EXPLORATIONS

Activity 4

Objective

- To use Geoboard to determine the areas of quadrilaterals between parallel lines

Materials

- TI-73
- Student Activity pages (pp. 41 – 45)

Slide Along

In this activity you will

- Find the areas of parallelograms and trapezoids.
- Discover shortcut ways of finding the areas of parallelograms and trapezoids.

Introduction

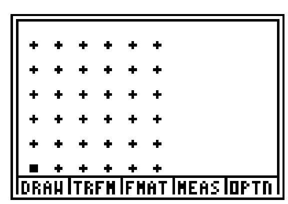
Earlier, you found the areas of triangles between parallel lines. In this activity, you will use a similar approach to find the areas of parallelograms and trapezoids. The activity suggests that the area of a parallelogram is a special case of the area of a trapezoid.

All parallelograms can be changed into rectangles with the same area. Finding the area of a rectangle is easy because we know it is the product of the base and the height. This activity explores the idea of changing parallelograms into rectangles to help make finding the area easier. In a similar way, we can change the area of any trapezoid into another trapezoid with two consecutive right angles having the same height and base of the original trapezoid.

Investigation

This investigation will help you find the area of quadrilaterals between parallel lines.

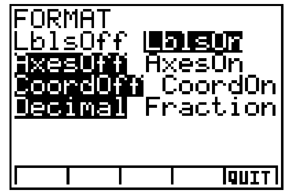
1. From the main Geoboard menu, select **2:6x6**.



2. To format the geoboard, select **FMAT** and make sure that the following settings are selected:

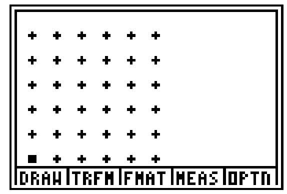
   LblsOn (Labels are on)
   AxesOff (Axes are off)
   CoordOff (Coordinates are off)
   Decimal (Measurement is in decimal form)

   Select **QUIT** to exit the FORMAT menu.



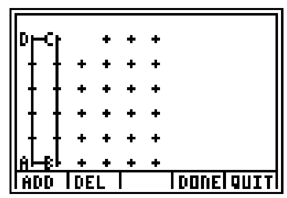
3. When you are ready to begin, your geoboard should look like the screen at the right.

   On this geoboard, consider the top row and bottom row to be two parallel line segments. Since parallel lines are always the same distance apart everywhere, the distance between the top row and bottom row will always be five units.



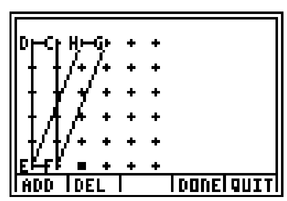
4. Between these two parallel lines, build three different parallelograms. Each parallelogram will have the same base and the same height.

   For the first parallelogram, start at the bottom left peg and select **DRAW, ADD** [►] **ADD** [▲] [▲] [▲] [▲] [▲] **ADD** [◄] **ADD** [▼] [▼] [▼] [▼] [▼] **DONE**. You have just constructed parallelogram ABCD. The information for this parallelogram is shown below.



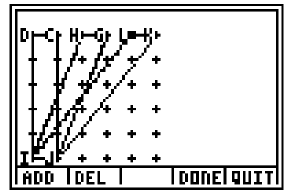
Parallelogram	Base	Base Length	Height	Area
ABCD	AB	1	5	5 square units

5. Now use AB as the common base to construct the next parallelogram. Starting at point A, move counterclockwise to create the parallelogram shown below. Complete the next row of the table for parallelogram EFGH. Use the **QUIT, MEAS, 2:Area** feature of your TI-73 to find the area of parallelogram EFGH.



Parallelogram	Base	Base Length	Height	Area
EFCD	EF	1	5	5 square units
EFGH	EF			

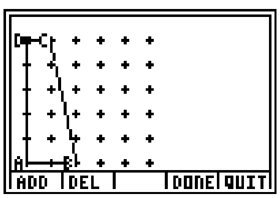
6. Now use EF as the common lower base. Starting at point E, move counterclockwise to construct the last parallelogram as shown at the right. Complete the next row of the table for parallelogram IJKL.

