Geometric Sequences & Series

ID: 8674

Time required
45 minutes

Activity Overview

In this activity, students begin by finding common ratios of geometric sequences on a spreadsheet and then create scatter plots of the sequences to see how each curve is related to the value of the common ratio and/or the sign of the first term of the sequence. Students then generate geometric sequences on a spreadsheet and develop a general explicit formula to find any term of the sequence. This is followed by discovering the geometric mean of two numbers and finding the sum of a geometric series, both on a spreadsheet and on a calculator page using sigma notation.

Topic: Sequences, Series, & Functions

- *Calculate the nth term of a sequence defined by an algebraic expression.*
- *Given several terms of a sequence, write an algebraic expression that generates the nth term.*
- *Graph the first n terms of a sequence.*
- *Derive and apply a formula for the sum of the first n terms of a geometric sequence.*

Teacher Preparation and Notes

- *This activity is designed to be used in an Algebra 2 classroom.*
- *Students should begin this activity knowing that a sequence is an ordered list of numbers that follows a pattern and that a series is an indicated sum of a sequence. For example, 1, 2, 3, 4 is a sequence and 1 + 2 + 3 + 4 is a series.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- *Information for an optional extension is provided at the end of this activity, both on the student worksheet and in the student TI-Nspire document. Should you not wish students to complete the extension, you may delete the extension from the student TI-Nspire file and have students disregard that portion of the student worksheet.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "8674" in the keyword search box.***

Associated Materials

- *GeomSeqSeries_Student.doc*
- *GeomSeqSeries.tns*
- *GeomSeqSeries_Soln.tns*

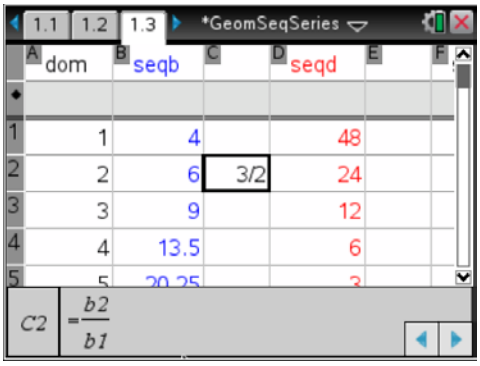
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Arithmetic Sequences and Series (TI-Nspire technology) — 8638*
- *Sum of an Infinite Geometric Sequences (TI-Nspire technology) — 13736*
- *Geometric Series (TI-Nspire technology) — 10224*

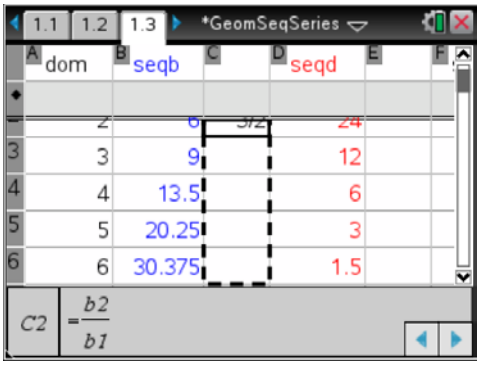
Problem 1 – Scatter Plots

Step 1: There are five finite sequences on page 1.3. They appear in Columns B, D, F, H, and J. Each has six terms. Instruct students to find the ratio of each term and its previous term by writing and copying formulas.



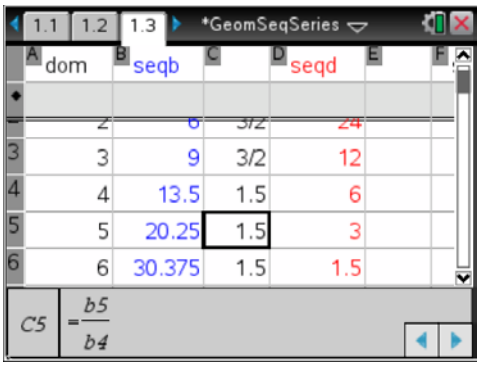
In cell C2, have students type **=b2/b1** to find the ratio between the first two terms. The cell displays 3/2; but notice that when the cell is selected, the formula appears at the bottom of the screen.

To copy this formula to find the ratios between the remaining terms of the sequence, tell students to go to cell C2 and select **MENU > Data > Fill**. Then, arrow down to highlight cells C2 through C6, and press enter.



Students should now arrow down through each cell, looking at the formulas used to determine the value of that cell.

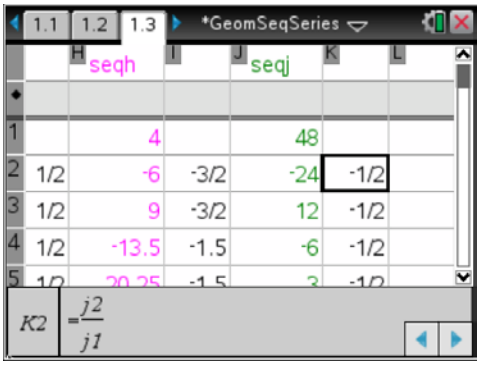
Notice that $\frac{3}{2} = 1.5$, and therefore all of these ratios are the same.



