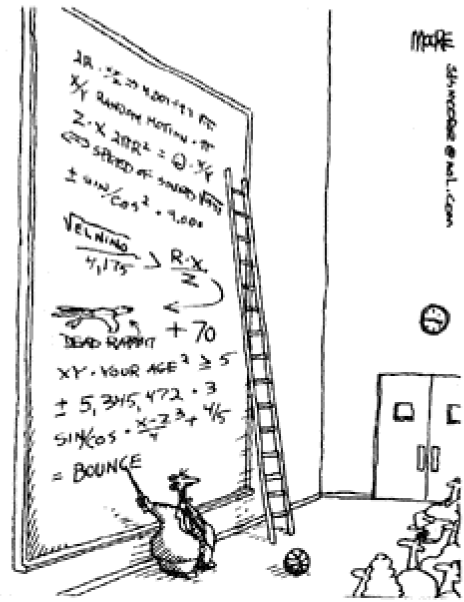
Collecting Ball Bounce data.

Introduction:

Many aspects relating to the motion of a bouncing ball can be modelled mathematically. The first stage in modelling the motion is to collect some data. The Calculator Base Laboratory or CBR™ is a motion detector that can be connected to a TI-83(plus) calculator. The position, velocity and acceleration at any time are recorded in the calculator's Stat. – List editor. The data can be used to draw graphs of the ball's motion.



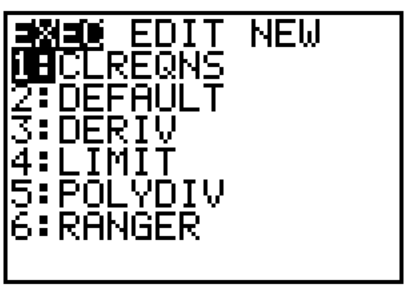
"And that, ladies and gentlemen, is the way the ball bounces."

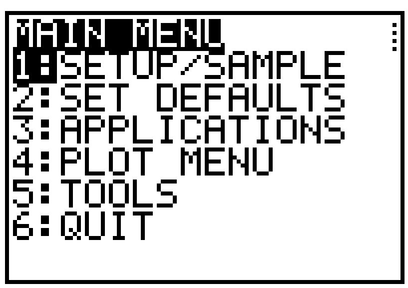
Equipment:

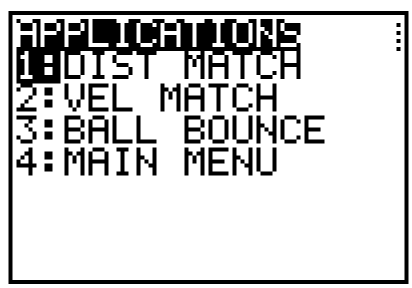
CBR
TI-83(Plus) Graphic Calculator
Calculator to calculator link cable
Medium – Large size ball

Getting Started – Collecting the data:

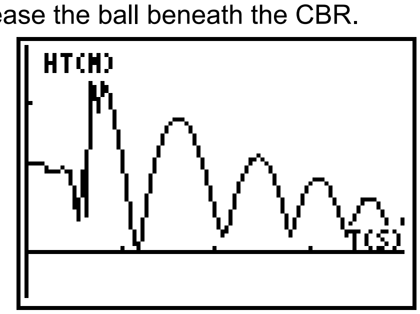
- Load the CBR *Ranger* program onto the graphics calculator.
- Make sure the ball is well inflated.
- Start the *Ranger* program.
- From the main menu select: `APPLICATIONS`.
- Select 'metres' for the unit of measurement
- Select '`BALL BOUNCE`' from the applications menu.







- Follow the instructions on the calculator screen. The CBR may be detached during the experiment and returned later for data transfer.
- Hold the CBR approximately 0.5 meters above the ball. Try to hold the CBR at a constant height.
- Press the **trigger** button on the CBR. The CBR will start ticking; release the ball beneath the CBR.
- When the CBR has stopped ticking the data collection is complete.
- Return the CBR to the calculator, plug it in and press [ENTER]; a graph of the data will be displayed. Press [ENTER] when you have finished viewing the graph.
- If the data/graph is not as clear as you would like, select option 5: `REPEAT SAMPLE`



