EXPLORATIONS

Activity 14

The Closer I Get to You

Objectives

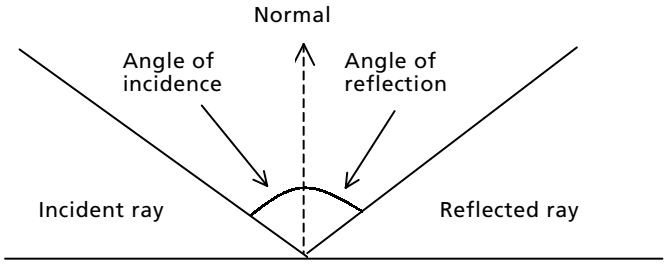
- To find a rational function
- To find the *x* value of a function, given the *y* value
- To find the *y* value of a function, given the *x* value
- To use technology to plot data

Materials

- TI-83 Plus
- Small mirror (one per group)
- Adding machine paper (at least 500 cm per group)
- Masking tape and markers
- Meter stick (one per group)

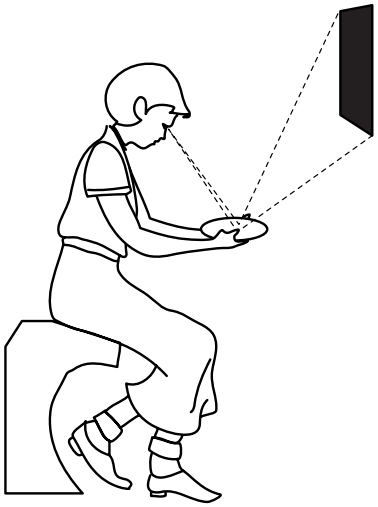
Introduction

If you drop a ball on the ground, it will return to you along the same path in which it was released. If you throw the ball so that it hits the floor at an angle, the ball will return in the opposite direction, but from the same angle with the floor. Light follows a similar pattern. When light rays strike a surface, the ray that strikes the surface is called the *incident ray*, and the ray that is reflected is called the *reflected ray*. The law that describes this phenomenon is called the *Law of Reflection*. This law states that the angle of incidence is equal to the angle of reflection. A line drawn forms the two angles perpendicular to the reflecting surface. (See diagram below.)



To explore the *Law of Reflection*, hold a mirror in the palm of your hand parallel to the ground and at a level that allows you to look into the mirror (see diagram on the next page). Find an object or classmate that is above your eye level (for example, a poster on the wall, a word on the blackboard, or a person in the room). Walk away from the object and observe what happens as you look in the

mirror. Walk towards the object and observe what happens as you look in the mirror. Your discovery might lead you to an interesting observation that can be used to find the height of an object.



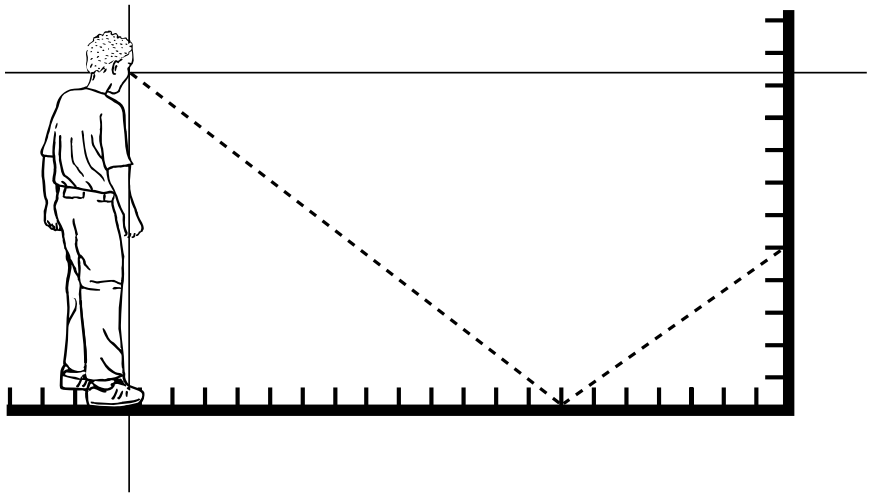
Problem

How can you use the *Law of Reflection* to find the height of objects that are impractical to measure using conventional techniques? Can you use a mirror to find the height of a classmate?

