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Time required: 30 minutes

Activity Overview

This lesson is intended to allow students to investigate the segment relationships when 2 chords in a circle intersect. Pages include a statement of the theorem, a dynamic geometry demonstration, several problems that apply the theorem, and a 2-column geometric proof of the theorem.

Teacher Preparation

This lesson is created for use in a middle school or high school geometry class.

- Similar triangles have corresponding sides that are proportionate.
- Segments of each of two intersecting chords will have equal products. This is the premise of this lesson.

Classroom Management

- This lesson is intended to allow students to investigate the chord-segment relationships using the TI-Nspire Geometry Application.
- Students need to read carefully due to the fact that sometimes they are given (or asked to find) segment measures, and at other times the lengths of the entire chord.
- The student worksheet contains additional diagrams that allow the student to work on each of the problems, as well as the geometric proof.

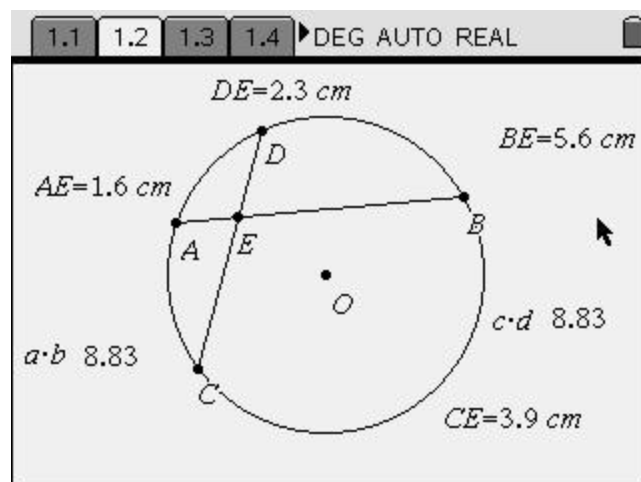
On page 1.1, the theorem is stated for the students. Most students will experience greater success working with these problems if they always deal with segment measures. In the case that they are given entire chord lengths, they should partition the length into segment measures, if possible.

1.1 1.2 1.3 1.4 ▶ DEG AUTO REAL

Segments Formed by Chords in a Circle

THEOREM: If two chords intersect within a circle, the product of the measures of the segments of one chord equals the product of the measures of the segments of the other.

On page 1.2, the Geometry Application allows the student to manipulate the measures of the segments by dragging the endpoints of the chords to different locations. As the measures of the segments change, so do the products, and the student is expected to be able to confirm that these products remain equal to each other.



The questions on pages 1.3-1.6 each require use of the theorem and correspond to the diagrams provided on the student worksheet.

Answers:

1.3 – $CE = 6$

1.4 – $CD = 10$

1.1 1.2 1.3 1.4 ▶ DEG AUTO REAL

Question

If $AE=2$, $EB=9$, and $ED=3$, find CE .

Answer ▼

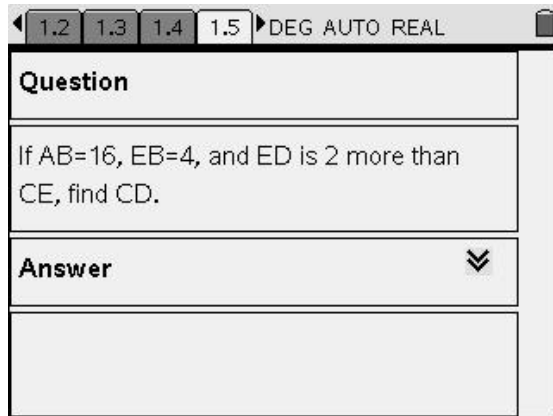
1.1 1.2 1.3 1.4 ▶ DEG AUTO REAL

Question

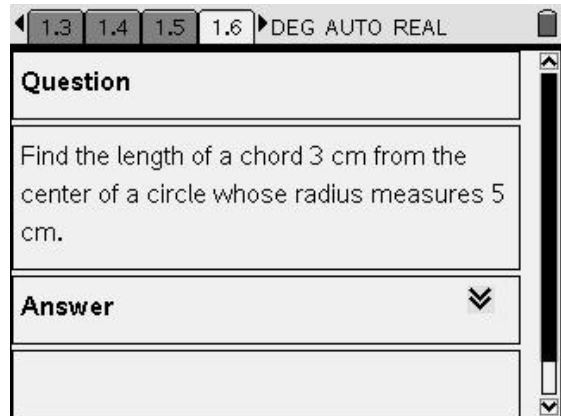
If $AB=14$, $EB=12$, and $ED=4$, find CD .

Answer ▼

1.5 – CD = 14



1.6 – the chord AB = 8



Page 1.7 instructs the student to complete the geometric proof that follows.

Page 1.8 illustrates a 6 step geometric proof of the theorem.

It is necessary to draw additional chords (BD & AC) in order to clearly identify the inscribed angles and triangles AEC & BED referred to in the proof.

The missing items are:

Reason #2 – Vertical angles are equal.

Reason #3 – Two angles inscribed in the same arc are equal.

Reason #4 – AA \cong AA.

Statement #5 – $\frac{AE}{DE} = \frac{CE}{BE}$.

Reason #6 – ... product of extremes.

Statements	Reasons
1. Circle O with chords AB and CD, that intersect at E.	1. Given
2. $m\angle AEC = m\angle BED$	2.
3. $m\angle BDC = m\angle CAB$	3.

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4. $\triangle AEC \sim \triangle BED$	4.
5.	5. Corresponding sides in similar triangles are proportionate.

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6. $AE \cdot BE = CE \cdot DE$	6. Product of means equals