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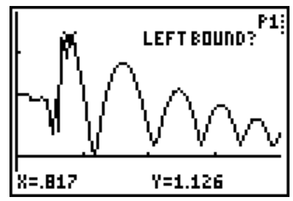
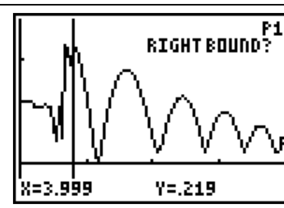
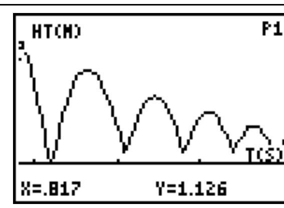
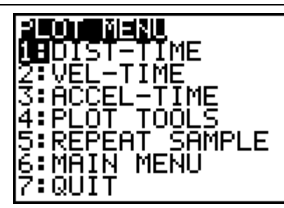
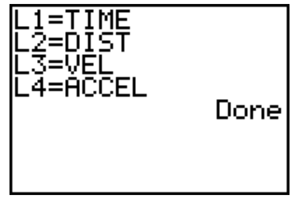
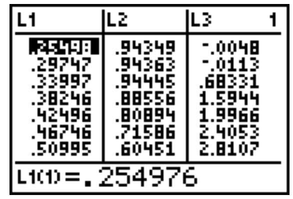
Working with the data:

Instruction	Screen Shot
In the graph shown previously, the CBR started measuring before the ball was released. This means there was a small amount of data collected that is not relevant. This data may be removed using the PLOT TOOLS. Select option [4]	
Selecting a domain allows irrelevant data to be removed from the data set. Select option [1]	
Use the [◄] [►] arrow keys to move the cursor to the start of 'useful' data. Press [ENTER]	
Use the [◄] [►] arrow keys to move the cursor to the end of the 'useful' data. Press [ENTER]	
The new graph will be displayed using the selected data. You can use the [◄] [►] arrow keys to move around through the data points collected.	
Press [ENTER] to return to the plot menu.	
Press [6] to return to the main menu.	

Select option [6] to quit back to the calculator home screen. Notice that the data is automatically saved to the calculator's lists. • L_1 = Time • L_2 = Distance • L_2 = Velocity • L_4 = Acceleration.	
Press [STAT] and select option [1] to edit or view the data collected. Note: Each set of data will be different depending on how high the ball was released, what data was removed and so on. To return to the home screen: [2nd] [MODE]	

Use either TI-Connect or the calculator's Flash memory (TI-83 Plus – only) to make a back up copy of the data. Many of the following exercises require parts of the data to be deleted. To avoid having to do the experiment again it is extremely important that a back up copy of the data is made.

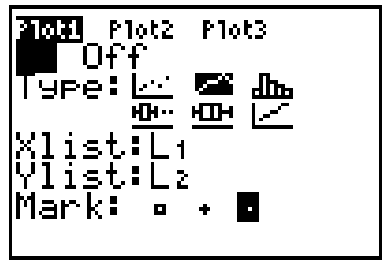
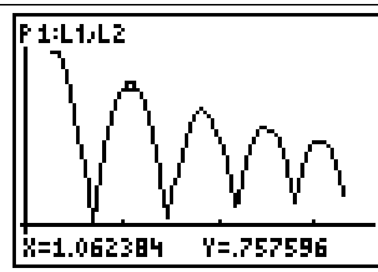
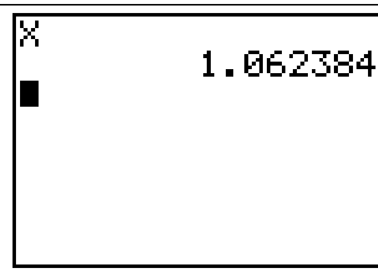
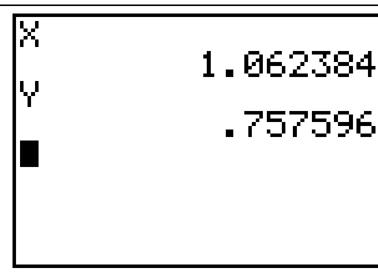
Translation of ball bounces.

Introduction:

The purpose of this investigation is to increase an understanding of the translation of functions and to demonstrate how useful this process can be in determining the equation to a function.

Finding the family of curves using translations.

Load the ball bounce data into the calculator from either the calculator's Flash memory or from the computer using TI – Connect. (For instructions on these processes refer to the Appendix.)