Collecting the data

1. Work in groups of three for this activity. Starting with zero, measure and write the following numbers on a strip of adding machine paper: 10 cm, 20 cm, 30 cm, 40 cm, and so on up to 100 cm. Make sure the numbers are at least 3 cm tall. Starting with zero, measure and write the following numbers on a second strip of adding machine paper: 10 cm, 20 cm, 30 cm, 40 cm, and so on up to 350 cm.
2. Tape the 10 cm to 100 cm paper strip to the wall. Make sure 0 is positioned on the floor.
3. Tape the 10 cm to 350 cm paper strip on the floor, with 0 in the same position as the paper strip in number 2. Make sure the two strips are perpendicular to one another.
4. Using a washable marker, draw a 2 cm x 2 cm square in the center of the mirror. Place the mirror on the floor such that the center of the 2 cm by 2 cm square is 20 cm from the wall.
5. Select one of the group members to be the spotter, one as the marker, and one as the scribe. Measure the *eye-level* height in centimeters of the spotter.
6. Instruct the spotter to face the paper strip on the wall. Ask the spotter to move away from the wall until the reflection of the numeral 10 can be seen in the mirror. Mark the position of the spotter's toe on the paper strip on

the floor. Record this position in the table on the **Data Collection and Analysis** page. (See diagram below.)



7. Ask the spotter to walk slowly towards the paper strip on the wall until the reflection of the numeral 20 can be seen in the mirror. Mark the position of the spotter's toe on the paper strip on the floor. Record this position in the table on the **Data Collection and Analysis** page.

   Note: *The spotter's distance from the wall is the only variable that changes.*

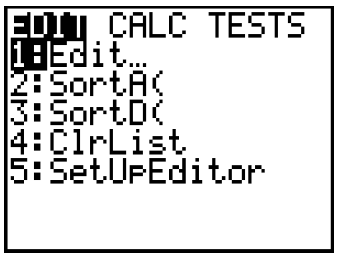
8. Continue this procedure until the spotter has seen the reflection of all the numerals on the paper strip.

Setting up the TI-83 Plus

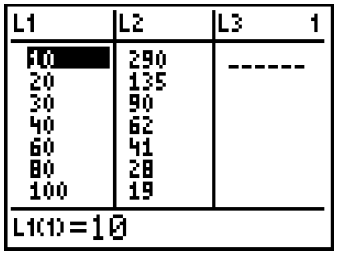
Before starting your data collection, make sure that the TI-83 Plus has the STAT PLOTS turned OFF, Y= functions turned OFF or cleared, the MODE and FORMAT set to their defaults, and the lists cleared. See the Appendix for a detailed description of the general setup steps.

Entering the data in the TI-83 Plus

1. Press [STAT] and select **1:Edit** by pressing [ENTER].

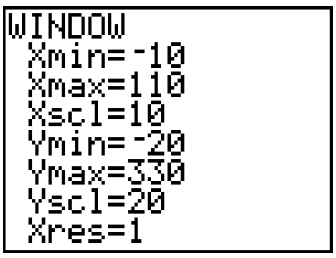


2. Enter the data for the height on the wall in **L1**.
3. Enter the data for the distance in **L2**.



Setting up the window

1. Press [WINDOW] to set up the proper scale for the axes.

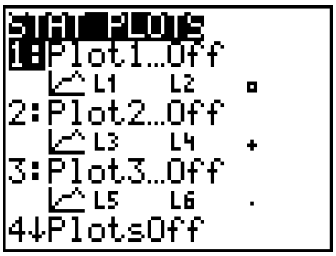


2. Set the **Xmin** value by identifying the minimum value in **L1**. Choose a number that is less than the minimum.
3. Set the **Xmax** value by identifying the maximum value in each list. Choose a number that is greater than the maximum. Set the **Xscl** to **10**.
4. Set the **Ymin** value by identifying the minimum value in **L2**. Choose a number that is less than the minimum.
5. Set the **Ymax** value by identifying the maximum value in **L2**. Choose a number that is greater than the maximum. Set the **Yscl** to **20**.

Graphing the data: Setting up a scatter plot

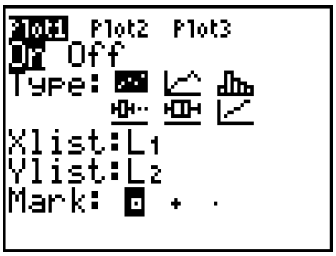
Plot a scatter plot using the data in the table on the **Data Collection and Analysis** page.

