

The Fruits of Your Labor - PLUMS and the Normal Distribution

Problems involving the normal distribution as a model are abundant throughout the NYS Math B, Precalculus, and AP Statistics curricula. This lesson is designed to introduce the students to the graphing calculator's capabilities in solving such problems using the built in `Solver` and `normalcdf` function.

Background Knowledge

Students are usually taught to use the Empirical Rule (68% - 95% - 99%) for the probabilities associated with the normal distribution when they are working without the use of tables.

Example: If the heights of the female students in Empirical High School are normally distributed with a mean of 63" and a standard deviation of 2.5", approximately what percent of the female student body is between 63" and 68" tall?

Answer: Since 68" is 5" above the mean (or 2 standard deviations), the associated probability should be half of 95%, or ~ 47.5%.

When the probability tables are introduced, students must learn to convert given raw values to z-scores in order to take advantage of the standard probabilities provided within the tables.

Example: If the GPA's of the male students in the Senior Class at Standard Normal High School are normally distributed with a mean of 2.90 and a standard deviation of 0.35, approximately what percent of the males in the Sr. Class have GPA's between 2.50 and 3.00?

Answer: Converting the given GPA's to z-scores results in $z_1 = -1.14$ and $z_2 = 0.28$, so the student must subtract the corresponding probabilities from the table. The difference of 0.6103 and 0.1271 yields the solution of 0.4832 or 48.32%.

The advantage to using the calculator's `normalcdf` function is that students do not need to convert to z-scores, subtract probabilities, or make use of the distribution's symmetry as is required to use some of the more condensed tables. Instead, students can simply input a lower bound (L), upper bound (U), mean (M), and standard deviation (S), and the function will return the associated probability (P). This function is found under the `DISTR` menu, option 2. Now refer back to the previous example on GPA's:

Answer:

```
normalcdf(2.50, 3
.00, 2.90, 0.35)
.4859024507
```

Note: Due to the rounding used in the previous solution (in both the z-scores and the table's values), the calculator's answer provides us with a higher degree of accuracy.

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The PLUMS Approach Within Solver

The calculator's built-in numeric solver allows the user to input an equation (in "0 =" form) involving several variables, assign known values to all but 1 of these variables, and numerically solve for the other. Solver can be found under the MATH menu, option 0:

```

MATH NUM CPX PRB EQUATION SOLVER
0: Frac          4: ∫(
1: Dec          5: *√
2: 3            6: fMin(
3: √(          7: fMax(
4: *√          8: nDeriv(
5: fMin(      9: fnInt(
6: fMax(     0: Solver...

```

The initial time that a student uses Solver the screen will look like the one above. However, if someone has previously used Solver, there will already be an equation there, and so students should then be instructed to arrow-up into the eqn: 0= line and hit CLEAR.

Next, we are going to type the PLUMS equation into Solver. Use the ALPHA key to access the alphabetic keys.

```

EQUATION SOLVER
eqn: 0=P-normalcdf
f(L,U,M,S)
                                Then hit ENTER.
P-normalcdf(L...=0
P=0
L=0
U=0
M=0
S=0
bound={ -1E99, 1...

```

Note: A student may have values other than 0 for their variables, depending on previous assignments.

In order to use PLUMS, students must know 4 of the 5 variables, enter them on the appropriate lines, move the cursor to the line of the unknown variable, and ALPHA SOLVE (ENTER) on that line. The calculator requires that the variable being solved for have a value entered as an initial guess and will not solve if left blank. Referring back to the previous GPA example:

| | | |
|---|---|---|
| <pre> P-normalcdf(L...=0 P=0 L=2.5 U=3 M=2.9 S=.35 bound={ -1E99, 1... </pre> | <pre> P-normalcdf(L...=0 P=█ L=2.5 U=3 M=2.9 S=.35 bound={ -1E99, 1... </pre> | <pre> P-normalcdf(L...=0 P=.48590245074... L=2.5 U=3 M=2.9 S=.35 bound={ -1E99, 1... left-rt=0 </pre> |
| Enter known values ... | Move cursor to unknown ... | ALPHA SOLVE to get answer! |

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The beauty of this approach is that answers can be found for any variety of normal probability problems, even those that would normally require solving a system of equations when both the mean and standard deviation are missing, assuming that the question is posed in a multiple choice format like those on Section I of the AP Statistics Exam. The multiple choice format allows the students to use a "guess-and-check" method to determine which choice is the correct one.