TI-Nspire Navigator Opportunity: *Live Presenter*

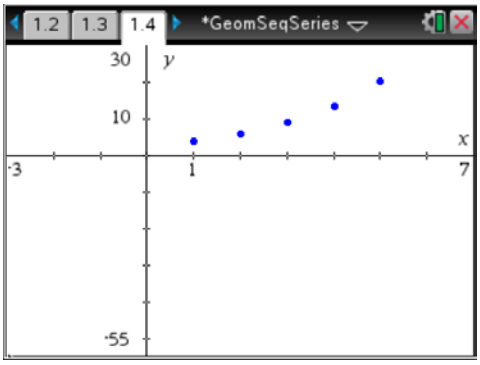
See Note 1 at the end of this lesson.

Step 2: Have students repeat the process of finding ratios between consecutive terms for the sequences in Columns D, F, H, and J.



Step 3: Tell students to advance to page 1.4 and make a scatter plot of the domain, (the natural numbers in Column A), and the terms in Sequence B.

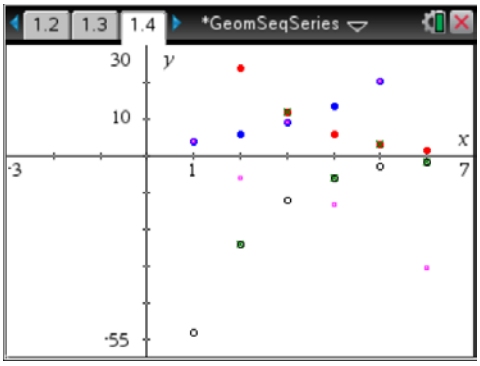
To create a scatter plot, select **MENU > Graph Entry/Edit > Scatter Plot**. Press var and select **dom** for *x* and **seqb** for *y*.



Step 4: Instruct students to create the scatter plots of the remaining four sequences in the same manner, on the same page. For each scatter plot, students should select **dom** (the title of Column A) as the domain, rather the **dom2**, which will be used later.

After the third sequence, students may wish to temporarily hide the three scatter plots to be better able to see the last two scatter plots. To hide them, select **MENU > Actions > Hide/Show**, select the plot(s) to hide, and press esc.

Students may need to adjust their viewing window to better see all the points in all the scatter plots.



TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 2 at the end of this lesson.

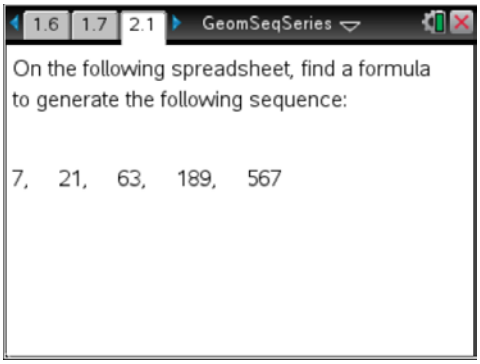
Step 5: Discuss that sequences with a common ratio, *r*, are called ***geometric sequences***. Have students make conjectures about the scatter plot of a geometric sequence, the common ratio, and/or the sign of the first term of the sequence.

More than saying the graph is a curve; students should conjecture when the graph will rise or fall from left to right and which quadrant(s) the graph will appear in. An additional spreadsheet (with **dom2** as Column A) and blank graph appear on pages 1.6 and 1.7 to allow for students to test and revise their conjectures.

Have students share their conjectures with the rest of the class.

Problem 2 – Explicit Formulas

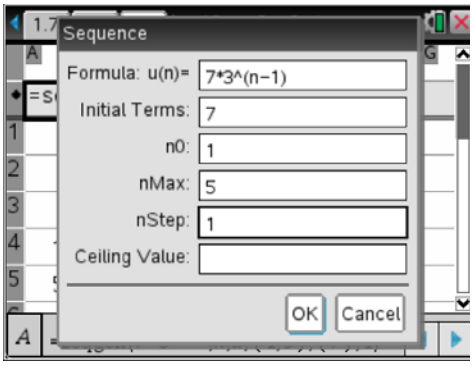
Step 1: A five-term geometric sequence appears on page 2.1. Ask students to find the common ratio (3). Then ask how each term can be written as an expression that includes a power with a base of this ratio ($7(3)^0$, $7(3)^1$, etc).



Lastly, ask how the exponents are related to the term numbers to derive the explicit formula $u_n = 7(3)^{n-1}$.

