Graphical Analysis – ID: 9988

By Holly Thompson

Time required
45 minutes

Activity Overview

Students will analyze graphs of polynomials finding intervals over which the function is increasing or decreasing and positive or negative, as well as the function's relative minimum and maximum values and x- and y-intercepts.

Concepts

- *Analysis of graphs of polynomial functions*
- *Hidden behavior of functions*
- *Difference between relative and absolute extrema*

Teacher Preparation

This investigation offers an approach for Precalculus students to analyze different aspects of polynomial graphs that they will use in Calculus.

- *This activity could be used in Precalculus as an introduction to graphs of polynomial functions. It could also be used in an advanced Algebra 2 class.*
- *Students should know how to place a point on a graph using the* ***Point On*** *tool in the Graphs & Geometry application, as well as how to drag the point along the graph. Students should also be familiar with interval notation.*
- *The screenshots on pages 2–4 demonstrate expected student results. Refer to the screenshots on page 5 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter "9988" in the quick search box.***

Classroom Management

- *This activity is intended to be mainly* ***teacher-led****, with breaks for students to work in pairs. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- *The student worksheet PreCalcAct05_GraphicalAnalysis_worksheet_EN helps guide students through the activity and provides a place for students to record their answers.*

TI-Nspire™ Applications

Graphs & Geometry, Notes

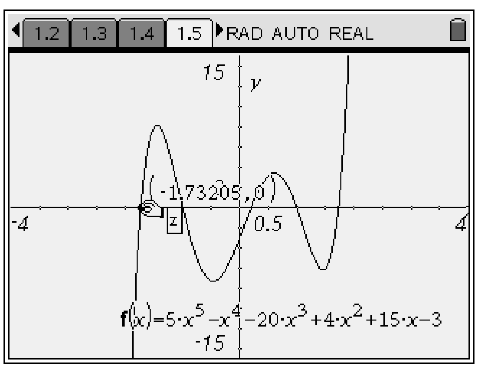
Problem 1 – Examining a complete graph

Listed on pages 1.2 and 1.3 are the important characteristics that students will look for in this activity:

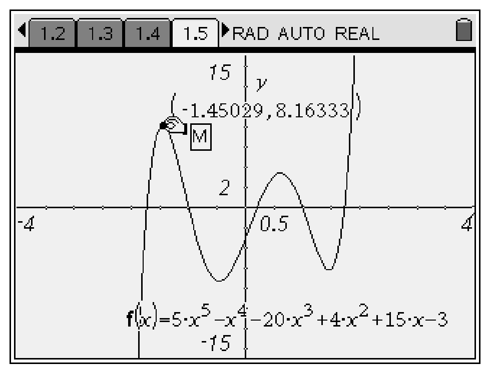
- intervals over which the graph is increasing or decreasing
- intervals over which the graph is positive or negative.
- the zeros or roots of the function
- the *y*-intercept of the function
- minimum and maximum points
- the end behavior of the function

Page 1.3 also provides students with the definition of a complete graph as a graph of a function that displays all of these important features.

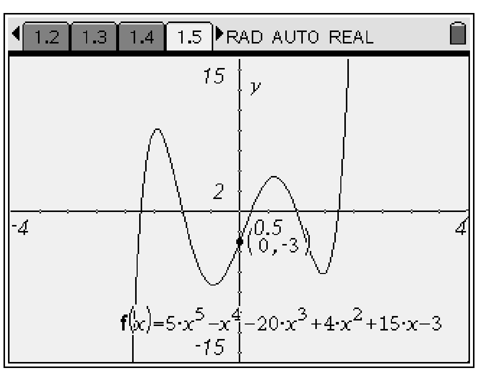
You may wish to walk through Problem 1 with students, helping those needing assistance with placing a point on the graph using the **Point On** tool and then grabbing and dragging this point to identify the important characteristics, some of which are picture below. (Students are asked to identify the zeros, *y*-intercept, and minimum and maximum points first to make it easier to then find the increasing/decreasing and positive/negative intervals.



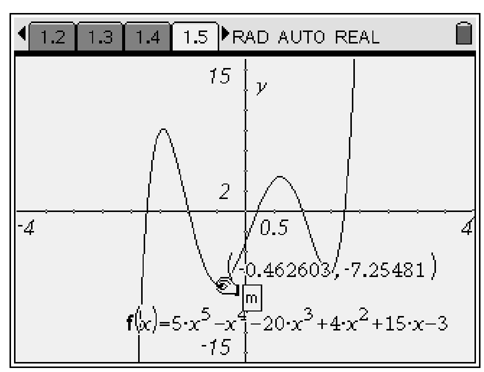
zero



maximum point



y-intercept



minimum point

Student worksheet solutions:

zeros: –1.732, –1, 0.2, 1, 1.732

y-intercept: –3

minimum points: (–0.463, –7.255), (1.461, –6.191)

maximum points: (–1.450, 8.163), (0.612, 3.383)

increasing intervals: (–∞, -1.450), (–0.463, 0.612), (1.461, ∞)

decreasing intervals: (–1.450, –0.463), (0.612, 1.461)

positive intervals: (–1.732, –1), (0.2, 1), (1.732, ∞)

