EXPLORATIONS™

Activity 5

Shortest Distance Between Points and Lines

Objectives

- To investigate the shortest distance between two points
- To investigate the shortest distance between a line and a point
- To investigate the shortest total distance between two points and a line

Cabri® Jr. Tools

Introduction

What is the shortest distance between two points in the plane? What is the shortest distance between a line and a point not on the line? The answer to these questions may seem intuitive, but can be difficult to explain analytically. In this exploration, you will investigate these minimization problems and support the analytic explanations with visual illustrations.

Part I: Shortest Distance Between Two Points

Construction

Draw two segments sharing an endpoint.

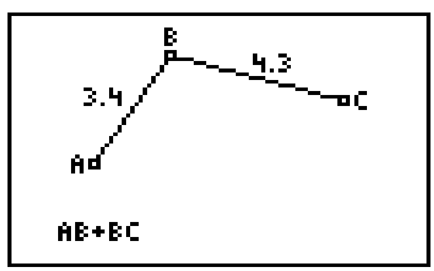
Draw a non-horizontal segment $\overline{AB}$ near the center of the screen.

Construct a second non-horizontal segment $\overline{BC}$.

Measure the length of segment $\overline{AB}$. Place the measurement near the segment.

Measure the length of segment $\overline{BC}$. Place the measurement near the segment.

Find the sum of *AB* and *BC*. Label the sum and place the sum near the bottom of the screen.



Not all measurements are shown.

Exploration

Drag point *A*, *B*, and/or *C* until the sum of *AB* and *BC* is at its lowest possible value.

Questions and Conjectures

Explain how this construction could be used to convince someone that the shortest distance between two points is a straight line.

Extension

Explain how this construction could be used to support the Triangle Inequality Theorem.

Part II: Shortest Distance Between a Line and a Point

Construction

Draw a line, a point not on the line, and a segment connecting them.

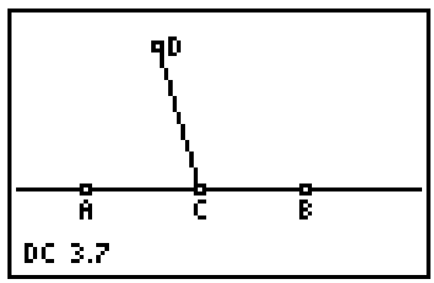
Clear the previous construction.

Draw a line $\overleftrightarrow{AB}$ near the center of the screen.

Draw a point *D* so that it is not on $\overleftrightarrow{AB}$.

Construct segment $\overline{DC}$ such that point *C* is on $\overleftrightarrow{AB}$.

Measure and label the distance from *D* to *C*. Consider this distance to be the distance from point *D* to the line.



Exploration

Since *C* was arbitrarily chosen, move point *C* along $\overleftrightarrow{AB}$ to find the shortest distance between *D* and $\overleftrightarrow{AB}$.

Move point *D* to a different location and again move point *C* until you find the shortest distance between *D* and $\overleftrightarrow{AB}$. Note any patterns.

Move $\overleftrightarrow{AB}$ to a different location and again move point *C* until you find the shortest distance between *D* and $\overleftrightarrow{AB}$. Note any patterns.

Questions and Conjectures

Make a conjecture about the location of point *C* and point *D* when the distance between *D* and $\overleftrightarrow{AB}$ is the shortest. It may be helpful to find the measure of $\angle ACD$ when the distance between *D* and $\overleftrightarrow{AB}$ is the shortest.

Part III: Shortest Total Distance

Construction

Draw a line, two points not on the line, and segments that connect the points to one point on the line.

Clear the previous construction.

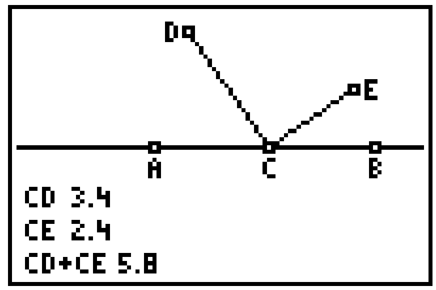
Draw a horizontal line $\overleftrightarrow{AB}$.

Draw two points, *D* and *E*, on the same side of $\overleftrightarrow{AB}$.

Construct a third point, *C*, on $\overleftrightarrow{AB}$.

Construct $\overline{CD}$ and $\overline{CE}$.

Measure the length of segments $\overline{CD}$ and $\overline{CE}$. Label the measurements and place them near the bottom of the screen.