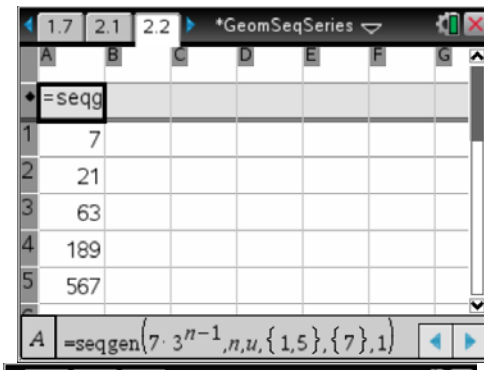
Note that many texts use the variable *a* instead of *u*, but the handheld uses *u*, so we will use *u* here.

Step 2: Show students how to enter a formula into a spreadsheet. With the gray box at the top of Column A selected, have students select **MENU > Data > Generate Sequence**.

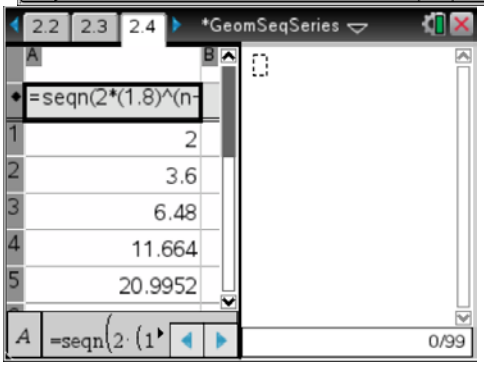


Tell them to enter the formula in the first box, the first terms in the next box, and the number of terms in the last box. (See the screenshot at right.)

After selecting **OK**, cells A1 through A5 are populated with the first 5 terms of the sequence.

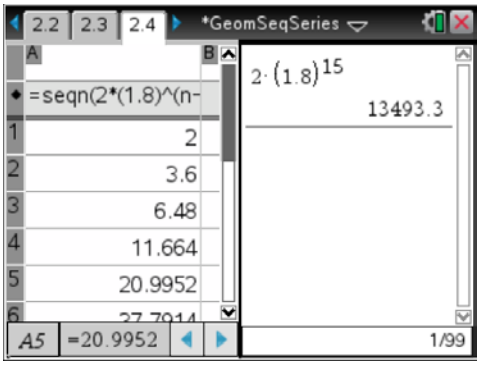


Step 3: Have students practice on their own to find the 16th term of a geometric sequence with a first term of 2 and common ratio, *r*, of 1.8 using the spreadsheet on page 2.4.



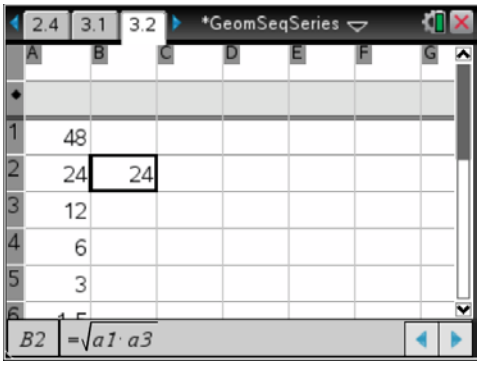
To view more decimal places, students may resize the column (**MENU > Actions > Resize > Resize Column Width**).

Next, tell students to look at the formulas they have written so far to determine a general explicit rule for finding any term of a geometric sequence $\left(u_n = u_1(r)^{n-1}\right)$. Then they can press ctrl + tab to move to the *Calculator* application to find the 16th term of the sequence by substituting 2 and 1.8 into their formula.



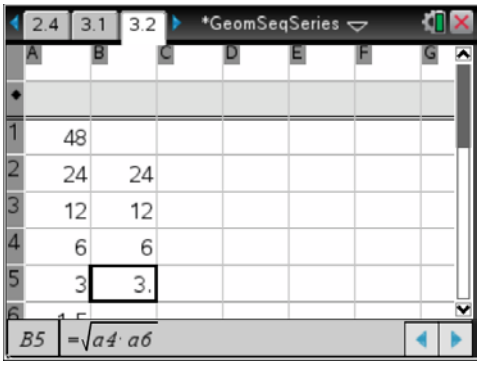
Problem 3 – An Interesting Observation

Step 1: A five-term geometric sequence appears on page 3.2. Have students to move to cell B2 and write a formula in the cell to find the square root of the product of the first and third terms.



Step 2: Next, they should copy this formula down through cell A5 (again, using **Fill**) and make a conjecture.

