Percentiles & Z-scores

ID: 11205

Time required
40 minutes

Activity Overview

In this activity, students will explore percentiles and z-scores of normal distributions by graphing and calculating. After students are shown how to complete each part of the activity, they are given problems that they can do own their own.

Topic: Normal Distributions

- *Standardized normal curves*
- *Z-scores*
- *Percentiles*
- *Probabilities using normal distributions*

Teacher Preparation and Notes

- *This can be used as a stand alone lesson on z-scores, percentiles, and probabilities using normal distributions. Homework problems are included. The student worksheet is needed if students are to work individually. It can be omitted if the lesson is teacher directed.*
- *The student TI-Nspire document contains the questions from the worksheet and the problems to try. If the activity is teacher directed, it can be paper-free. If students are working individually they will need the student worksheet.*
- *Students can easily use Scratchpad in place of Graphs & Geometry and Calculator pages throughout the lesson.*
- *The activity can also be completed without the given student TI-Nspire document. Starting from a new TI-Nspire document, students can follow the directions on the worksheet, inserting a new problem for each part.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11205" in the quick search box.***

Associated Materials

- *PercentileZScores_Student.doc*
- *PercentileZScores.tns*
- *PercentileZScores_Soln.tns*

Suggested Related Activities

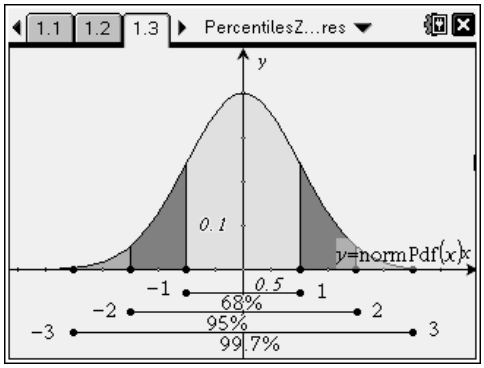
To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Areas In Intervals (TI-Nspire technology) — 9472*
- *Percentiles (TI-Nspire technology) — 9539*

Before beginning the activity, review with students that a **density curve** has the following properties:

- Smooth curve.
- Always on or above the horizontal axis.
- Has area exactly 1 underneath it.
- Area under the curve within a range of values is the proportion of all observations that fall in the range.

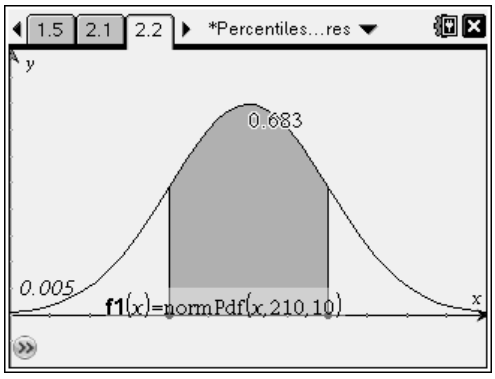
The top half of the first page of the student worksheet is meant as a review of the properties of normal curves along with a visual representation, also on page 1.2 of the tns file, of the corresponding percentages.



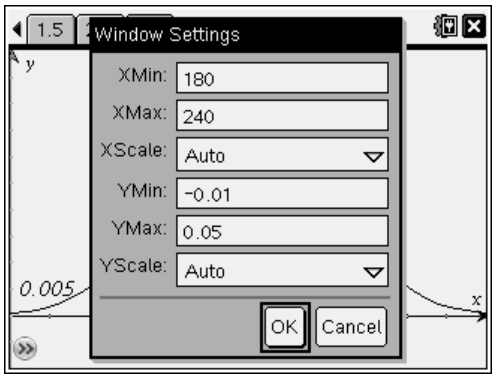
Students should know this diagram to allow them to make connections and see relationships. It then introduces *z*-scores and percentiles. It can be given to students as a resource or used in teacher instruction.

Part 1 – Given *x*-values, finding probabilities

Students are given a range of *x*-values and asked to find the probability (or percent) of data that fall within the interval. The worksheet provides step-by-step instructions to find the answer using a *Graphs* page and then using a *Calculator* page.



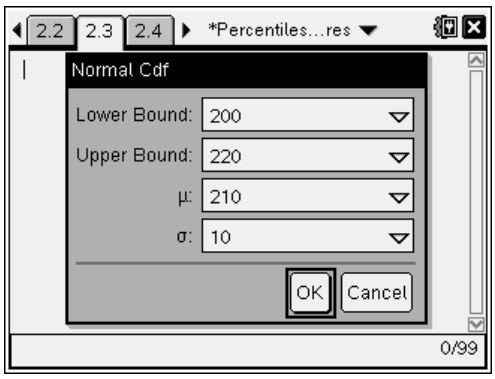
A graphical representation (solution) is shown. It is a normal distribution with a mean of 210 and a standard deviation of 10. This visual is helpful as it allows students to visualize a difficult topic for many.



The window is a challenge for students to set up. The *x*-values should be the mean + or – 3 standard deviations. The *y*-values need to be small. Encourage students to think about a rectangle with the base of *x*-length and an area of 1. This can become the starting point for the YMax value. Use a YMin that is negative and with a similar magnitude as the YMax.

Students will then use the **Integral** tool from the Analyze Graph menu to calculate the area under the curve. It may be important to give a short explanation as to why “integral” is used and not “area”. Students do not need to understand the calculus concepts.

A numerical solution is also presented. The values students are to enter are shown at the right.



This problem could have been done without a calculator if one realizes that the values are both 1 standard deviation from the mean.