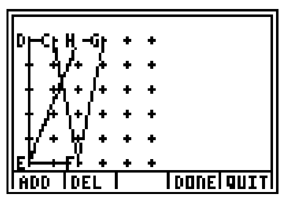


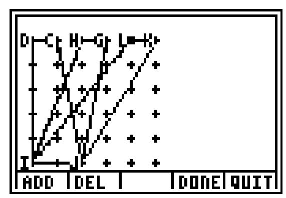
Parallelogram	Base	Base Length	Height	Area
IJCD	IJ	1	5	5 square units
IJGH	IJ			
IJKL				

Although all three parallelograms look quite different, they all have the same base and the same height. Whenever this is true, the parallelograms always have equal areas. This means that parallelograms between the same pair of parallel lines with a common base always have the same area. Since this is true, every parallelogram between the same parallel lines can be shifted to a special shape (a rectangle) with the same base and height. Since parallelograms IJGH and IJKL are between the same parallel lines, they can be shifted into rectangle IJCD having the same area.

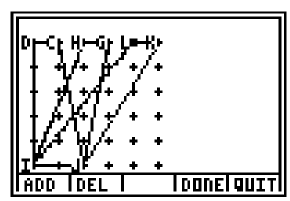
7. Erase the geoboard. Now build the following three trapezoids consecutively. Make sure that you completely draw each trapezoid.







8. Using this picture of all three trapezoids, complete the table below.



Trapezoid	Bottom Base Length	Top Base Length	Height	Area
IJCD	2 units	1 unit	5 units	7.5 square units
IJGH				
IJKL				

All three trapezoids have bases and heights of the same length. All such trapezoids between the same parallel lines have the same area. Since trapezoids IJGH and IJKL are between parallel lines, they can be shifted into trapezoid IJCD having the same area. Since trapezoid IJCD has two consecutive right angles, its area is easier to find.

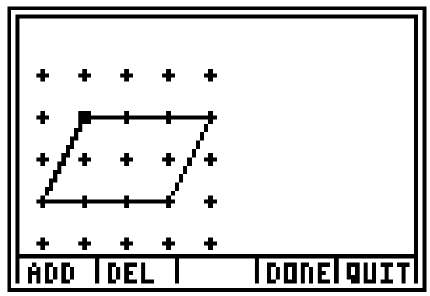
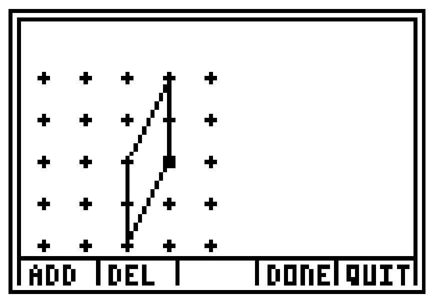
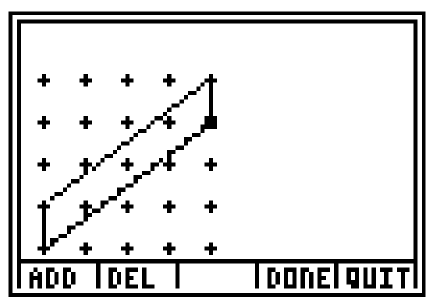
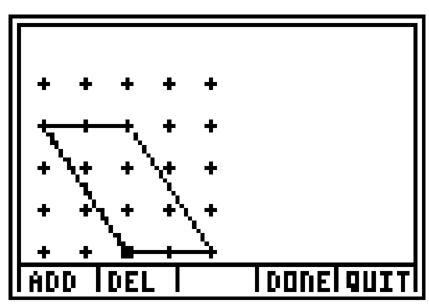
Student Activity

