NUMB3RS Activity: The Leaf Drops
Episode: “Finders Keepers”

Topic: Experimental Probability, Histograms **Grade Level:** 9 - 10
Objective: Students will estimate the experimental probability of various events and use the results to make decisions.
Time: 20 - 25 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator, meter stick, and paper

Introduction

In “Finders Keepers,” a world-class racing yacht named the Cheetah sinks in the ocean. The FBI determines that the Cheetah may have been used to smuggle advanced weapon systems to terrorists, and wants to find the sunken yacht as soon as possible. The NSA (National Security Agency) has the exact location where the Cheetah’s homing beacon shut down. It is assumed that the ship sank at this same location. The two NSA agents are puzzled when they discover that the ship is not directly below this location. Charlie is just as mystified until he sees a leaf falling to the ground. The NSA agents assumed that when the boat sank it went directly to the bottom of the ocean. Charlie explains that the boat, like a falling leaf, probably did not descend straight down like an anchor. In this activity, students will try to determine the most probable resting location of objects as they descend.

Discuss with Students

In this activity, Question 1 asks the students to calculate the maximum search area for the Cheetah if it drifts up to 3 meters horizontally for every meter it sinks. The direction of the sinking ship is not known, so the search area would be a circle at the bottom of the ocean. The area of the circle is the maximum search area for the Cheetah. You may want to have the students draw the cone shape to help them find the radius of this circle.

You should discuss the experiment in Question 2 with your students. Make sure you discuss that the results of the experiment will be more accurate if they are consistent with their dropping procedures (i.e., toss from the same height each trial, hold the paper parallel to the floor in the same way before dropping, etc.). Students may use any type of paper to do their drops and the paper can be cut into any size. It may be easiest if students use a typical 8.5” × 11” piece of paper. You could have the students cut a piece of paper into a triangle to make it look more like a ship so that it better matches the context of the show. If you decide to aggregate the data, you may want to have all your students use the same shape. Emphasize that students should practice their drops a few times before they collect the data. You may want to have the students collect data in pairs, because it could take a long time for each student to complete 50 drops individually.

Questions 3 through 6 ask students to use the data to make calculations using experimental probability. Most of their answers will be different since they will all have different data. You may want to aggregate the data and ask the same or similar questions using this data to check for understanding and to compare results.

Question 7 asks students to compare the maximum search area (i.e., the area of the circle that would contain all their experimental results) with the search area that would contain 90 percent of the experimental measurements. The sample data will likely show that the maximum search area is almost double the area of the 90-percent search area. You should discuss as a class the various implications this finding has on how the NSA conducts the search. Students will get

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different results, which should lead to an interesting discussion. You may want to have students compare their results with the aggregate data.

The Extensions page introduces the "Find the Cheetah" game. Make sure your students understand the rules of the game before playing. The goal of the game is to help them use the histogram to make decisions involving probability. Make sure that students use the same intervals that they used when they made their histograms to answer Questions 3 through 7 in the student activity. You should discuss their strategies for winning the game as a class. The students should realize that the intervals correspond to different sections of a circle and the area of each section is different. The farther the interval is from the anchor point, the larger the area will need to be balanced with the frequency of the measurements shown in the histogram.

Student Page Answers:
1. *The radius for the search area is 12 km; the area is approximately 452 km^2.* **2.** *Data will vary. Sample results are shown below:*

Distance from Anchor Point to Cheetah A (centimeters)

49	35	116	12	13	84	55	38	100	19
29	46	23	24	36	76	25	25	48	9
33	46	72	47	72	16	98	44	79	41
114	34	75	61	28	79	56	71	27	109
53	25	23	28	55	16	11	58	55	36

3. *Answers will vary. Using the sample results shown above there are 6 distances longer than 80 cm out of 50 trials, so the experimental probability is* $\frac{6}{50} = 0.12$ *.* **4.** *Answers will vary. The sample data has 23 measurements within 40 cm so the probability is* $\frac{23}{50} = 0.46$ *.* **5.** *Answers will vary. There are 5 data points above 90 cm in the sample data so at least 90 percent of the measurements are within 90 cm.*
6. *Answers will vary. In the sample data, the maximum distance was 116, so the maximum search area for the sample data is* $3.14 \times 116^2 = 42{,}252$ *cm^2.* **7.** *Answers will vary. For the sample data, 90 percent of the measurements fall within a search area of* $3.14 \times 90^2 = 25{,}434$ *cm^2. The maximum search area is almost double the area of the 90-percent search area.*

Terry Wyberg, University of Minnesota, Minneapolis, MN

Name: ___________________________________ Date: _________________

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1. Suppose the Cheetah drifts up to 3 meters horizontally for every meter it descends, and assume that the depth of the water is 4,000 meters. Determine the maximum search area in square kilometers.

Experimental Probability

For the NSA to search such a large area, it would still take a long time. The NSA would not need as much time to search if the area was smaller. You will now carry out an experiment to narrow the search for the Cheetah.

Suppose you are holding a piece of paper parallel to the ground. Imagine how the paper will drift to the ground if you let go of it. Will it drop straight down? Will it drift in a predictable path? How far will it drift from where you dropped it? Will it fall to the same spot each time? You will be able to answer these questions and more after doing an experiment. To do this experiment, you will need a piece of paper to drop, a piece of paper to denote the "anchor point," and a meter stick. It may be easier for you to do this experiment with a partner.

