Math Objectives

- Students will recognize the types of quadrilaterals formed when opposite sides are congruent.
- Students will recognize that there are special quadrilaterals in which consecutive angles are supplementary and opposite angles are congruent.
- Students will recognize that for some special quadrilaterals, opposite sides are parallel.
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

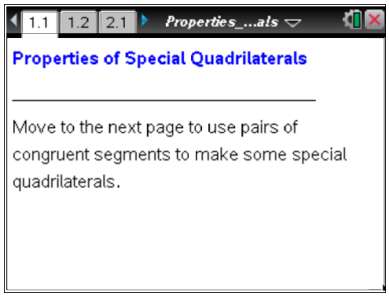
- parallelogram
- square
- rhombus
- rectangle

About the Lesson

- This lesson involves properties of parallelograms. Students manipulate endpoints of a set of segments or manipulate angles formed by two segments.
- As a result students will:
  - Make a quadrilateral (parallelogram) when opposite sides are congruent.
  - Investigate a given quadrilateral (parallelogram) to observe that consecutive angles are supplementary and opposite angles are congruent.
  - Investigate a given quadrilateral formed by parallel lines.
  - Visualize different types of quadrilaterals.
  - Infer about the properties of different special quadrilaterals.
  - Describe consequences of all segments being congruent.

TI-Nspire™ Navigator™ System

- Use Class Capture to assess students' control of the TI-Nspire Navigator System.
- Use Live Presenter to demonstrate proper procedures.
- Use Quick Poll to check students' answers.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point
- Move from one application to another
- Click on the ∧ and ∨ icons of a slider

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can show the entry line by pressing ctrl G.

Lesson Files:

Student Activity
Properties_of_Special_Quadrilaterals_Student.pdf
Properties_of_Special_Quadrilaterals_Student.doc

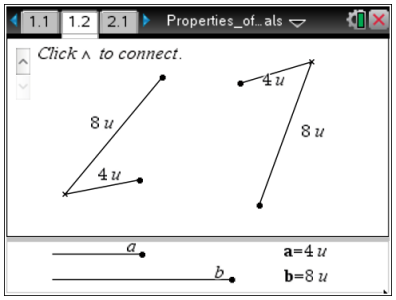
TI-Nspire document
Properties_of_Special_Quadrilaterals.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (☜). Press ctrl ☜ to grab the point and close the hand (✊).

Move to page 1.2.



1. Drag point *a* in the bottom portion of the screen to change the value of **a,** and describe what happens in the top portion of the screen.

   Answer: When the value of ***a*** is changed, this changes the lengths of two of the segments at the top.

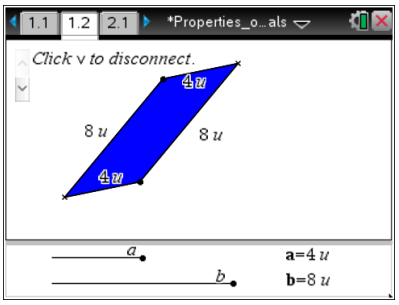
2. Drag point *b* in the bottom portion of the screen to change the value of **b,** and describe what happens in the top portion of the screen.

   Answer: When the value of ***b*** is changed, this changes the lengths of two of the segments at the top.

   Teacher Tip: The above two questions are certainly not math related but are included so students see that what they do with the segments in the bottom portion of the screen controls what is happening in the top portion of the screen.

3. a. Move the segments to form a quadrilateral by following the steps.
   - Pairs of segments can be moved by dragging the x found at their point of intersection (common endpoint).
   - The angle between the connecting segments can be changed by dragging an endpoint that looks like this: •.
   - Final placement of segments can only be end-to-end.
   - Move the segments and change the angle to get as close as you can to form a quadrilateral. Then press ∧ to connect.
   - The goal is to have all four segments connected to one another to form a closed figure.

b. Sketch your results on the screen at the right.



Sample Answer: (to the right)

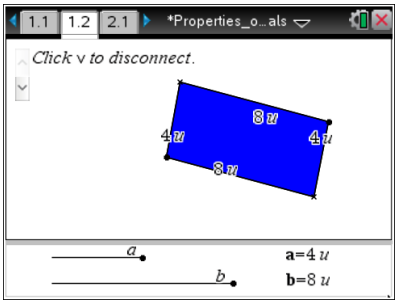
Teacher Tip: The premise of this investigation is that there are two pairs of congruent segments to use when forming the quadrilateral. This file is designed to build only parallelograms and will not connect the segments with congruent adjacent sides to form kites. A segment with length **a** will not attach to another segment of length **a**.