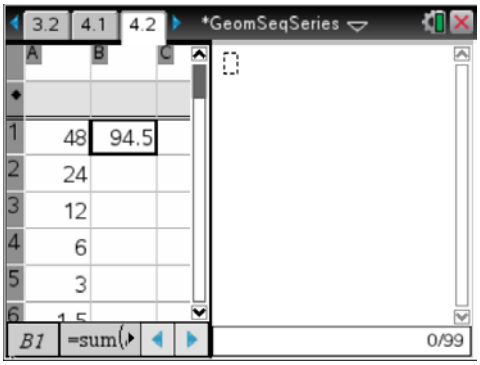
Share that the square root of the product of two numbers is called the *geometric mean* of those two numbers.



Problem 4 – Sums of Series

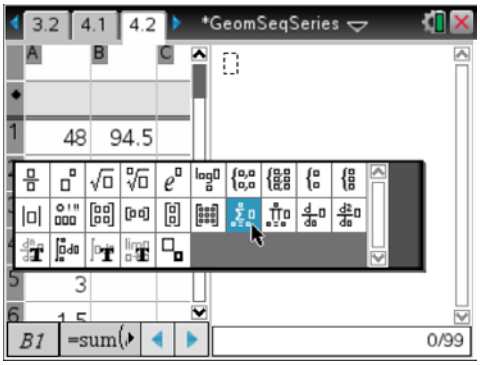
Step 1: Tell students that the expression consisting of summing the terms of a sequence is called a ***series***. On page 4.2, students will find the sum of the series of numbers in Column A in two ways.

First, they can use the **sum(** command in cell B1 of the spreadsheet. To find the sum of the six terms in the sequence enter **=sum(a1:a6)**.



Step 2: Moving to the *Calculator* application on the right side of the page, students will find the sum using sigma notation.

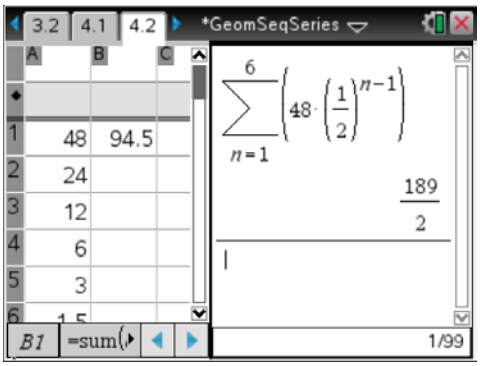
Press [ctrl] [π] to access the math templates and select the Sigma template shown to the right.



Step 3: Have students enter the needed information to see that the sum is the same sum found on the spreadsheet.

Since $\frac{189}{2} = 94.5$, the sums are the same.

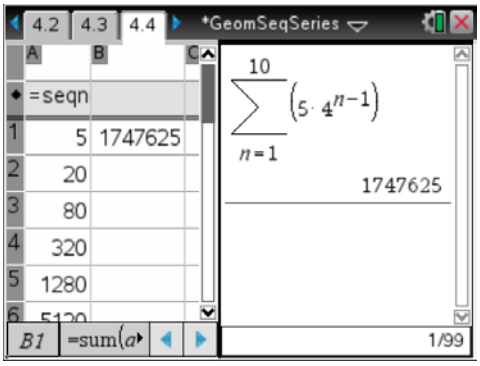
If desired, students can change the document settings to display the decimal. To do so, press [doc] and select **Settings & Status > Settings > Document Settings** and change the **Calculation Mode** from **Auto** to **Approx**. A decimal approximation is also obtainable by pressing [ctrl] + [enter] to evaluate the expression.



TI-Nspire Navigator Opportunity: *Screen Capture*

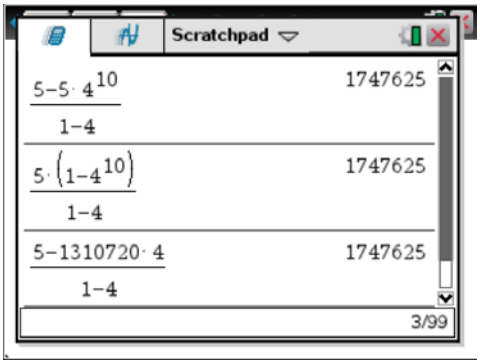
See Note 3 at the end of this lesson.

Step 4: Have students work independently on page 4.4 to find the sum of the first 10 terms in the sequence shown on page 4.3 (shown here).



Extension

Using *Scratchpad*, have students show that each of the three methods of finding sums explained on page 5.1 gives the same sum they found on page 4.4.



TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Live Presenter*
To start off the activity, you could have one student demonstrate how to use the Fill Down command in the TI-Npsire document. Although very similar to the feature on the computer, ensure students are using it correctly because it will be used several times in this activity.

Note 2
Problem 2, *Screen Capture*
Use Screen Capture to ensure students have graphed all the correct relationships. Having the correct scatter plots will help ensure they arrive at the correct ratios.

Note 3
Problem 3, *Quick Poll*
You may choose to use Quick Poll to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.