2. Follow the steps below:

i.) Mark a dot on a piece of paper and place it on the ground. Label this point the "anchor point." This point denotes the place to which the ship would sink if it sank straight down like an anchor.

ii.) Take another piece of paper to represent the Cheetah. Mark a dot in the center of this sheet and label this point Cheetah A.

iii.) Hold the Cheetah piece of paper 150 cm (about 5 ft) above the anchor point, parallel to the ground, and make sure the point Cheetah A is directly over the anchor point.

iv.) Drop the square piece of paper. Once the paper comes to rest on the ground, use your meter stick to measure the distance point Cheetah A is from the anchor point. Repeat this experiment 50 times. Record the measurements, rounded to the nearest centimeter, in the table on the next page. Repeat a trial if the piece of paper hits you or some object on the way down. Make sure you do this experiment in an area that is open enough to allow the piece of paper to fall. Do a few practice drops to test the area and to help you be consistent.

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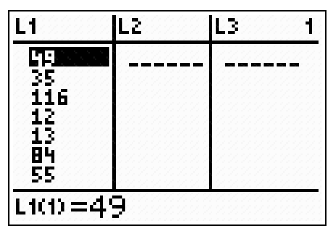
Distance from Anchor Point to Cheetah A (centimeters)

3. Suppose you are going to drop the piece of paper again. Use your data to estimate the probability that Cheetah A will be more than 80 centimeters from the anchor point.

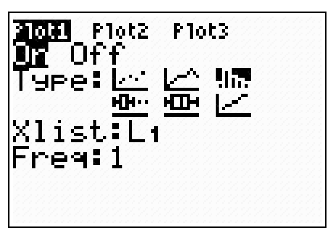
You used **experimental probability** to answer Question 3 above. There are a large number of variables affecting the situation, and it is not clear how to sort them out. In a situation like this, many scientists use experimental probability to determine the likelihood of certain events that are difficult to determine theoretically. For example, the paper you dropped did not end up at the same place each time you dropped it, even though you tried to drop it the same way each time. Many variables that are difficult to control affect the flight of the paper, but if you systematically record the results you may begin to notice patterns in the way it falls.

It is easier to calculate experimental probabilities if the data are organized. Use your graphing calculator to make a histogram of your data.

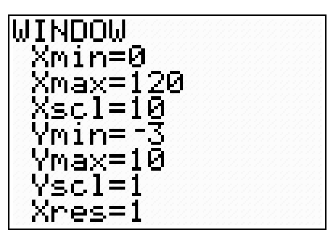
Press [STAT], then select **1:Edit…**. Enter your data from the experiment in L_1. The numbers shown at the right are partial results from one experiment.



Press [2nd] [STAT PLOT] and select **1:Plot1**. Use the settings shown to the right to create a histogram of the data.



Make appropriate window settings for your data. The **Xscl** sets the widths of the bars on the histogram.



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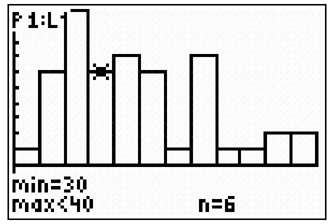
Press [GRAPH], and then press [TRACE] to find how many data points are in each interval. Note that the trace on the 4th interval shows there are 6 measurements (n = 6) greater than or equal to 30 cm and less than 40 cm. Compare your graph with other students' graphs.



4. Use your graph to estimate the probability that Cheetah A will land within 40 cm of the anchor point.

5. Use your graph to estimate the distance from the anchor point within which at least 90 percent of the measurements fall.

6. Estimate the maximum search area for finding Cheetah A using your data.

7. Use your data to determine the smallest search area needed to find Cheetah A 90 percent of the time. Compare this area with the maximum search area. Explain how you could use this result to give advice to the NSA as they search for the Cheetah.

Terry Wyberg, University of Minnesota, Minneapolis, MN

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Find the Cheetah

Play the game “Find the Cheetah” with a partner. The object of this game is to correctly guess how far a piece of paper (Cheetah A) ends up from the anchor point. When you are not looking, have your partner drop the piece of paper as done previously and secretly record the distance from the anchor point. You “Find the Cheetah” by guessing an interval (e.g., 50 cm to 60 cm). Keep guessing intervals until you find the one that contains the correct distance. The intervals you guess should match the ones you used in your histogram above. Determine how many intervals you need to guess in order to find the Cheetah. Repeat the turn with another classmate. The person who finds the Cheetah by guessing the fewest number of intervals wins.

Play the game a few times and explain how the graph you created influences your strategy for winning the game.

Experimental Probability

- In the episode, Charlie explains that the sophisticated design of the Cheetah’s hull would make it fly through the water and the boat would drift horizontally 4 or 5 feet for every foot that it descends. Make a paper airplane (this will have a more sophisticated design than the piece of paper you used above) and carry out an experiment similar to the one in the previous activity. Throw the airplane from the same height along a line taped to the floor. Try as much as possible to make your tosses of the airplane consistent. Measure both the distance the plane travels as well as the perpendicular distance the plane is away from the tape on the floor. Try to determine the best search area for this plane.

- Carry out the same experiment as above but place a fan in different locations to represent air currents, and see how it affects the path of the plane.

- Go to **http://illuminations.nctm.org/ActivityDetail.aspx?ID=79** to investigate the connections between experimental and theoretical probability.

Terry Wyberg, University of Minnesota, Minneapolis, MN