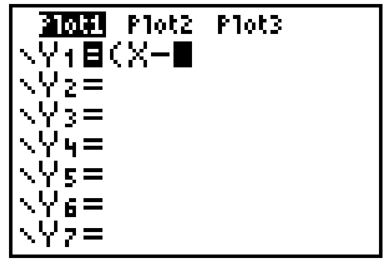
Instruction	Screen Shot
Press [2nd] [Y=] and copy the settings shown to set up `Stat-Plot 1`:	
Press [GRAPH] to display the ball bounce data. Use the [TRACE] button to find the co-ordinates of the maximum height of the first ball bounce. This is the turning point of the first parabola. Write down the co-ordinates of the turning point.	
The translational form of a parabola is given by y = a(x – b)^2 + c where b and c give the co-ordinates of the turning point. Press [2nd] [MODE] to return to the calculator home screen. Press [X,T,Θ,n] then [ENTER]	
To recall the value for Y, press [ALPHA] [1] [ENTER]	

☞ When the trace function has been used on the graphing screen the values for x and y are temporarily stored in the calculator in the variables x and y. If x or y are used in another computation or another graph is drawn then the values for x and y will be changed.

Press [Y=] and enter the first part of the quadratic equation:

y = a(x –

This can be done by: [(] [X,T,Θ,*n*] [–]

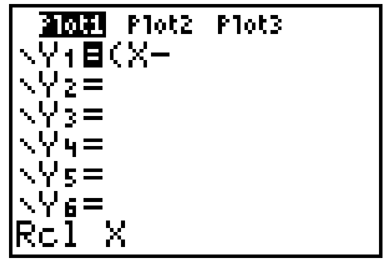


You can either type the x ordinate for the turning point or press:

[2nd] [STO▶] [X,T,Θ,*n*]

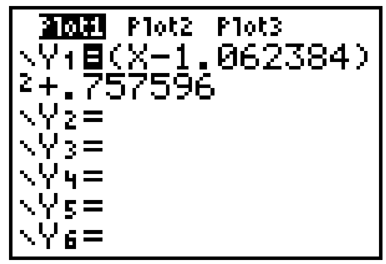
This recalls the value for x from when the ball bounce was traced.

Press [ENTER] to insert the value for x.

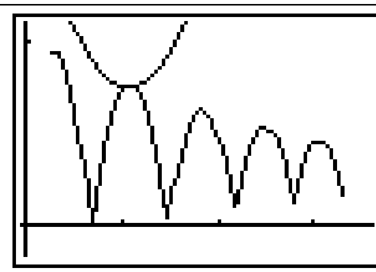


Finish off the next stage of the equation: [)] [x^2] [+] then recall the value for y by pressing:

[2nd] [STO▶] [ALPHA] [1] [ENTER]



Press [GRAPH] to display the graph so far.

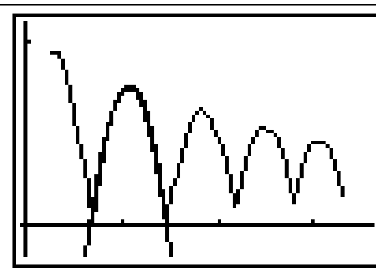


The parabola needs to be turned upside down and dilated before it matches the data correctly. You can refer to your equations from previous investigations to determine an appropriate value for ‘a’.

Alternatively you can experiment with the value for ‘a’ until the graph matches the ball - bounce data.

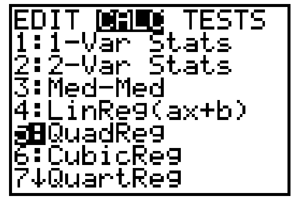
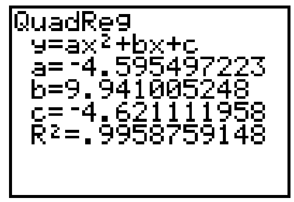
Return to the [Y=] screen and press [2nd] [DEL] at the start of the equation to insert the value for ‘a’. Then press [GRAPH] to display the graph.

If you are experimenting with the value of ‘a’ continue alternating between the [Y=] screen and the [GRAPH] screen. Use the “Insert” editing feature and “delete” as though the calculator were functioning as a word processor.



