Solutions to Differential Equations – ID: 8998

By Steve Ouellette

Time required
45 minutes

Activity Overview

*In this activity, students will be introduced to solutions of differential equations. Students will use the **deSolve(** command to find a general solution to a differential equation. This general solution will be investigated graphically and verified analytically. Students will then look at some particular solutions to this same differential equation, investigate them graphically, and verify them analytically.*

Concepts

- *differential equations*
- *general and particular solutions*

Teacher Preparation

This investigation offers an opportunity to introduce the concept of solutions, both general and particular, to differential equations.

- *Prior knowledge of solutions to differential equations is not required for this activity.*
- *Students should have some familiarity with antidifferentiation. In particular, they should know that the antiderivative of a differential equation results in a family of solutions.*
- *The screenshots on pages 2 and 3 demonstrate expected student results. Refer to the screenshots on page 4 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter "8998" in the quick search box.***

Classroom Management

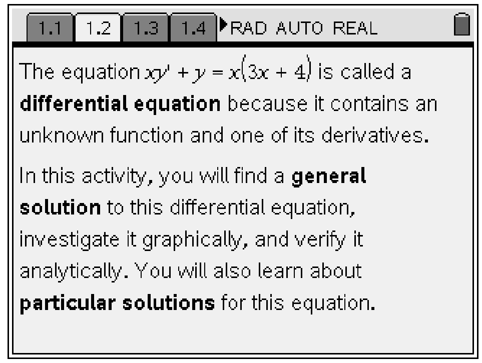
- *This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. The student worksheet is intended to guide students through the main ideas of the activity and provide a place to record their observations.*
- *For the most part, students will manipulate pre-made sketches, rather than constructing the diagrams themselves. Therefore, a basic working knowledge of the TI-Nspire handheld is sufficient.*
- *The example used in this activity was intentionally chosen because it cannot be solved by taking antiderivatives, nor by separation of variables. As an extension to this activity, you might want to have students construct a slope field for the differential equation $xy' + y = x(3x + 4)$ and use the slope field to draw some particular solutions.*
- *Provide students with additional practice verifying the general solutions to other differential equations. Have students also find some particular solutions.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the activity is completed successfully.*

TI-Nspire™ Applications

Calculator, Graphs & Geometry, Notes

Two focus questions define this activity:

- *How can you find and verify the general solution to a differential equation?*
- *How can you find and verify a particular solution to a differential equation?*

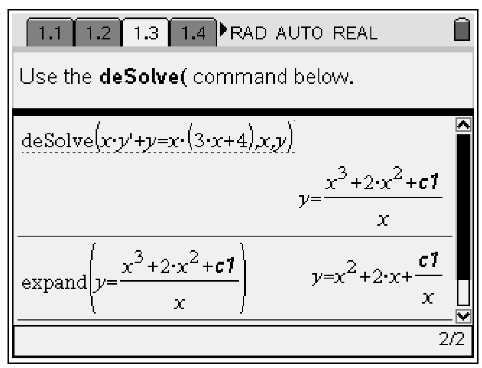


Use page 1.2 to explore the meaning of a "differential equation." Then use the equation $\frac{dy}{dx} = 2x$ as an example of a differential equation that students should be able solve by finding the antiderivative of 2*x*. This will provide a nice segue to the exploration of the differential equation $xy' + y = x(3x + 4)$, which cannot be solved as easily.

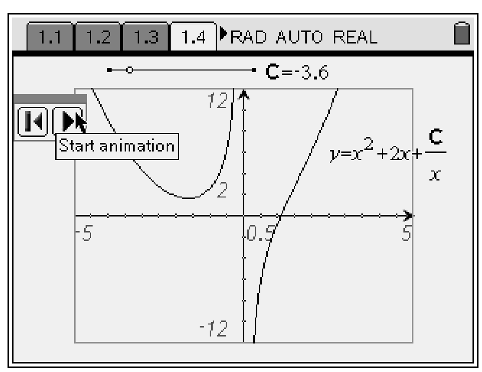
Be sure students realize that they will not learn how to find solutions to differential equations in this activity. Rather, they will investigate and verify a solution obtained using the **deSolve** command.