Name ____________________________

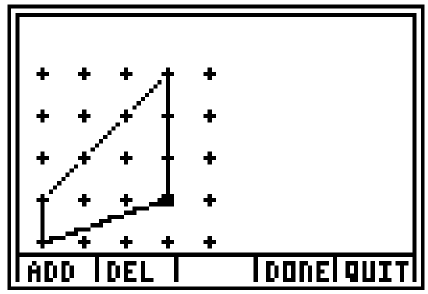
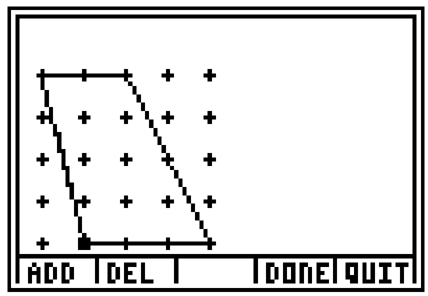
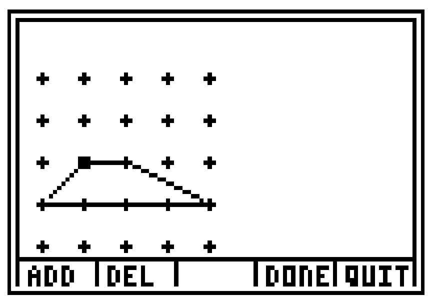
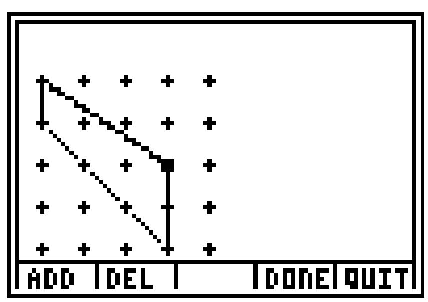
Date ____________________________

Activity 4.1: Slide Along

Shift each parallelogram below between parallel lines in order to make a rectangle with the same area as the original parallelogram. Sketch the parallel lines and a picture of each new rectangle. Each rectangle constructed can be made using one side of the original parallelogram. Find the area of each new rectangle. Then use your TI-73 to check the area of the original parallelogram.

1. Area: __________ square units	
2. Area: __________ square units	
3. Area: __________ square units	
4. Area: __________ square units	

Shift one side of each trapezoid below between parallel lines in order to make another trapezoid with two consecutive right angles. Sketch the parallel lines and pictures of each trapezoid. Find the area of each new trapezoid. Then use your TI-73 to check the area of the original trapezoid.

5. Area: __________ square units	
6. Area: __________ square units	
7. Area: __________ square units	
8. Area: __________ square units	

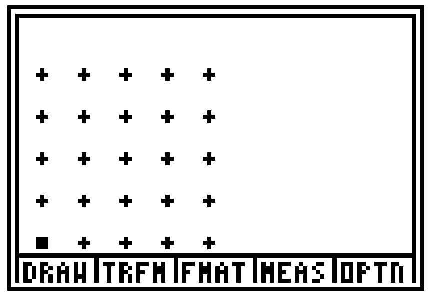
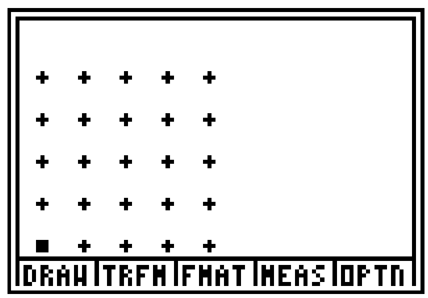
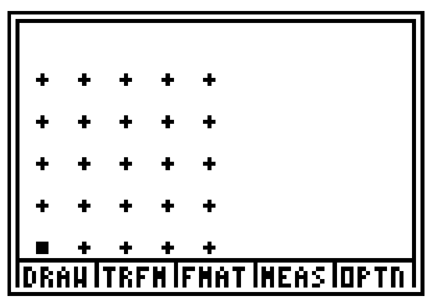
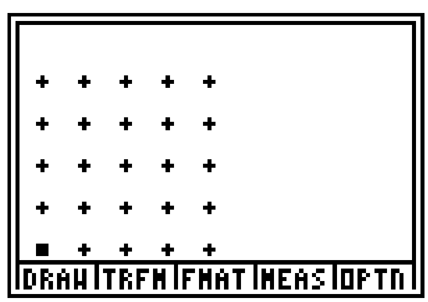
Student Activity