Example: If 75% of all commuters spend more than \$75 per week for gas, while 15% spend more than \$150, what is the mean weekly gas expense and corresponding standard deviation?

- (A) $\mu = 83.33, \sigma = 12.44$
- (B) $\mu = 56.26, \sigma = 11.85$
- (C) $\mu = 118.52, \sigma = 56.26$
- (D) $\mu = 104.39, \sigma = 43.86$
- (E) $\mu = 139.45, \sigma = 83.33$

Answer: Usually students would find the associated z-scores, create 2 equations involving μ and σ , then solve them simultaneously. Now with PLUMS, a student can input the given information for the first set of P, L, and U values, along with the M from one of the choices, and solve for S. If the S value is consistent with the multiple choice selection, the student should then also verify that the same result is obtained from the second set of values for P, L, and U.

```
P-normalcdf(L...=0
P=.75
L=75
U=1E99
M=0
S=0
bound={-1E99,1...
left-rt=0
```

Input known P, L, & U values...

(Note: $U = +\infty$ is 1EE99)

```
P-normalcdf(L...=0
P=.75
L=75
U=1E99
M=83.33
S=12.3500806104
bound={-1E99,1...
left-rt=0
```

Use M from (A), solve for S,

& since S is close to (A)'s value ...

```
P-normalcdf(L...=0
P=.15
L=150
U=1E99
M=83.33
S=64.326367053...
bound={-1E99,1...
left-rt=0
```

Use other P, L, & U values

with same M. The S changed so (A) is the incorrect choice.

Repeat this process until correct answer of choice (D) is found.

```
P-normalcdf(L...=0
P=.75
L=75
U=1E99
M=104.39
S=43.573693774...
bound={-1E99,1...
left-rt=0
```

Note: Although the s values are not exact and have changed slightly, this choice (D) is still the best available.

```
P-normalcdf(L...=0
P=.15
L=150
U=1E99
M=104.39
S=44.006683685...
bound={-1E99,1...
left-rt=0
```

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For students who would benefit from seeing the graphs of the normal distribution in addition to the numeric solutions from Solver, the calculator's ShadeNorm(function will produce the appropriate graph and shade the region which represents the corresponding probability. If the user goes directly from Solver to WINDOW, they can use the values currently stored in the "L U M S" variables to set-up an appropriate viewing window, as well as define the shaded region on the home screen.

Using the GPA example:

```

WINDOW
Xmin=M-4S
Xmax=4
Xscl=1
Ymin=-.5
Ymax=1.5
Yscl=1
Xres=1
    
```

Set Xmin to 4S below M ...

```

WINDOW
Xmin=1.5
Xmax=4
Xscl=1
Ymin=-.5
Ymax=1.5
Yscl=1
Xres=1
    
```

... and ...

```

WINDOW
Xmin=1.5
Xmax=M+4S
Xscl=1
Ymin=-.5
Ymax=1.5
Yscl=1
Xres=1
    
```

Set Xmax to 4S above M ...

```

WINDOW
Xmin=1.5
Xmax=4.5
Xscl=1
Ymin=-.5
Ymax=1.5
Yscl=1
Xres=1
    
```

Use Ymin=-.5 and Ymax=1.5 values.

```

2nd VARS DRAW
1: normalpdf(
2: normalcdf(
3: invNorm(
4: tpdf(
5: tcdf(
6: X²pdf(
7: X²cdf(
    
```

Go to 2nd VARS (DISTR) ...

```

DISTR DRAW
1: ShadeNorm(
2: Shade_t(
3: ShadeX²(
4: ShadeF(
    
```

Right arrow to DRAW ...

```

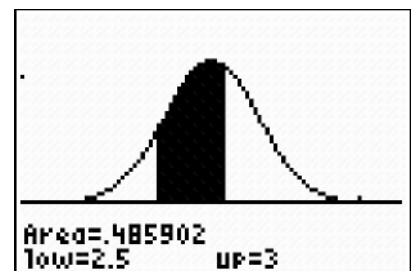
ShadeNorm(
    
```

Select option 1 ...

```

ShadeNorm(L, U, M, S)
    
```

Input the "L U M S" variables ...



Hit ENTER to go to graph.

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