1. Press [2nd] [STAT PLOT] and select **1:Plot1** by pressing [ENTER].

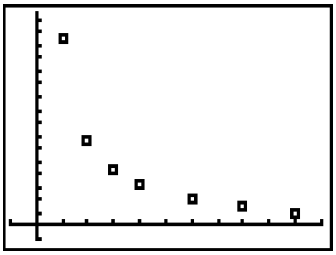


2. Set up the plot as shown by pressing [ENTER] [▼] [ENTER] [▼] [2nd] [L1] [ENTER] [2nd] [L2] [ENTER] [ENTER].

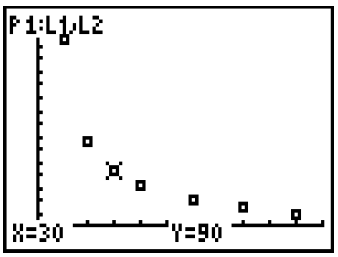
   Note: *Press* [▼] [▼] *if* **L1** *and* **L2** *are already displayed.*



3. Press [GRAPH] to see the plot.



4. Press [TRACE] [▶] [▶] [▶] to see the heights and distances.



5. Find a function to model the data that you plotted.

Answer questions 1 through 7 on the **Data Collection and Analysis** page.

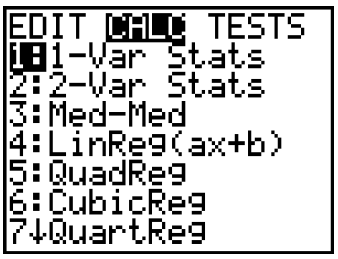
Analyzing the data: Finding a best fit line

The line of best fit that models the data collected is a power regression (**PwrReg**). This model is a rational function. The power function, however, is written in a different form. The variable, *x*, that is in the denominator of the function is written using a negative exponent. You will discuss negative exponents in more advanced Algebra courses. Your teacher will give you a brief explanation of negative exponents.

Find the power regression for the data.

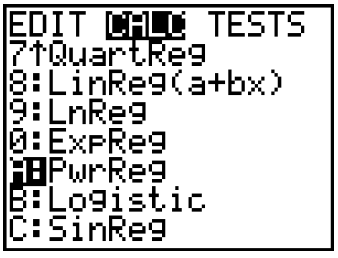
1. Press [2nd] [QUIT] [CLEAR] to return to the Home screen.

2. Press [STAT] and move the cursor to the **CALC** menu.

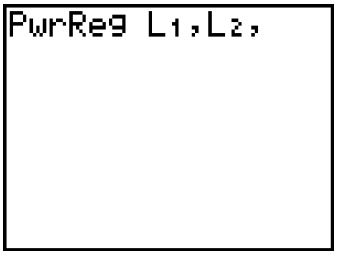


3. Press the up arrow [▲] or the down arrow [▼] to select **A:PwrReg** and press [ENTER].

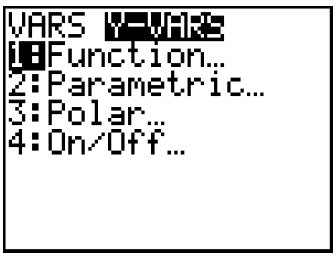
Note: *Place the function in* **Y2** *so that you do not erase the function in* **Y1**.



4. Press [2nd] [L1] [,] [2nd] [L2] [,].

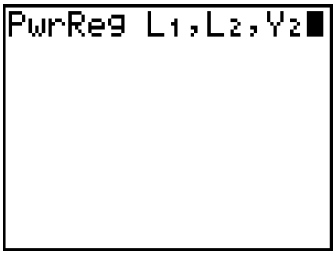


5. Press [VARS] and move the cursor to the **Y-VARS** menu.

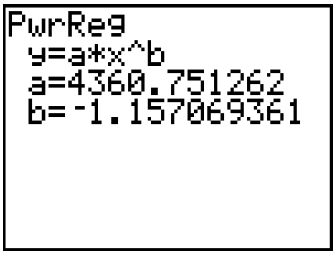


6. Select **1:Function** by pressing [ENTER].

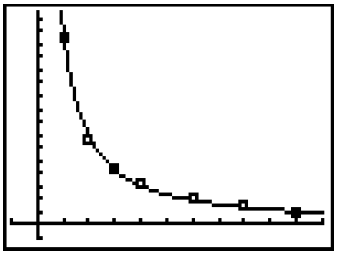
7. Select **2:Y2** and press [ENTER].



8. Press [ENTER] to calculate the power regression and paste the function in **Y2**.



9. Press [GRAPH] to see the power regression.



Answer questions 8 through 11 on the **Data Collection and Analysis** page.

Data Collection and Analysis

Name _____

Date _____

Activity 14: The Closer I Get to You

Collecting the data

Record your data in the table below.

Height (cm)	Distance from mirror (cm)
10	
20	
30	
40	
60	
80	
100	

Analyzing the data

Use the equation you find in number 1 to answer questions 1 through 7.