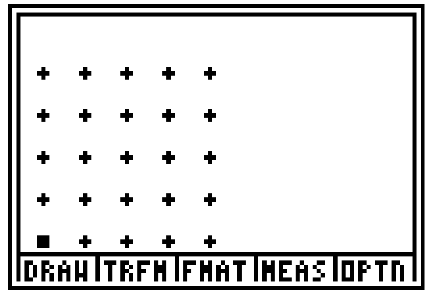
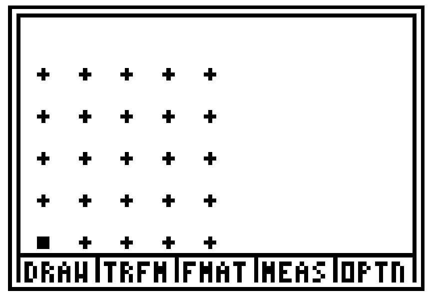
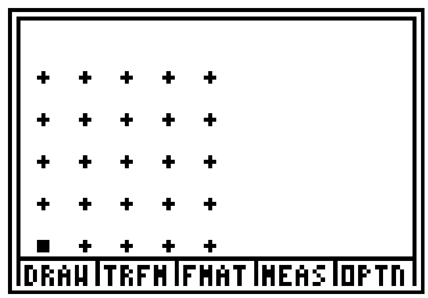
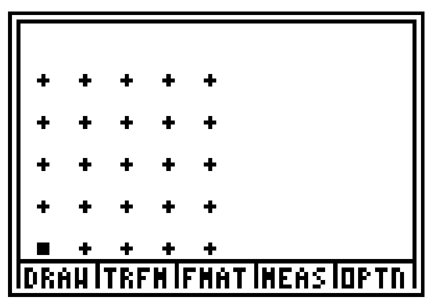
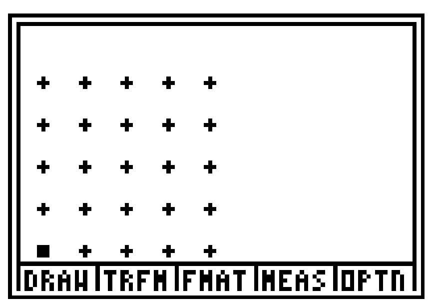
Name ______________________________

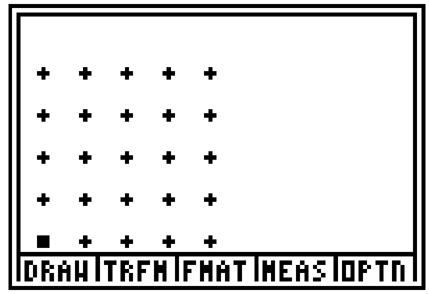
Date ______________________________

Activity 4.2: Build Your Own Shapes

Construct the following shapes on your geoboard and record a dot picture of each shape. Check the area using your TI-73.

1. A square with area 2	
2. A triangle with area 1½	
3. A triangle with area 2½	
4. A parallelogram with area 3	

5. A triangle and a square with equal areas	
6. A rectangle twice as long as it is wide with area 4	
7. A trapezoid with area 5	
8. A hexagon with area 3	
9. A square with area 10	

10. A trapezoid with area 7½	

④ The quadrilateral has 8 boundary points	④ The shape has 2 consecutive obtuse angles
④ One base is 3 times the length of the other base	④ The trapezoid has an area of 6 square units
❹ One angle has a measure of 45 degrees	❹ There are 3 interior points

Teacher Notes

Activity 4

Slide Along

Objective

- To use Geoboard to determine the areas of quadrilaterals between parallel lines

NCTM Standards

- Select and apply techniques and tools to accurately find…area…to appropriate levels of precision

Investigation

Moving a parallelogram's base or bases between parallel lines is sometimes called "sheering." Students should become familiar with the idea that moving a parallelogram's base along a line that is parallel to the other base never changes its height nor its area. By shifting a parallelogram's base between parallel lines, we can change any parallelogram into a rectangle with the same area. This is true for either base of the parallelogram.

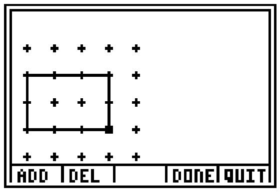
Although the area of every parallelogram can be determined by multiplying its base times its height, these values are *not* always easy to determine. On the geoboard, the base lengths of parallelograms are frequently irrational numbers (which can be readily found) but the altitudes often have one endpoint that is *not* a geoboard pin point, which makes it difficult to determine these lengths. However, the area can be determined without the formula by using techniques investigated in earlier activities, such as the surrounding rectangle. Similar comments apply equally well to trapezoids since their altitudes may not be easily determined.

Answers to Student Activity pages

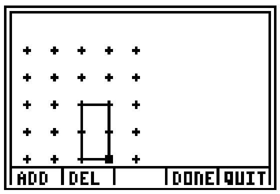
Activity 4.1: Slide Along

There are two possible diagrams for each problem.