TI-Nspire Navigator Opportunity

Use *Class Capture* to check to see that students are having success moving and connecting the segments.

Make someone the *Live Presenter* to demonstrate how to move and connect the segments if students are having difficulty creating a quadrilateral.

4. a. Use the point of intersection x to drag and change the quadrilateral. Sketch your new quadrilateral.



Sample Answer: (to the right)

b. Compare and contrast your quadrilaterals in questions 3 and 4.

Sample Answer:

Similarities	Differences
Opposite sides in both quadrilaterals are congruent.	Quadrilaterals have different shapes.

Teacher Tip: The quadrilateral "disappears" when the distance between the two connection points exceeds the possible length of a diagonal for the selected lengths of the segments (**a** and **b**) forming the sides. This is due to the triangle inequality theorem and might deserve more discussion.

Teacher Tip: Check to see that students have explored a variety of parallelograms including rectangles, rhombi, and squares. Be sure they consider characteristics such as size, shape, and orientation when comparing figures. Rhombi and squares are possibilities if students began with **a** = **b**. Depending on your textbook definition of *kite,* you might also explore those, although they are not being emphasized here.
Students may also begin to make observations concerning relationships between sides or angles (congruent, supplementary, parallel). Encourage students to work in pairs or groups and compare quadrilaterals.

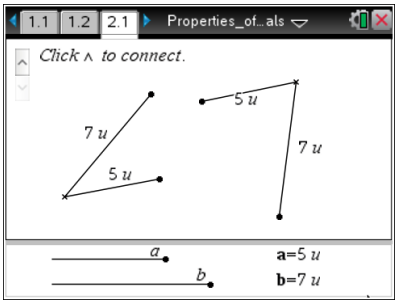
TI-Nspire Navigator Opportunity

Use *Class Capture* to examine a variety of parallelograms including rectangles, rhombi, and squares. You might have certain groups of students make each of the figures so that all students can view through *Class Capture*.

5. In questions 1–4, you determined the lengths of the pairs of segments and then made a quadrilateral. Press ctrl tab to move to the bottom portion of the screen. Change the values of **a** and **b,** and observe the changes in the quadrilateral. Describe what happens to the quadrilateral as the values of **a** and **b** are changed.

   Answer: The quadrilateral gets larger or smaller. Sometimes it disappears.

Move to page 2.1.



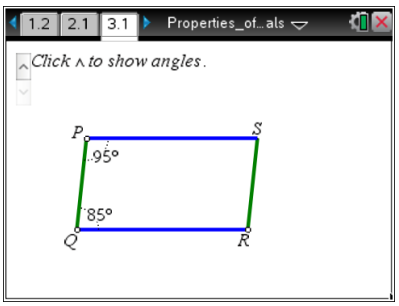
6. Change the lengths in the bottom portion of the screen so that **a** = **b**. Press ctrl tab to move to the top of the screen and move the segments to form a quadrilateral. Sketch your results. What types of quadrilaterals can be made when all four segments are equal?

   Answer: squares and rhombi

TI-Nspire Navigator Opportunity

Have students answer this question through an *Open Response Quick Poll*. Use this opportunity to discuss whether the quadrilaterals that are formed are ever also considered to be parallelograms (always) and rectangles (sometimes).

Move to page 3.1.



7. A special quadrilateral has been constructed that includes angle measurements.
   a. Drag a vertex other than *S* and record angle measurements in the chart at the right. Make a conjecture about the relationship of consecutive angles ∠*P* and ∠*Q*.

   Sample Answer: Answers in the chart will vary. The sum of the measures of the angles is 180°. (It is possible for both angles to be 90°.)

∠*P*	∠*Q*
95°	85°
109°	71°
47°	133°
70°	110°

Teacher Tip: Students may change the quadrilateral by moving points *P*, *Q*, or *R*. It is not possible to move point *S*.

   b. To display the measurements of the other two angles, press ∧ on the screen. Then drag a vertex. Make a conjecture about the measures of opposite angles ∠*P* and ∠*R*.

   Answer: Opposite angles have the same measure. The angles are congruent.

Teacher Tip: As a result of page 3.1 and question 7, students should recognize that in this special quadrilateral the adjacent angles are supplementary and opposite angles are congruent.