1. Find an equation for the data by using the formula:

$$y\ (distance) = \frac{\text{Height of Spotter from Eye to Floor} \bullet 20}{x\ (\text{Height on the Wall})}$$

or

$$y = \frac{\text{Height of Spotter from Eye to Floor} \bullet (20)}{x}$$

Equation: _____

2. Enter your equation in **Y1** in the TI-83 Plus. Use your equation to find the distance from the mirror if the height visible in the mirror is:

a. 45 cm _____

b. 110 cm _____

3. Use your equation to find the height on the wall if the spotter stands the following distances from the mirror:

a. 100 cm _____

b. 40 cm _____

c. 200 cm _____

Hint: Use 2nd [CALC] **5:intersect** on the TI-83 Plus.

4. What will happen to the graph of the data if the spotter is taller?

_____

_____

5. What will happen to the graph of the data if the spotter is shorter?

_____

_____

6. Determine the height of a classmate. Place a piece of masking tape on the wall to mark the classmate's height. Walk away from the mirror until you see the masking tape in the mirror. Use your equation to find the classmate's height.

_____

7. Find a classmate's height, in centimeters, using the meter stick. Determine where you would have to stand to see the top of the classmate's head.

_____

Use your power equation to answer questions 8 through 11.

8. Find the distance from the mirror if the height visible in the mirror is:

a. 45 cm _____

b. 110 cm _____

How much do the values differ from the values that you found above?

_____

_____

9. Find the height on the wall if the spotter stands the following distances from the mirror:

a. 100 cm _____

b. 40 cm _____

c. 200 cm _____

Hint: Use 2nd [CALC] **5:intersect** on the TI-83 Plus.

10. Determine the height of a classmate. Place a piece of masking tape on the wall to mark the classmate's height. Walk away from the mirror until you see the masking tape in the mirror. Use your power equation to find the classmate's height.

_____

11. Find a classmate's height, in centimeters, using the meter stick. Determine where you would have to stand to see the top of the head of the classmate's head. Use your power equation to find the classmate's height.

_____

Teacher Notes

Activity 14

The Closer I Get to You

Objectives

- To find a rational function
- To find the *x* value of a function, given the *y* value
- To find the *y* value of a function, given the *x* value
- To use technology to plot data

Materials

- TI-83 Plus
- Small mirror (one per group)
- Adding machine paper (at least 500 cm per group)
- Masking tape and markers
- Meter stick (one per group)

Preparation

- Use a marker for the numbering on the tape.
- Make sure students are not bending to see the numeral in the mirror.
- It is recommended that you have students find their own equation before using the power regression to find a power function. The section on finding a power function can be omitted without losing any continuity in this activity. If the power function is found, some explanation of negative exponents might be necessary.

Answers to Data Collection and Analysis questions

Collecting the data

- Sample data.

Height (cm)	Distance from mirror (cm)
10	290
20	135
30	90
40	62
60	41
80	28
100	19

Analyzing the data

1. Find an equation for the data by using the formula.

For the sample data $y = \frac{146(20)}{x}$.

2. Enter your equation in **Y1** in the TI-83 Plus. Use your equation to find the distance from the mirror if the height visible in the mirror is:

a. 45 cm is approximately 65 cm from the mirror.

b. 110 cm is approximately 27 cm from the mirror.

3. Use your equation to find the height on the wall if the spotter stands the following distances from the mirror:

a. 100 cm = 29.2 cm in height.

b. 40 cm = 73 cm in height.

c. 200 cm = 14.6 cm in height.

4. What will happen to the graph of the data if the spotter is taller?

The graph will be shifted up if the spotter is taller.

5. What will happen to the graph of the data if the spotter is shorter?

The graph will be shifted down if the spotter is shorter.

6. Determine the height of a classmate. Place a piece of masking tape on the wall to mark the classmate's height. Walk away from the mirror until you see the tape in the mirror. Use your equation to find the classmate's height.

Answers will vary.

7. Find a classmate's height, in centimeters, using the meter stick. Determine where you would have to stand to see the top of the classmate's head.

Answers will vary.

8. Find the distance from the mirror if the height visible in the mirror is:

a. 45 cm is approximately 53 cm from the mirror.

b. 110 cm is approximately 19 cm from the mirror.

How much do the values differ from the values that you found above?

9. Find the height on the wall if the spotter stands the following distances from the mirror:

a. 100 cm = 26.1 cm in height.

b. 40 cm = 57.7 cm in height.

c. 200 cm = 14.3 cm in height.

10. Determine the height of a classmate. Place a piece of masking tape on the wall to mark the classmate's height. Walk away from the mirror until you see the tape in the mirror. Use your power equation to find the classmate's height.

Answers will vary.

11. Find a classmate's height, in centimeters, using the meter stick. Determine where you would have to stand to see the top of the classmate's head. Use your power equation to find the classmate's height.

Answers will vary.