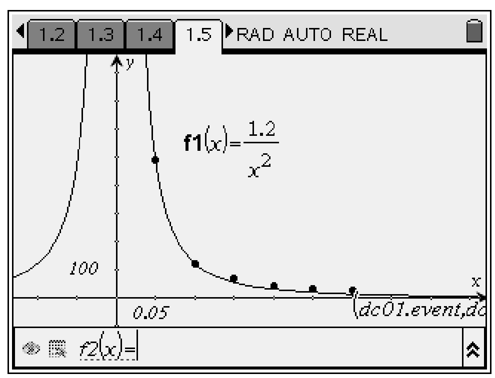
Q7. Use substitution to find the value of *k* that makes the equation above true for the leftmost point on the graph. Show your work.

A. *Answers will vary. The calculation below is for the sample dataset shown.*

$$I = \frac{k}{d^2}$$

$$489.378 = \frac{k}{(0.05)^2}$$

$$k = (489.378)(0.0025) = 1.22$$



Step 2: Next, students graph their calculated functions on the graph of the data. They should vary the value of *k* to force the curve to fit the data.

Q8. What value of *k* makes the curve fit the data best?

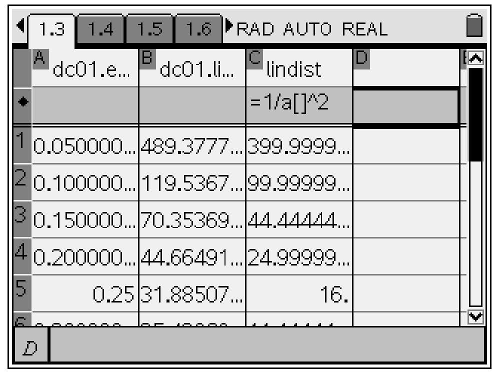
A. *The exact values that students will come up with will vary.*

Q9. How does this value of *k* compare to the value of *a* calculated in problem 1?

A. *The best-fit value of* k *calculated here will probably be different from the value of* a *from problem 1. You may wish to discuss this result with students in more detail. If they have not used the best-fit exponent, they should expect that the best-fit coefficient for this equation will be different from that calculated in the power regression.*

Part 3 – Another confirmation of the inverse-square relationship

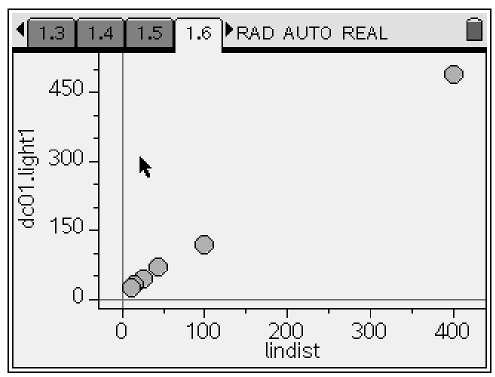
Step 1: In the final part of this activity, students examine the relationship between intensity and $\frac{1}{d^2}$. They are first asked to define a new variable, **lindist**, which is equal to $\frac{1}{d^2}$, as shown. Students may have difficulty understanding why the graph of intensity vs. **lindist** should be a straight line. If necessary, review the equations with them to help them grasp the concept before continuing.



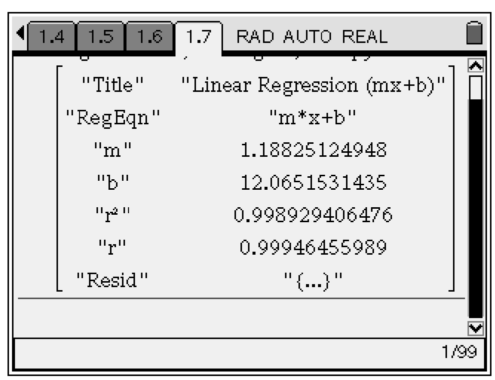
Step 2: Next, students plot intensity vs. **lindist** on a scatter plot, as shown.

Q10. Describe the shape of the graph.

A. *The data appear to lie on a straight line.*



Step 3: Finally, students use the *Calculator* application to determine the linear equation that best relates intensity to $\frac{1}{d^2}$. (Note that students could also calculate the linear regression on these variables using the **Stat Calculations** menu in the *Lists & Spreadsheet* application on page 1.3.)



Q11. Does the r^2 value for the linear regression support the statement that intensity is directly related to $\frac{1}{d^2}$? Explain your answer.

A. *Answers will vary. An* r^2 *value near 1 indicates a good fit between the equation and the data.*

Q12. Use each of the three equations relating *I* and *d* that you found in this activity to calculate the intensity of this light at a distance of 0.35 m from the source. Show your work.

A. *The calculations below are based on the sample data set. Using the equation from part 1:*

$$I = 3.0d^{-2}$$
$$I_{0.35} = (3.0)(0.35)^{-2}$$
$$I_{0.35} = (3.0)(8.16)$$
$$I_{0.35} = 24.5$$

Using the equation from part 2:

$$I = 1.22d^{-2}$$
$$I_{0.35} = (1.22)(0.35)^{-2}$$
$$I_{0.35} = (1.22)(8.16)$$
$$I_{0.35} = 9.96$$

Using the equation from part 3:

$$I = (1.19)\left(\frac{1}{d^2}\right) + 12.1$$
$$I_{0.35} = (1.19)\left(\frac{1}{(0.35)^2}\right) + 12.1$$
$$I_{0.35} = (1.19)(8.16) + 12.1$$
$$I_{0.35} = 21.8$$

Q13. How could you test your calculations from question 12 to determine which value is most accurate?

A. *One way to test the accuracy of the values would be to repeat the experiment, but include an additional data point at 0.35 m away from the light sensor.*

Q14. Using the equation for the surface area of a sphere, explain why the relationship between intensity and distance follows an inverse-square law.

A. *The equation for the surface area of a sphere,* $SA = 4\pi r^2$*, shows that the surface area of a sphere is directly related to the square of the radius of the sphere. The intensity of the light is given by* $I = \frac{E}{SA} = \frac{E}{4\pi r^2}$*, where* E *is the energy of the light. As this equation shows, the intensity of the light is directly proportional to the inverse square of the radius.*

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(Student)TI-Nspire File: *PhyAct02_light_intensity_EN.tns*