Exploration 2

Students use a geometry utility to continue their investigations of reflections in a line. **Note:** For best results, students should set the measurement preferences on their geometry utilities so that angles are measured to the nearest degree and lengths to the nearest 0.1 units.

a. Student constructions should resemble the one shown in Figure **11**.

b. From Step **9**, students should observe that $m\angle QEB = 90^\circ$. From Step **11**, they should observe that $EQ = EQ'$. This should lead to the conjecture that $\overline{AB}$ is the perpendicular bisector of $\overline{QQ'}$.

c. Students should note that, even as the lengths of the segments and positions of the points change, the relationships among them do not. For example, the measures of the incoming and outgoing angles remain congruent and $\overline{AB}$ remains the perpendicular bisector of $\overline{QQ'}$.

d. Students should discover that the shortest total distance occurs when S and C are concurrent.

Discussion 2

a.
1. Students should conjecture that $\overline{AB}$ is the perpendicular bisector of $\overline{PP'}$.
2. Yes. Since $\overline{DP} \cong \overline{DP'}$ and $\angle PDA$ is a right angle, $\overline{AB}$ is the perpendicular bisector of $\overline{PP'}$.

b. Sample response: Since the shortest total distance occurred when S was at the same position as C, the path with the shortest total distance always passes through the reflection point on the mirror line.

c.
1. Since $m\angle QEB$ is 90°, the x-coordinates of Q and Q' are the same.
2. Since $\overline{EQ}$ and $\overline{EQ'}$ are congruent, the y-coordinates of Q and Q' have the same absolute value, but are opposites.
3. $\overline{DP}$ and $\overline{DP'}$

d.
1–2. The x-coordinates are the same, while the y-coordinates are opposites.
3. Yes, since the mirror was placed along the x-axis.

e.
1. Under a reflection in the x-axis, the image of a point (x, y) has coordinates $(x, -y)$.
2. Under a reflection in the y-axis, the image of a point (x, y) has coordinates $(-x, y)$.

f.
1. Student measurements should uphold the conjecture that the incoming and outgoing angles are congruent.
2. Yes, this conjecture becomes a theorem only if proved in general for all cases. One approach might be to demonstrate that ΔPDC is similar to ΔQEC, then show that $\angle PCD \cong \angle QCE$.