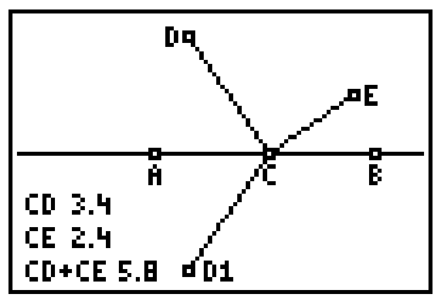


Calculate the sum of the lengths of segments $\overline{CD}$ and $\overline{CE}$. Label the calculation and place it near the bottom of the screen.

Exploration

Drag point *C* along $\overleftrightarrow{AB}$ until the sum *CD* and *CE* is as small as possible.

Use the **Reflection** tool to reflect $\overline{CD}$ across $\overleftrightarrow{AB}$ by selecting $\overline{CD}$ and then selecting $\overleftrightarrow{AB}$. Label this new segment $\overline{CD1}$. Use various measurement tools (**Distance and Length** and **Angle**) to investigate the relationship between $\overline{CD}$ and $\overline{CD1}$. Observe this relationship for different locations of points *D* and *C*.



Move points *D* and/or *E* and repeat the first Exploration. Notice the relationship between the points *D*1, *C*, and *E* when the sum of the lengths of the segments is at a minimum.

Measure $\angle ACD$ and $\angle ECB$ and repeat the previous Exploration. Notice any relationship that appears to exist between these angles when the sum of the lengths of the segments is at a minimum.

Questions and Conjectures

1. Write a conjecture describing the conditions under which the sum of the distances *CD* and *CE* is minimized.
2. Make a conjecture about the relationship between $\overline{CD}$ and $\overline{CD1}$.

3. Write a paragraph to support the following: Given that *D* and *E* are two points on the same side of $\overleftrightarrow{AB}$ and point *C* is on $\overleftrightarrow{AB}$, such that $CD + CE$ is at a minimum, then $\angle ACD$ is congruent to $\angle ECB$.

4. Not every example you investigated showed that $\angle ACD$ and $\angle ECB$ have the same measure. Explain why this might be true.

Teacher Notes

Activity 5

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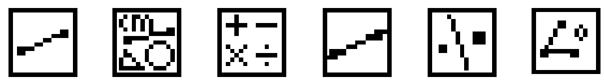
Cabri® Jr. Tools



Additional Information

Part I of this exploration could be done as a classroom demonstration to establish the Triangle Inequality Theorem.

Part I: Shortest Distance Between Two Points

Answers to Questions and Conjectures

Explain how this construction could be used to convince someone that the shortest distance between two points is a straight line.

Let the sum of *AB* and *BC* represent the distance from point *A* and *C*. This sum can be minimzed when point *B* is on a segment connecting points *A* and *C*. In Activity 1, it was found that if a point is on a segment between two points, then the sum of the parts is equal to the original segment. Therefore, the shortest distance between *A* and *C* is the straight line distance between *A* and *C*.

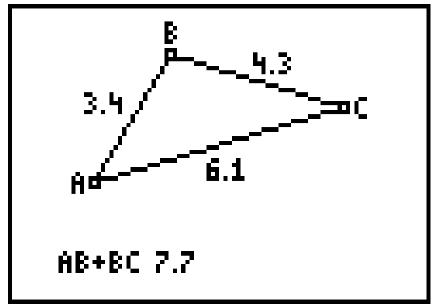
Extension

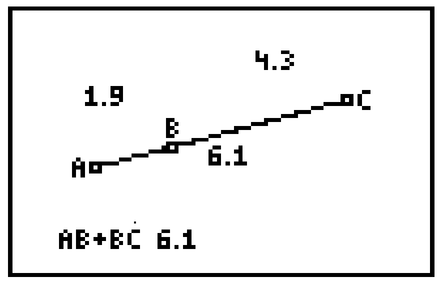
Explain how this construction could be used to support the Triangle Inequality Theorem.

For a figure to be a triangle, the sum of the lengths of two of the sides must be larger than the length of the third side.

This rule is called the Triangle Inequality Theorem. This means that the length of segment $\overline{AC}$ must be shorter than the sum of the lengths of segments $\overline{AB}$ and $\overline{BC}$ for any position of point *B* not on $\overline{AC}$ or an extension of $\overline{AC}$.