Quadratic Regression:

The graphic calculator can determine the equation of best fit for a set of data that follows a parabolic path. Before the calculations can be performed it is necessary to remove the other ball bounces. Use the Select command on the calculator to isolate a single ball bounce and store the data in L_3 and L_4.

Instruction	Screen Shot
Press [STAT] [►] select option [5] for quadratic regression.	
Press: [2nd] [3] [,] for L_3 and [2nd] [4] [,] for L_4 follow the equation with Y_1 by pressing: [VARS] [►] [1] [1] then [ENTER] to execute the command.	

Note: There is a large range of options for the statistics section of the calculator. The screen shot for the QuadReg shows R^2, Pearson's correlation. If you do not wish this value to be shown use the catalogue menu to switch diagnostics off. Press [2nd] [0] [x^{-1}], use the [▼] arrow key to locate Diagnostic Off and press [ENTER] to select this menu option, press [ENTER] again to activate it. Alternatively, if R^2 is not displayed and you wish it to be displayed, turn the diagnostics on. Follow the process above to locate Diagnostics On.

Ball Bouncing with Matrices – Echelon Form

Introduction:

A single bounce of a ball can be represented by a quadratic equation. The equation can be determined using a number of methods. In this investigation three points on the path of the ball need to be selected. Matrices will be used to determine the quadratic equation that passes through these points and therefore describes the path of the ball. Finally, the equation for each ball bounce in the data will be determined to see if there are any patterns amongst the equations.

Preliminary Questions:

The process for using matrices for real data and contrived data is the same, however, calculations for the real data will be more complicated due to the nature of the figures generated. It is therefore useful to practice solving a set of equations before attempting the practical data.

Question 1:

Consider the following two equations 2x + 3y = 14 and 6x – 2y = 20. These equations can be solved simultaneously or they can be solved using matrices.

a) Consider the following three matrices: $A = \begin{bmatrix} 2 & 3 \\ 6 & -2 \end{bmatrix}$ $B = \begin{bmatrix} x \\ y \end{bmatrix}$ $C = \begin{bmatrix} 14 \\ 20 \end{bmatrix}$

i) Determine the resultant matrix: A × B

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ii) Given A × B = C show that 2x + 3y = 14 and 6x – 2y = 20

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b) Given Eq$^{\underline{n}}$ 1: 2x + 3y = 14 and Eq$^{\underline{n}}$ 2: 6x – 2y = 20 complete the following:

i) Multiply Eq$^{\underline{n}}$ 1 by 3.

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ii) Subtract Eq$^{\underline{n}}$ 2 from 3 × Eq$^{\underline{n}}$ 1.

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iii) Using your result from b ii) determine the value for y.

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iv) Determine the value for x in the simultaneous equations.

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c) The following matrix is in Echelon from: $\begin{bmatrix} 2 & 3 & 14 \\ 6 & -2 & 20 \end{bmatrix}$

The numbers in row one are the coefficients from equation 1. The numbers in row two are the coefficients from equation 2. Alternatively, the numbers in column 1 represent the coefficients of x from both equations and the numbers in column 2 represent the coefficients of y from both equations.

i) Multiply all the numbers in the top row (first equation) by 3 and write down the new matrix.

ii) In each column subtract the numbers in row 2 from the numbers in row 1. The numbers in row 1 remain the same but write the answers as the new values for row 2.

iii) Divide the numbers in row 2 by 11 and write the answers as the new values for row 2.

iv) What is the value of y in the solution to these simultaneous equations?

v) Multiply the values in row 2 by -9 and add them to row 1, write the result as the new row 1.

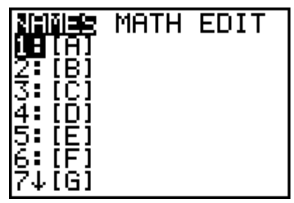
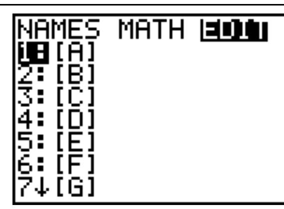
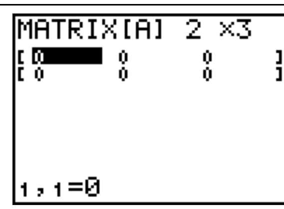
vi) Divide the new values in row 1 by 6 and write the result as the new row 1.

vii) Write down the complete solution to the simultaneous equations and explain how these are read from the matrix:
$2x + 3y = 14$ and $6x - 2y = 20$.