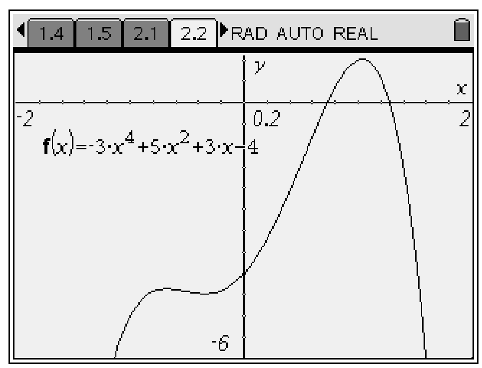
negative intervals: (–∞, –1.732), (–1, 0.2), (1, 1.732)

end behavior: as $x \to +\infty$, $\mathbf{f}(x) \to +\infty$; as $x \to -\infty$, $\mathbf{f}(x) \to -\infty$

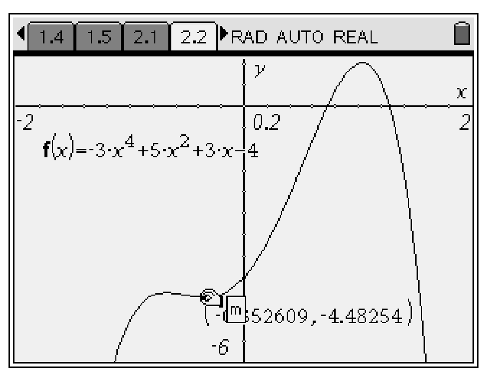
- To find increasing/decreasing intervals, you use the *x*-coordinates of the minimum/maximum points, since that is where the graph changes from increasing to decreasing, and vice versa.
- To find positive/negative intervals, you use the zeros, since that is where the graph changes for positive to negative, and vice versa.

Problem 2 – Hidden behavior

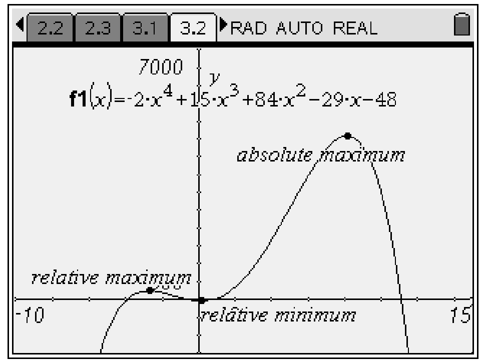
In Problem 2, students are not provided a complete graph of the function, and they will need to adjust the **Window Settings** to obtain a complete graph. Allow students to experiment with the window and then show an example of what an appropriate window would look like.



For this particular problem, it is highly suggested to discuss with students the “hidden behavior” occurring on the approximate interval (–0.8, –0.1), where it initially appears that a saddle point occurs, but turns out to be both a maximum and minimum close together.

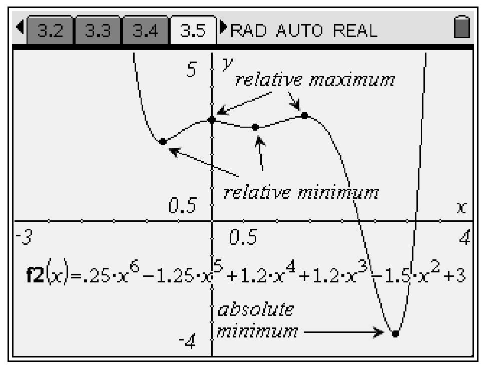


Problem 3 – Minimum and maximum points



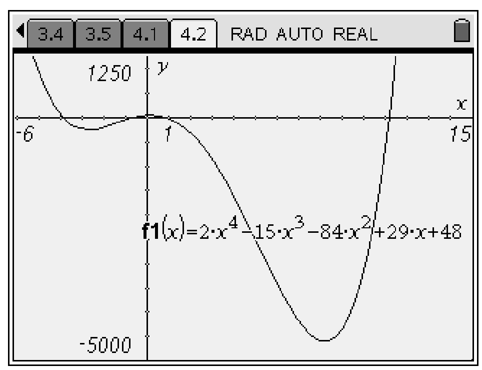
In this problem, students examine the difference between relative and absolute extrema. By looking at the graph on page 3.2, they should determine that a relative maximum/minimum occurs when the graph has a "peak/valley," and the absolute maximum/minimum occurs when there is no value of *x* for which *f*(*x*) is greater/less than that value.

