Fundamental Algebra

7 8 9 10 11 **12**

TI-Nspire | Investigation | Student | 45 min

Introduction

The quadratic equation $f(x)=x^2+8x+12$ has two distinct linear factors: $x+2$ and $x+6$ which means it has two distinct roots: $x=-2$ and $x=-6$. The quadratic equation $g(x)=x^2+8x+16$ has one 'repeated' linear factor: $x+4$ and therefore one distinct (but repeated) root: $x=-4$. The quadratic equation: $h(x)=x^2+8x+17$ has no real distinct factors or roots but does have two distinct complex factors: $x+4-i$ and $x+4+i$, and therefore two distinct complex roots $x=-4+i$ and $x=-4-i$.

This investigation looks at factors and roots of larger polynomials with real coefficients.

Real Preliminary Questions

Question: 1

a. Where possible, factorise each of the following quadratics over R.

$$f(x)=x^2+8x+12 \qquad g(x)=x^2+8x+16 \qquad h(x)=x^2+8x+17$$

$$a(x)=x^2-4x-5 \qquad b(x)=x^2-6x+9 \qquad c(x)=x^2-10x+29$$

b. Each of the quadratic functions above is used to produce a polynomial (below). Complete the table with factors and roots determined over the real number system.

Equation:	Degree:	Factors:	Distinct Roots:
$p(x)=f(x)\cdot a(x)$			
$q(x)=f(x)\cdot g(x)$			
$r(x)=f(x)\cdot h(x)$			
$s(x)=g(x)\cdot b(x)$			
$t(x)=h(x)\cdot c(x)$			

c. The functions $r(x)$ and $s(x)$ have the same number of distinct roots yet they are significantly different. Explain the difference with reference to the number and type of roots.

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Complex Preliminary Questions

Question: 2

a. Factorise each of the following quadratics over C.

$$f(z)=z^2+8z+12 \qquad g(z)=z^2+8z+16 \qquad h(z)=z^2+8z+17$$

$$a(z)=z^2-4z-5 \qquad b(z)=z^2-6z+9 \qquad c(z)=z^2-10z+29$$

b. Each of the quadratic functions above is used to produce a polynomial (below). Complete the table with factors and roots determined over the complex number system.

Equation:	**Degree:**	**Factors:**	**Distinct Roots:**
$p(z)=f(z)\cdot a(z)$			
$q(z)=f(z)\cdot g(z)$			
$r(z)=f(z)\cdot h(z)$			
$s(z)=g(z)\cdot b(z)$			
$t(z)=h(z)\cdot c(z)$			

c. Comment on the number of factors for each of the polynomials.

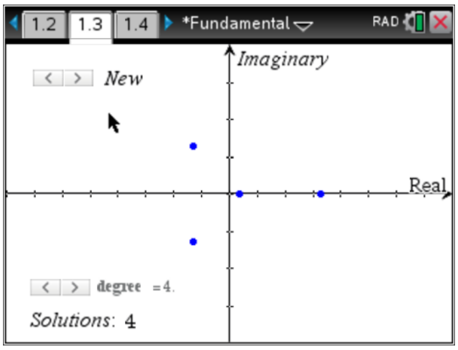
Multiplicity

The polynomial $p(z)=z^2-6z+9$ has one distinct root: $z=3$. The root $z=3$ has multiplicity 2 since it occurs twice. The polynomial $p(z)$ is said to have 2 roots when degenerate roots are included, that is the repeated roots are counted with their multiplicity.

d. Comment on the number of roots for each of the polynomials including degenerate roots.

Investigation

Open the TI-nspire document: "Fundamental" and navigate to page 1.3. The 'degree' slider produces a random polynomial of the nominated degree with **real coefficients**. The 'new' slider can be used to generate another polynomial of the same degree. The polynomial is factorised and the roots plotted on the Argand plane. The example shown opposite is for a polynomial of degree 4 that has two real roots and two imaginary roots.



Question: 3

Generate at least 10 more polynomials of degree 4 and describe the number and type of roots for each polynomial.

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Question: 4

Generate at least 10 polynomials of degree 5 and describe the number and type of roots for each polynomial. Comment on the number of real and complex roots.

Question: 5

Generate at least 10 more polynomials of degree 6 and describe the number and type of roots for each polynomial.

Question: 6

Generate polynomials of various degrees and identify any patterns with respect to the following:

- Type and number of roots (Real / Complex)
- Nature of complex roots

Question: 7

A polynomial of degree 6 with real coefficients has the following roots: $z=3+i$, $z=2-i$ and $z=6$. Which one of the following could represent the remaining roots:

a) $z=3+i$, $z=2-i$ and $z=4$

b) $z=3-i$, $z=2+i$ and $z=4$

c) $z=-3+i$, $z=-2-i$ and $z=4$

d) $z=3+i$, $z=2-i$ and $z=-4$

e) $z=3-i$, $z=2+i$ and $z=4-i$

Question: 8

The polynomial $p(z)=z^6+az^5+bz^4+cz^3-18z^2-dz-1500$ has roots: $z=-3+i$, $z=-4-3i$ and $z=2$. Determine the values of *a*, *b*, *c* and *d*.

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