EXPLORATIONS

Activity 9

Hey, That's Not Fair! (Or is it?)

Students will use the calculator to simulate dice rolls to play two different games. They will decide if the games give each player an equally likely chance of winning.

Concept

- Probability and statistics
- Number sense

Skills

- Determining fair and unfair games
- Collecting and organizing data
- Predicting possible outcomes
- Examining sums and products of even and odd numbers
- Calculator skills: MATH, rand (random number generator)

Materials

- Student Activity sheets (page 46)
- TI-73 calculator for each group of 2 students

Activity

***Tip**: You may want to do this activity with real dice first to get students acquainted with probability and dice rolls.*

Ask students:

- *What does "fair" mean?*
- *Have you ever played a game that you felt wasn't fair?*
- *What made it unfair?*

Tell them the definition of fair and unfair games:

- When everyone playing a game has an equally likely chance of winning, it is said to be a *fair* game. (For example, if 5 people play, each has a $\frac{1}{5}$ or 20% chance of winning.)
- If someone has a clear advantage to win due to the rules of the game, it is an *unfair* game. (For example, if 2 people play, one has a greater than 50% chance of winning.)

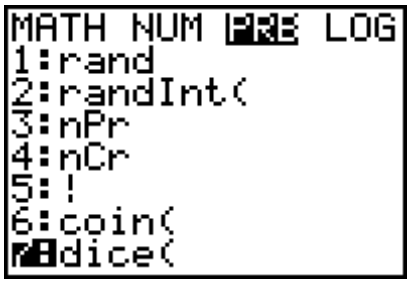
Have the students pair up in groups of two. One player will be player A and one will be player B. (They will decide.) They will be playing two different games involving dice rolls on the calculator. They will need one calculator and a Student Activity sheet to record the rolls.

***Note**: If you have never used the calculators for any random numbers before this activity, seed the random sets in each calculator before you begin by selecting a different number for each calculator and storing it to **rand**. Otherwise, all the calculators will select the same numbers in the same sequence.*

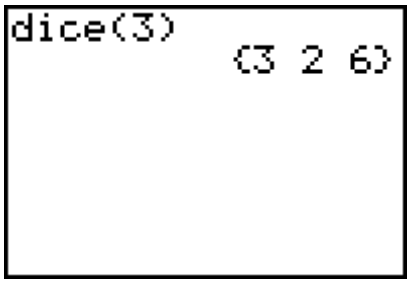
To seed a calculator:

1. Press any number, then press [STO▸]. (Each calculator should have a different number.)
2. Press [MATH], press [▸] to move to **PRB**, select **1:rand**, and press [ENTER]. (**1:rand** is the default selection.)

Using a ViewScreen unit, show students how to set up the calculator to roll dice.



1. Make sure you are in the Home screen. (If necessary, press [2nd] [QUIT] and [CLEAR] to get a blank screen.)
2. Press [MATH] and press [▸] to move over to **PRB**.
3. Press [7] to choose **dice**. This will paste **dice(** on the Home screen.
4. Since the games require rolling 3 dice, press [3] [)] and [ENTER]. Three numbers will appear in brackets. These are the three numbers from the first simulated dice roll. Continue pressing [ENTER] for each roll.



Game 1:

Use the calculator to roll 3 dice at once. If the sum of the three numbers is odd, player **A** gets a point, if the sum is even, player **B** gets a point. Do 20 "tosses" on the calculator. The winner is the player with the most points at the end. Students should record their points on the Student Activity sheet.

To verify the validity of games, have students write the 3 numbers and their sum in the appropriate cell. For example, write **3+2+6=11** in the **Player A** column.

Game 2:

Use the calculator to roll all 3 dice at once. Multiply all three numbers. If the answer is odd , player A scores a point and if it is even, player B scores a point. Do 20 "tosses" on the calculator. The winner is the player with the most points in the end. Students should record their points on the Student Activity sheet.

Ask the groups:

- *Who won game 1, player A or B? (Write the class results on the board in form of a T-chart.)*
- *Do you think Game 1 is fair or unfair? Does each player have an equally likely chance of winning?*
- *Why or why not?*
- *How can we determine if this is a fair game? (By writing out all the possible outcomes or doing a probability tree.)*
- *Who won game 2, player A or B? (Write the class results on the board.)*
- *Do you think Game 2 is fair or unfair? Why or why not?*
- *How can we determine if this is a fair game? (By writing out all the possible outcomes.)*
- *How can we list all the possible outcomes? Should we make a chart? Should we use a particular order?*
- *What elements should we include in our chart?*