You may wish to discuss with students that all *absolute* extrema are also *relative*, but all not relative extrema are absolute.



Page 3.5 displays a graph for which students should obtain a complete graph, as well as identify all extrema as relative or absolute.

Problem 4 – More practice



This problem simply provides students with another polynomial graph on which to practice what they have learned throughout the activity.

A complete graph is shown to the right, with solutions below.

Student worksheet solutions:

zeros: –3.879, –0.634, 0.876, 11.137

y-intercept: (0, 48)

minimum points: relative minimum at (–2.688, –241.144);
absolute minimum at (8.148, –4591.395)

maximum points: relative maximum at (0.165, 50.432); no absolute maximum

increasing intervals: (–2.688, 0.165), (8.148, ∞)

decreasing intervals: (–∞, –2.688), (0.165, 8.148)

positive intervals: (–∞, –3.879), (–0.634, –0.876), (11.137, ∞)

negative intervals: (–3.879, –0.634), (0.876, 11.137)

end behavior: as $x \to +\infty$, $\mathbf{f}(x) \to +\infty$; as $x \to -\infty$, $\mathbf{f}(x) \to +\infty$

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(Student)TI-Nspire File: *PreCalcAct05_GraphicalAnalysis_EN.tns*

GRAPHICAL ANALYSIS

Precalculus

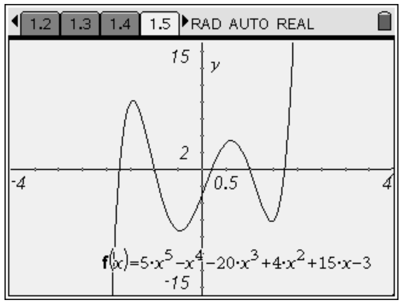
Analyzing graphs of functions

When analyzing graph of a function, there are certain aspects of the graph you should consider. Some of these aspects are:

- intervals over which the graph is increasing or decreasing
- intervals over which the graph is positive or negative
- the zeros or roots of the function
- the y-intercept of the function
- minimum and maximum points
- the end behavior of the function

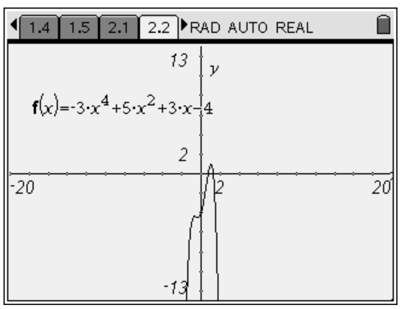
A graph of a function is said to be a **complete graph** if it shows all of these important features. (Obviously any graph cannot *show* the end behavior, but a complete graph *suggests* it.)

A complete graph of a polynomial function is shown on the next page. Use the **Point On** tool to place a point on the curve and drag the point along the graph to find the information requested on your student worksheet.



The graph on the next page shows the graph of another polynomial function $f(x)$. This, however, is not a complete graph.

Adjust the **Window Settings** until you think you have a complete graph. Sketch it on your worksheet and indicate the viewing window.

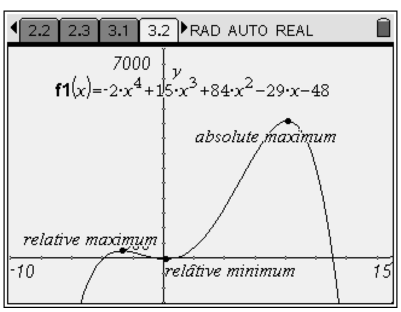


The graph of this function had some "hidden behavior." Where does this behavior occur and how did you find it?

What key features might you have missed if you did not adjust the **Window Settings** to obtain a complete graph?

The extrema (its minimum and maximum points) of a graph are said to be either **relative** or **absolute.**

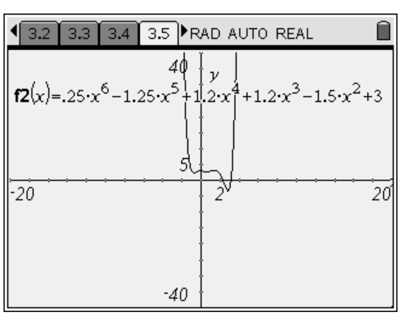
The complete graph of a polynomial function is shown in page 3.2. Each minimum and maximum is labeled as relative or absolute.



Based on your observations, explain the difference between a *relative* minimum or maximum and an *absolute* minimum or maximum.

For the graph on the next page, adjust the **Window Settings** until you have a complete graph. Sketch it on your worksheet and indicate the viewing window.

Find all extrema and identify them as relative or absolute.



For the graph on the next page, adjust the **Window Settings** until you think you have a complete graph. Sketch it on your worksheet and indicate the viewing window.

Find all key features, including identifing extrema as relative or absolute.