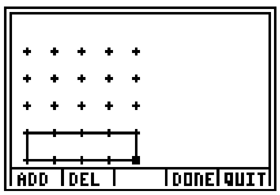
1. 6 square units



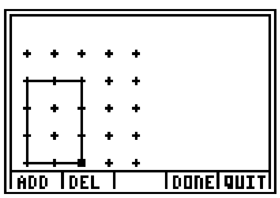
2. 2 square units



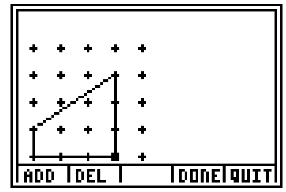
3. 4 square units



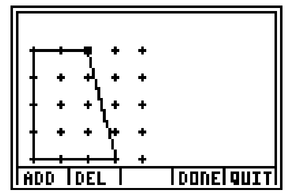
4. 6 square units



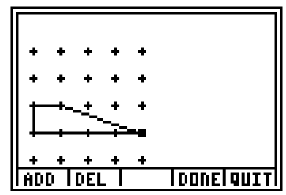
5. 6 square units



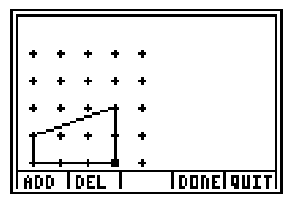
6. 10 square units



7. 2.5 square units

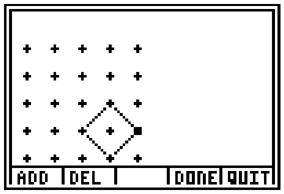


8. 4.5 square units

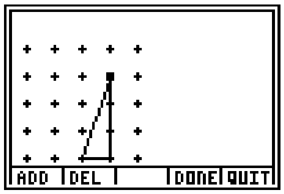


Activity 4.2: Build Your Own Shapes

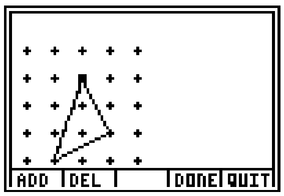
1.



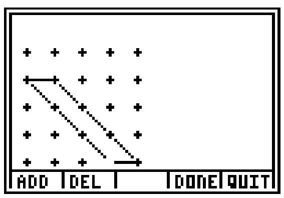
2.



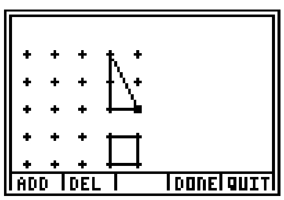
3.



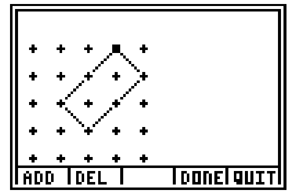
4.



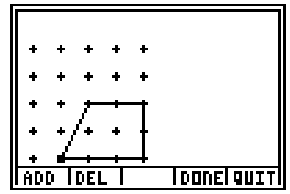
5.



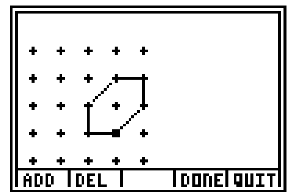
6.



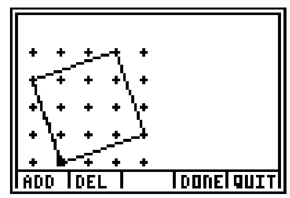
7.



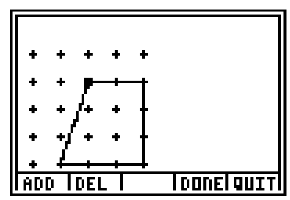
8.



9.



10.



Group Problem Solving: ***The area of quadrilaterals between parallel lines***

The Group Problem Solving cards are challenge problems that can be used alone or with the individual sections of this book. The problems are designed to be used in groups of four (five or six in a group are possibilities using the additional cards) with each person having one of the first four clues. Students can read the information on their cards to others in the group but all should keep their own cards and not let one person take all the cards and do the work.

The numbers at the top of the cards indicate the lesson with which the card set is associated. The fifth and sixth clues (the optional clues) have the lesson number shown in a black circle.

The group problems can be solved using the first four clues. The fifth and sixth clues can be used as checks for the group's solution or they can be used as additional clues if a group gets stuck. Some problems have more than one solution. Any shape that fits all the clues should be accepted as correct.

With a little experience, students should be able to design their own group problems. They could then switch problems with other groups for additional problem solving practice.

One solution for this problem solving exercise: