



### Things to Remember...

This model is similar to a hot air balloon and provides a way to visualize adding and subtracting integers.

- *Positive* integers are represented by helium bags. They *raise* the balloon. *Negative* integers are represented by sand bags. They *lower* the balloon.
- Addition is the operation of *putting bags on* the balloon. Subtraction is the operation of *taking bags off* of the balloon.

Always reset the balloon's vertical position to 0 before beginning another calculation.

### Problem 1 – Integer addition

Refer to pages 1.2 and 1.3

To find the sum  $2 + (-6)$ , understand what the sum represents in terms of the balloon. The 2 signifies adding two helium bags to the balloon, which raises it 2 units. Adding  $-6$  means you then add six sandbags, which lowers the balloon 6 units. The balloon's resulting position is the sum.

To view this addition  $a + b$  using the model:

- Reset the balloon at ground level by entering **0** into cells A1 and A2 of the spreadsheet.
- Move the balloon to the position of the first addend—in this case, enter **2** into cell A1. Watch the balloon rise 2 units.
- Enter **-6** for the second addend into cell A2 and watch the balloon fall 6 units.

The balloon is now positioned at  $-4$ , which is the sum  $2 + (-6)$ .

With your partner, translate the following expressions into “balloon language” and use the model to find the sum.

1.  $-4 + 7 = \underline{\quad}$       2.  $7 + 3 = \underline{\quad}$       3.  $5 + (-7) = \underline{\quad}$       4.  $-5 + (-3) = \underline{\quad}$

### Problem 2 – Missing addend

Refer to pages 2.1 and 2.2

In this model, you are given the value of  $a$  and the value of the sum  $a + b$ , but the value of  $b$  is unknown.

Suppose you want to find the value of  $b$  such that  $-4 + b = 3$ .

First think about what this means in terms of the balloon. You know that the balloon ends up at 3, and you added 4 sand bags. How many and which type of bag do you need to add to have a resulting position of 3?



To find  $b$  using the model,

- Reset the balloons' vertical position by entering 0 for cells A1, A3, and A4.
- Set the balloon on the left to the value of the sum (in cell A1); in this case, the sum is 3.
- Enter the value of  $a$  (in cell A3) to move the right balloon to the position of the known addend.
- The task is now to find the value of  $b$  that is needed to move the balloon on the right to the same position as the target balloon on the left. Enter values for  $b$  (into cell A4) until the balloons line up. For this example,  $b = 7$ .

With your partner, translate the following equations into "balloon language" and use the model to find the missing addend.

1.  $2 + b = -3$   
 $b = \underline{\quad}$

2.  $-6 + b = -1$   
 $b = \underline{\quad}$

3.  $5 + b = 1$   
 $b = \underline{\quad}$

4.  $-2 + b = 4$   
 $b = \underline{\quad}$

### Problem 3 – Integer subtraction

Refer to pages 3.1 and 3.2

The model also provides a way of visualizing the subtraction of integers. As with addition, positive integers are represented by helium bags and negative integers by sand bags. However, subtraction is the operation of *taking off* a bag. For example, the expression  $-2 - 5$  translates to "put on 2 sand bags and then take off 5 helium bags."

Use the model in the same manner as in Problem 1.

- Reset the balloon at ground level by entering **0** into cells A1 and A2.
- Enter the minuend, **-2**, into cell A1; the balloon falls to  $-2$ .
- Enter the subtrahend, **5**, into cell A2.

Watch as the balloon falls another 5 units to end at  $-7$ , which is the difference  $-2 - 5$ .

With your partner, translate the following expressions into "balloon language" and use the model to find the difference.

1.  $2 - 7 = \underline{\quad}$

2.  $-3 - 1 = \underline{\quad}$

3.  $5 - (-2) = \underline{\quad}$

4.  $-4 - (-7) = \underline{\quad}$



## Problem 4 – Missing subtrahend

Refer to pages 4.1 and 4.2

This model shows two balloons side by side—like the model from Problem 2, except that it is used to find a missing *subtrahend* rather than a missing addend.

For example, find the value of  $b$  such that  $-3 - b = 8$ . In terms of the balloon, this means that the balloon ends up at 8, and you have used 3 sand bags. You need to find how many and which type of bag you must *remove* to have a resulting position of 8.

Use the model as in the same manner as in Problem 2:

- Enter the difference, **8**, into cell A1 to set the left balloon at the target position
- Enter the minuend, **-3**, into cell A3 and adjust the value of  $b$ , in cell A4, as needed to align the right balloon with the left.

For this example, you should find that  $b = -11$ .

With your partner, translate the following equations into “balloon language” and use the model to find the missing subtrahend.

1. $6 - b = 9$	2. $5 - b = -3$	3. $-4 - b = -1$	4. $-2 - b = 6$
$b = \underline{\quad}$	$b = \underline{\quad}$	$b = \underline{\quad}$	$b = \underline{\quad}$

## Problem 5 – Addition and subtraction exploration

Refer to page 5.1

In this model, the balloon on the left is for subtraction and the one on the right is for addition. For each of the following expressions, use what you learned from Problems 1 and 3 to translate into “balloon language” and then find each sum or difference.

1. $-2 - 4 = \underline{\quad}$	2. $-2 + (-4) = \underline{\quad}$	3. $5 - (-6) = \underline{\quad}$	4. $5 + 6 = \underline{\quad}$
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For each of the following equations, use what you’ve learned from Problems 2 and 4 to translate into “balloon language” and then find each missing addend or subtrahend.

5. $-1 - b = 5$	6. $-1 + b = 5$	7. $3 - b = -4$	8. $3 + b = -4$
$b = \underline{\quad}$	$b = \underline{\quad}$	$b = \underline{\quad}$	$b = \underline{\quad}$

Complete the following statements.

9. Taking off 8 sand bags is the same as putting on 8 \_\_\_\_\_ bags.
10. Taking off 5 helium bags is the same as putting on 5 \_\_\_\_\_ bags.
11. If  $a$  and  $b$  are any two integers, then  $a - b = a + \underline{\quad}$ . That is, subtracting a number is equivalent to adding its \_\_\_\_\_.