Problem 1 – Finding and verifying a general solution to $xy' + y = x(3x + 4)$

Students will first use the **deSolve** command to solve the differential equation $xy' + y = x(3x + 4)$. The result shown on the screen at right shows the general solution to this equation. The **Expand** command can also be used to write the general solution as three separate terms.



On page 1.4, students will view an animation that produces a series of curves that are part of the general solution to this differential equation. Students should be encouraged to pause the animation periodically to look at particular solutions. For example, the screen at right shows the particular solution $y = x^2 + 2x - \frac{3.6}{x}$.



For the differential equation

$$xy' + y = x(3x+4) \qquad \textbf{(1)}$$

students will verify that the general solution is

$$y = x^2 + 2x + \frac{C}{x} \qquad \textbf{(2)}$$

by performing the following steps.

- Differentiate **(2)** with respect to x to find y':

$$y' = 2x + 2 - \frac{C}{x^2} \qquad \textbf{(3)}$$

- Substitute y and y' from equations **(2)** and **(3)** in to equation **(1)** to transform the differential equation so it is written strictly in terms of x. Simplifying both sides of the equation to show they are equal verifies that the solution is correct.

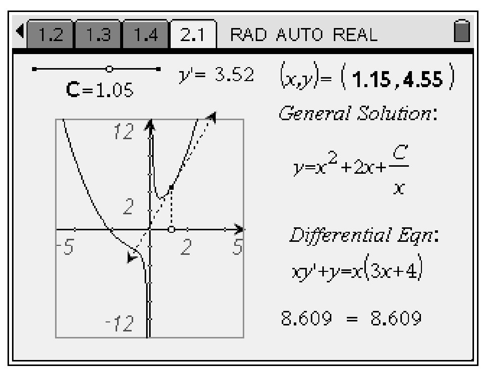
$$xy' + y = x(3x+4) \;\rightarrow\; x\left(2x + 2 - \frac{C}{x^2}\right) + \left(x^2 + 2x + \frac{C}{x}\right) = x(3x+4)$$

$$\left(2x^2 + 2x - \frac{C}{x}\right) + \left(x^2 + 2x + \frac{C}{x}\right) = 3x^2 + 4x$$

$$3x^2 + 4x = 3x^2 + 4x$$

Problem 2 – Investigating particular solutions to *xy′* + *y* = *x*(3*x* + 4)

Students will use page 2.1 to investigate particular solutions and verify that the particular solutions are, in fact, solutions to the differential equation.



The specific values of **C**, y', x, and y shown on the screen at right are used to obtain the true statement at the bottom of the screen, 8.609 = 8.609, in the following manner: (***Note***: Due to rounding, the values may vary slightly, as they do here.)

$$xy' + y = x(3x+4)$$

$$(1.15)(3.52) + (4.55) \approx (1.15)(3(1.15)+4)$$

$$8.598 \approx 8.5675$$

It should be emphasized that verifying this particular solution $\left(y = x^2 + 2x + \frac{1.05}{x}\right)$ for a specific (x, y) pair does not constitute a proof. Manipulating the tangent line provides additional evidence that this particular solution is in fact correct.

Next, students change the value of **C** by adjusting the slider. Make sure that students understand that this changes the particular solution but not the *x*-coordinate of the point of tangency. They should recognize that they can manipulate to position of the tangent line to obtain evidence that this new particular solution is also true.

In this step, students also gain some experience solving for *C*, given an ordered pair through which the solution passes. Students can solve for *C* as follows:

$$y = x^2 + 2x + \frac{C}{x}$$
$$10 = (2)^2 + 2(2) + \frac{C}{(2)}$$
$$\vdots$$
$$C = 4$$

Students do not need to verify this particular solution $y = x^2 + 2x + \frac{4}{x}$ to the differential equation because the general solution $y = x^2 + 2x + \frac{C}{x}$ has already been verified for any value of *C*, including *C* = 4.

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(Student)TI-Nspire File: *CalcAct15_DifferentialEqns_EN.tns*