If *B* is on $\overline{AC}$, then no triangle is formed since there will not be three distinct sides. In this case, $AC = AB + BC$.



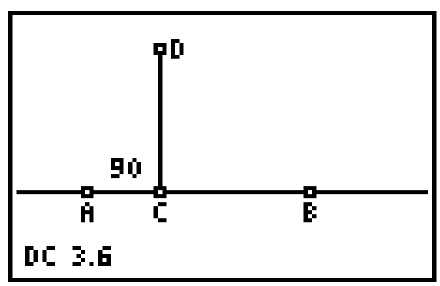


Part II: Shortest Distance Between a Line and a Point

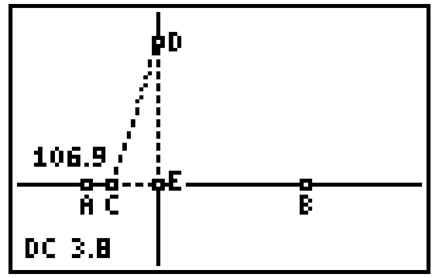
Answers to Questions and Conjectures

Make a conjecture about the location of point *C* and point *D* when the distance between *D* and $\overleftrightarrow{AB}$ is the shortest. It may be helpful to find the measure of $\angle ACD$ when the distance between *D* and $\overleftrightarrow{AB}$ is the shortest.

$\overline{CD}$ is shortest when the measure of $\angle ACD$ is 90°. This implies that $\overline{CD}$ is perpendicular to $\overleftrightarrow{AB}$. The distance between a point and a line is equal to the length of the segment that is perpendicular to the line having one end point on the line and the other as the given point.



To prove this conjecture analytically, move point *C* toward point *A.* Construct the perpendicular from *D* to $\overleftrightarrow{AB}$. Label the intersection of the perpendicular point *E.* This forms the right triangle ΔCDE. As long as point *C* is not coincident with point *E*, right triangle ΔCDE exists. Since $\overline{CD}$ is the hypotenuse of ΔCDE, $\overline{CD}$ will be longer than $\overline{DE}$. Therefore, $\overline{DE}$ is the shortest segment that can be constructed having one endpoint on $\overleftrightarrow{AB}$.



Part III: Shortest Total Distance

Additional Information

Because the Cabri® Jr. application rounds values to the nearest tenth, the minimum sum may exist in several locations on the screen. Remind students that the minimum sum occurs near the center of the locations that produce the minimum value.

Answers to Questions and Conjectures

1. Write a conjecture describing the conditions under which the sum of the distances *CD* and *CE* is minimized.

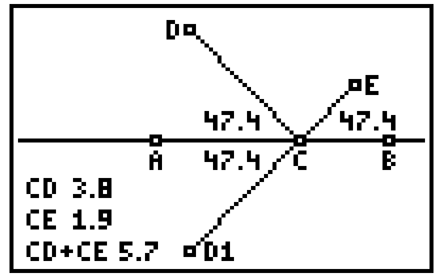
   From Part I, we know that the shortest distance between two points is a straight line. $CD + CE$ is minimized when points *C*, *D*1, and *E* are collinear. When the three points are collinear, the distance from *D*1 to *E* equals the sum of *CD* and *CE*.

2. Make a conjecture about the relationship between $\overline{CD}$ and $\overline{CD1}$.

   $\overline{CD1}$ is congruent to $\overline{CD}$ and $\angle ACD$ is congruent to $\angle ACD1$. The properties of a reflection are studied in more detail in Activity 7, Reflections in the Plane.

3. Write a paragraph to support the following: Given that *D* and *E* are two points on the same side of $\overleftrightarrow{AB}$ and point *C* is on $\overleftrightarrow{AB}$, such that $CD + CE$ is at a minimum, prove that $\angle ACD$ is congruent to $\angle ECB$.

   Since angle measures are preserved in a reflection, $\angle ACD$ is congruent to $\angle ACD1$. When point *C* is located as the minimized sum of the distances $CD + CE$, point *D*1, *C* and *E* are collinear. $\angle ACD1$ and $\angle ECB$ are vertical angles and are congruent. Therefore, $\angle ACD$ and $\angle ECB$ are congruent.



4. Not every example you investigated showed that $\angle ACD$ and $\angle ECB$ have the same measure. Explain why this might be true.

   The Cabri® Jr. application measures each angle to two decimal places and displays to only one decimal place; therefore, the measurement displayed may appear to be inaccurate. See “A Specific Cabri Jr. Issue” on page vi.