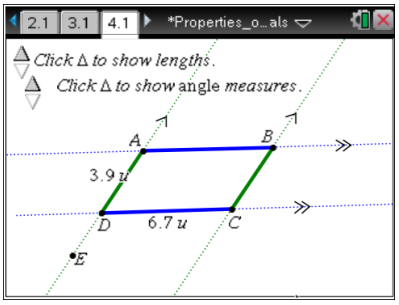
TI-Nspire Navigator Opportunity

Send students an *Open Response Quick Poll:*

In quadrilateral *PQRS*, if m∠*S* = 45, what is m∠*Q* and m∠*P*?

Answer: m∠*Q* = 45 and m∠ *P* = 135.

Move to page 4.1.



8. Find a quadrilateral that has been constructed using the intersection points of two pairs of parallel lines. Drag vertex *A*, *B*, or *C*. Observe the lengths of the sides.
   a. What seems to be true about opposite sides of quadrilateral *ABCD*? To display the measurements of the other two sides, press ∧ on the screen. Make a conjecture about opposite sides.

      Answer: A figure that has four sides so that the two opposite sides are parallel and the other two sides are parallel to each other is a parallelogram.

   b. Press ∧ on the screen to display angle measurements. Explain why consecutive angles ∠*BAD* and ∠*ADC* are supplementary.

      Sample Answer: ∠*BAD* ≅ ∠*CDE*: Line *AB* is parallel to line *DC* with transversal *AD*. If two parallel lines are cut by a transversal, then corresponding angles are congruent. ∠*CDE* and ∠*CDA* are supplementary because they form a linear pair. Therefore, ∠*BAD* and ∠*CDA* are supplementary by substitution.

      Teacher Tip: Students may change the quadrilateral by moving points *A*, *B*, or *C*. Be sure to remind students that a picture is not a proof.

   c. Explain why opposite angles ∠*BAD* and ∠*DCB* are congruent.

      Sample Answer: ∠*BAD* ≅ ∠*CDE*: Line *AB* is parallel to line *DC* with transversal *AD*. If two parallel lines are cut by a transversal, then corresponding angles are congruent.

      ∠*CDE* ≅ ∠*DCB*: Line *DA* is parallel to line *CD* with transversal *DC*. If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

      ∠*BAD* ≅ ∠*DCB*: By substitution.

9. The quadrilaterals you have been exploring on all the pages of this activity are parallelograms.

   a. Why do you think they are called parallelograms?

      Answer: They are called parallelograms because opposite sides are parallel.

   b. Renata says that in a parallelogram, the opposite sides are always parallel and congruent. Jerome says that in a parallelogram, each pair of consecutive angles is supplementary and opposite angles are congruent. Who is correct? Renata? Jerome? Both? Neither? Explain your reasoning.

      Answer: Both Renata and Jerome are correct. In a parallelogram, opposite sides are parallel and congruent. In addition, consecutive angles are supplementary and opposite angles are congruent.

TI-Nspire Navigator Opportunity

Send students a *Multiple Choice Quick Poll* to answer question 9b where Renata is choice A, Jerome is choice B, Both is choice C, and Neither is choice D.

Teacher Tip: Be sure that students recognize and articulate these properties of parallelograms. You might want to have them work in pairs or groups as they consider examples, and also some counterexamples.

Teacher Tip: Be sure that students see the difference between questions 7 and 8. In question 7, students are given angles but nothing about the lines forming them. In question 8, students are given parallel lines first, and then the congruent sides and angles result from that.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- Opposite sides of a parallelogram are congruent.
- Opposite sides of a parallelogram are parallel.
- Four sets of consecutive angles of a parallelogram are supplementary.
- Two sets of opposite angles of a parallelogram are congruent.

Extension

Teachers may want to have students formally state theorems discovered and perhaps prove them as well. For example, "If a quadrilateral has opposite sides that are congruent, then the quadrilateral is a parallelogram." Or, "If a quadrilateral has consecutive angles that are supplementary, then the quadrilateral is a parallelogram." Or, "If a quadrilateral is a parallelogram, then opposite sides are congruent, opposite angles are congruent, and adjacent angles are supplementary."

Teachers may make a counterpoint to help students see how special parallelograms are by comparing them to kites or trapezoids, the other two types of quadrilaterals. Using non-examples is a good way to solidify a concept. For example, "What type of figure would be formed if the two pairs of congruent sides were adjacent rather than opposite?" Or, "What type of figure would be formed if only one pair of opposite sides is parallel?"