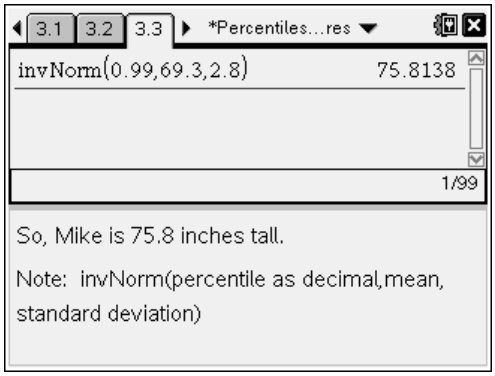
Discussion Questions:

- Is there a difference between the probability of $x > 200$ and $x \geq 200$? Why not? What is the probability $x = 200$?
- What should the endpoint be when only one is given? Can any value to chosen or will any "big" number suffice?
- Why is there not a 100th percentile? Why is there not a 0th percentile?

Part 2 – Given percentiles, finding *x*-values

This part begins with questions that should be answered using the diagram on the worksheet as a reference. The answers are to be written in terms of the mean and standard deviation. Answers are:

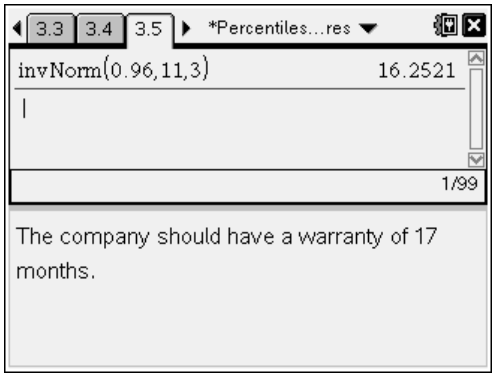
- Mean
- A value one standard deviation below the mean
- A value one standard deviation above the mean



Students are given a percentile and asked to find the corresponding *x*-value.

Try It! problems:

1. It is important to discuss the first problem. Drawing a diagram helps to visualize what is happening. Students want to use 4% as the percentage, but they need to recognize that the warranty is more than the average life expectancy, so the percentage is actually 96%. The answer should also be "rounded" up since that can afford to replace up to 4%. So, the answer is 17 months.



2. Again a diagram is helpful.
   As – top 8% means 92nd percentile.
   Bs – next 20% is 72nd – 92nd.
   Cs – next 42% is 30th – 72nd.
   Ds – next 18% is 12th – 30th.
   Fs – bottom 12% is 0 – 12th.

Part 3 – Given *z*-scores, finding percentiles and *x*-values

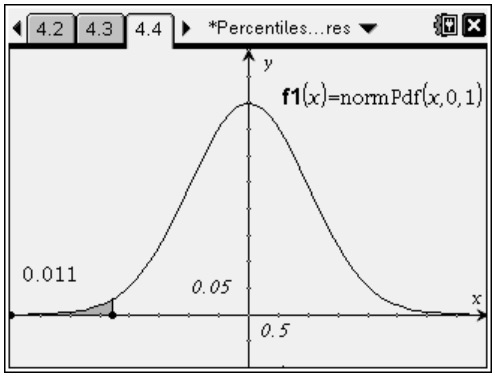
Students are given a *z*-score and asked to find the corresponding percentile. This begins with questions that should be answered using the diagram on page one.

Answers are: 50th, 0.15th ,98.75th

They also describe *x*-values with given z-scores in terms of mean and standard deviation.

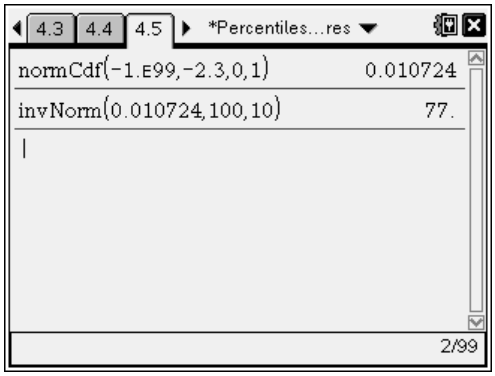
Answers are:

- mean
- 3 standard deviations below the mean
- 2 standard deviations above the mean



A *Calculator* page and *Graphs* page can be used to find corresponding percentiles and *x*-values when given *z*-scores. Students will find the percentile and *x*-value for a z-score = –2.3.

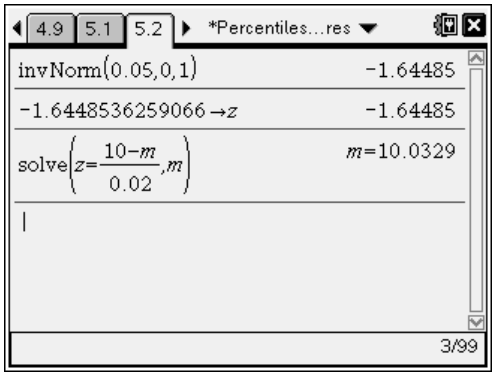
After students graph the normal curve and plot the point (–2.3, 0) explain to students that the area under the curve from the left to –2.3 is the corresponding percentile. This value should be found as before using the **Integral** tool. However, the left bound is the **xmin**.



Finding the percentile can be done without graphing. On a *Calculator* page students will need to type **normcdf(–1E99, –2.3, 0, 1)**.

Homework problems:

For these problems students have to find the *z*-score and then solve for *x* using the z-score formula.



1. This problem is not straight-forward and requires a student to think about what is being asked and make connections. A picture helps students visualize what is being asked. Begin by calculating the z-score for the 5th percentile and then use the formula $z = \frac{x - \mu}{\sigma}$ to find the mean. Note that for the **invNorm** command the mean is 0 and the standard deviation is 1. Discuss the reasonableness of the answer. This is very important in aiding understanding.
2. A picture will again help. A similar method is used as above. The solution is shown in the tns file.