Matrix operations on the TI-83(Plus).

Matrix operations can be performed easily on the calculator. Follow the instructions below to perform the matrix operations on the calculator.

> **Note: The TI-83 and TI-83Plus menus, with the exception of the memory management menu, are the same. The screen shots for both calculators are also the same. The calculator keys are essentially the same with the exception of the [APPS] button on the TI-83Plus. The inclusion of this button required the relocation of some buttons on the TI-83Plus compared with the TI-83; the [MATRX] button is one of those buttons.**
>
> **Most of the instructions here are specifically for the TI-83Plus. If you are using a TI-83 whenever the instruction [2nd] [x^{-1}] appears, simply press the [MATRX] button.**

Instruction	Screen Shot
On a TI-83Plus press [2nd] [x^{-1}] On a TI-83 press [MATRX] In this screen shot none of the matrices have been defined. If a matrix has been defined, it can be redefined in the edit menu.	
Use the [▸] arrow key twice or the [◂] arrow key once to move across to the matrix edit menu.	
Press [ENTER] to edit matrix [A]. Matrix [A] needs to be defined as a 2 row by 3 column matrix. Press [2] then [ENTER] followed by [3] and [ENTER] The matrix is formed as per the dimensions. The cursor then moves to the first row – first column. The location of the cursor is specified at the bottom of the calculator screen: Element 1,1 = 0.	
Enter the elements of matrix [A] by entering the number followed by [ENTER]. $\begin{bmatrix} 2 & 3 & 14 \\ 6 & -2 & 20 \end{bmatrix}$ Once the information has been entered press [2nd] [MODE] to return to the home screen.	

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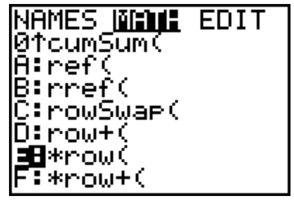
The calculations to be performed on the calculator follow the calculations performed in question 1 part c; keep the results of these calculations close by as you perform them on the calculator.

Recall the first operation when solving simultaneous equations using the matrices was to multiply the first row by 3. To perform this operation:

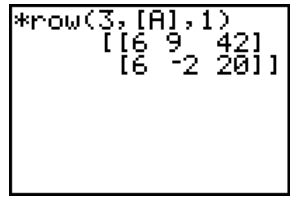
[2nd] [x^{-1}] [▶] [▲] [▲]

When you press [ENTER] the calculator pastes this operation back to the home screen.



The syntax for this operation is: ✱row(value, matrix, row)

[3] [,] [2nd] [x^{-1}] [1] [,] [1] [)] [ENTER]



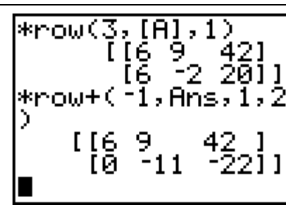
Note: The previous screen shot shows the result after row 1 has been multiplied by 3. The result of this calculation has been stored as the new row 1 on the home screen. The original matrix [A] is unaltered. To change the original matrix you would need to store the result as matrix [A].
ie: [STO▸] [2nd] [x^{-1}] [1] [ENTER]
The process used in these instructions uses the ANS function to recall the result of previous calculations. The advantage of this method is that it preserves the original matrix and takes less key strokes. The disadvantage of this method is that intermediate calculations or operations not involving the matrix change the value of ANS.

The next step is to subtract the numbers in row 2 from the numbers in row 1 and store the result in row 2. This option does not appear in the matrix menu. To get around this problem multiply row 2 by – 1, add the result to row 1 and store the result in row 2.

The syntax for multiplying a row followed by adding it two another is:
`*row+(value, matrix, rowA, rowB)`

[2nd] [x^{-1}] [▶] [▲] [ENTER] [(-)] [1] [,] [2nd] [(-)] [,] [1] [,] [2] [)] [ENTER]

Note in this case the "ANS" operation was used. When a matrix operation has been performed the answer is returned to the home screen. The result could be stored to another matrix.

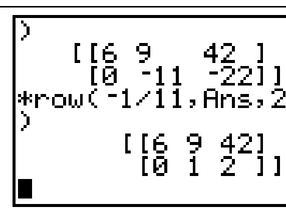


At this stage it is possible to see that –11y = -22, therefore y = 2; however the complete operations can be continued using the calculator. The objective is to get a matrix that appears as follows:

$$\begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \end{bmatrix}$$ where x and y represent the solution.