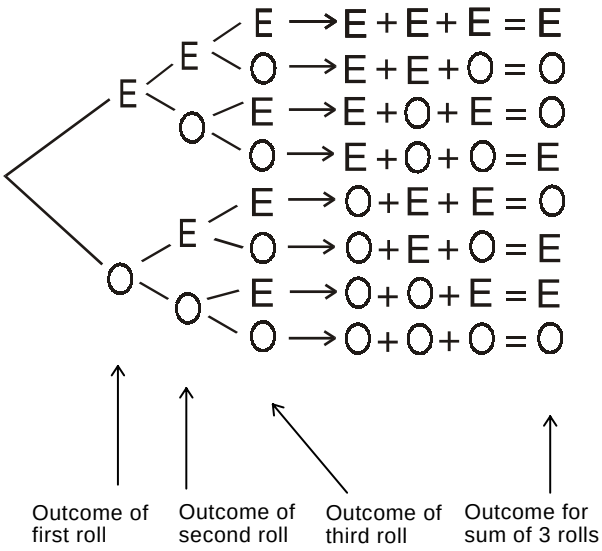
One way to analyze the game to determine fairness is to list all of the possible outcomes. Then you can find the fraction of outcomes that are odd and even. This fraction then is the *probability* of Player A or Player B winning.

You may want to guide the class through the process of writing out all the possible outcomes for Game 1. To list all the possible outcomes, encourage them to use an organized list beginning with a roll of all 1's, and so on. All outcomes need to be considered, for example: 1 1 2, 1 2 1, and 2 1 1 are three different rolls.

Rolls	Sum	Odd/Even
1 1 1	3	Odd
1 1 2	4	Even
1 1 3	5	Odd
1 1 4	6	Even
1 1 5	7	Odd
1 2 2	5	Odd
…….		

Students will soon see that there are a lot of outcomes (216 to be exact) and listing them all would be difficult. (If you would like to have all outcomes, divide the task among several groups of students.)

Another way to analyze the game is to think of the individual rolls of the die and whether they are odd or even. Then, look at the sum or product of 3 rolls. An efficient method for organizing the information is a tree diagram. For example:



Die 1	Die 2	Die 3	Sum	Product
E	E	E	even	even
E	E	O	odd	even
E	O	E	odd	even
E	O	O	even	even
O	E	E	odd	even
O	E	O	even	even
O	O	E	even	even
O	O	O	odd	odd

The **E**'s represent *even* and the **O**'s represent *odd*. For example, **E + E + E** gives an even sum. So based on the tree diagram (or the accompanying table), $\frac{4}{8}$ of the possible outcomes are even and $\frac{4}{8}$ are odd, making Game 1 a fair game.

A discussion of how students know whether or not the sums are odd or even can be very interesting as students provide their convincing arguments.

Wrap-Up

After students have listed the outcomes for each game, ask:

- *How many possible ways are there for you to get an even sum in game 1? (108 if you list all outcomes, 4 if you use a tree diagram.)*
- *What about an odd sum? (108 if you list all outcomes, 4 if you use a tree diagram.)*
- *Do you think Game 1 is fair? (Yes.)*
- *Why or why not? (Same chance of getting an even or odd sum.)*
- *How many possible ways are there to get an even answer in game 2? (189 if you list all outcomes, 7 if you use a tree diagram.)*
- *Odd answer? (27 if you list all the outcomes, 1 if you use a tree diagram.)*
- *Is game 2 fair? (No.)*
- *Why or why not? (There is a clear advantage for the player receiving points for even outcomes.)*

Assessment

In a math journal, have students define *equally likely outcomes*. Have them give names of games they have played that were fair and unfair and explain the fairness based on probability.

Extension

- Have students create two of their own number games, one fair and one unfair. Encourage them to think of other ways to categorize numbers other than odd or even (for example, divisible by 5, divisible by 10, prime, greater than 6, and so forth). Have them present their games to the class and have classmates play their games to determine the fairness.

Student Activity

Name _____

Date _____

Activity 9

Hey, That's Not Fair

*Use the calculator to roll 3 dice at once. For Game 1, if the sum of the three numbers is odd, player **A** gets a point, if the sum is even, player **B** gets a point. For Game 2, if the product is odd, player **A** gets a point, and if it is even, player **B** gets a point. Do 20 "tosses" on the calculator and record each player's points. The winner is the player with the most points at the end.*

	Game 1		Game 2	
Toss	Player A	Player B	Player A	Player B
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
Totals				

Tree diagram for Game 1:

Die 1	Die 2	Die 3	Sum

Tree diagram for Game 2:

Die 1	Die 2	Die 3	Sum