If row 2 is multiplied by -1/11 the result will have the bottom row appearing as it should:

[2nd] [x^{-1}] [▶] [▲] [▲] [ENTER] [(-)] [1] [÷] [1] [1] [,] [2nd] [(-)] [,] [2] [)] [ENTER]



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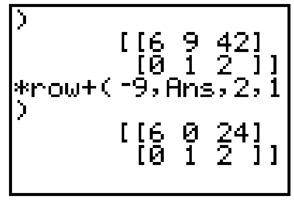
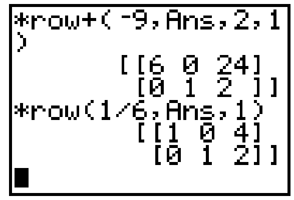
If row 2 is multiplied by -9 and added to row 1 it will continue to reduce the matrix. [2nd] [x^{-1}] [►] [▲] [ENTER] [(-)] [9] [,] [2nd] [(-)] [,] [2] [,] [1] [)] [ENTER]	
The final operation is to divide the first row by 6. It is only possible to multiply rows on the calculator, the first row can be multiplied by 1/6. [2nd] [x^{-1}] [►] [▲] [▲] [ENTER] [1] [÷] [6] [,] [2nd] [(-)] [,] [1] [)] [ENTER] The final answer has been produced and can be read as x = 4 and y = 2	

Question 2

a) Use the 'row reduction method' outlined above to solve the following pair of simultaneous equations: 5x + 3y = 16 and 10x + 2y = 44.

Remember the objective is to generate a matrix of the form: $\begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \end{bmatrix}$

i) Write the matrix in Echelon form: (Store it as matrix [A] on the calculator.)

ii) Write down the result of your first calculation below and the calculator operation used to perform this calculation:

iii) Write down the result of your second calculation below and the calculator operation used to perform this calculation:

iv) Write down the result of your third calculation below and the calculator operation used to perform this calculation:

v) Write down the result of your remaining calculations below and the calculator operations used to perform these calculations:

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vi) Write down the solution to the simultaneous equations:

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b) The matrix menu on the calculator has another operation that is exceptionally useful. Start from the home screen, select “rref” from the `MATRIX` – `MATH` menu and insert [A] as the matrix.
ie: `rref([A])`, press [ENTER] to execute the command. What does the “rref” command do?

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c) Solve the following equations: 3x + 2y = -1 and -2x + 4y = 22. Write down your solution and the calculator operation(s) used to determine the solution.

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d) Solve the following equations: 3x + 2y – 3z = -12, 2x – 4y + z = 20 and x – 3y + 5z = 31. Write the matrix in Echelon form, the calculator operation(s) used to determine the solution and the solution to the simultaneous equations.
Hint: You will need to set matrix [A] to a 3 x 4 matrix.

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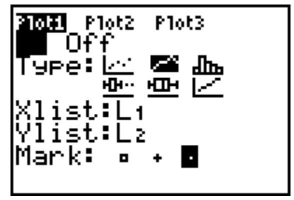
Finding quadratic equations for ball bounces using matrices.

Use either the calculator's Flash memory or TI-Connect to return the complete data set for the ball bounces to the calculator. For instructions on this process refer to the Appendix. Set up the distance time plot either using the Ranger program or by matching the settings for `Plot1` shown:
Press [2nd] [Y=] to enter the plot set-up, match the settings and press [ZOOM] [9].



Question 3

a) Use the [TRACE] button and the ◄ ► arrow keys to locate three points from the first bounce, record them in the table below:

Bounce 1:	X ordinate	Y ordinate
Point 1:		
Point 2:		
Point 3:		

b) Use the first point and substitute the values for x and y into the general equation: $y = ax^2 + bx + c$. Write down the equation in terms of a, b and c.

c) Repeat the above process for points 2 and 3. Write down the second and third equations in terms of a, b and c.

d) Write down the matrix in Echelon form that can be used to solve these equations.

e) Solve these equations on the calculator. Write down the values obtained for a, b and c and the corresponding quadratic equation for the data.

f) Use Y_1 to enter the equation, use TI-Connect to get a picture of your graph and data. Paste the picture into the space provided.

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