

Activity 4

Objectives

- Use translation and dilation to change the general parabola
- Develop models using the parabola to fit approximately a data set

Exploring Quadratic Data

In Activity **3**, you explored the effect of the parameters A , B , and C on the graph of the general quadratic function in vertex form: $y = a(x - b)^2 + c$. In this activity you will practice analyzing the vertex form of a parabola and finding an approximate fit of a model. You will continue your study of the quadratic function (parabola) and its properties by developing quadratic models.

When you move to Data Model **2**, think about the function properties of domain and range and their importance in modeling.

If you have not completed Activity **3**, you should do so before starting this activity.

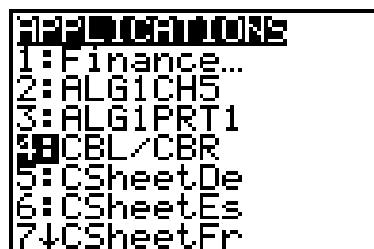
Data Model 1—Ball Bounce

Equipment Needed

- Graphing handheld with CBL™/CBR™ and Transformation Graphing applications installed
- CBR unit
- Ball—a racquetball or a basketball are recommended for this activity. Do not use a fuzzy ball such as a tennis ball.

Collecting Data

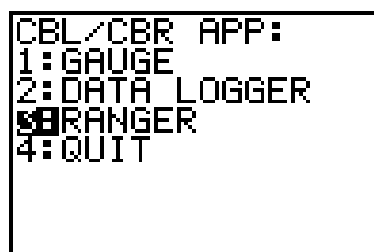
1. Attach the CBR™ unit to the graphing handheld with a unit-to-unit link cable.



2. Press [APPS] and select **CBL/CBR**.
3. Press any key to advance past the introduction screen.



4. Select **RANGER**. Press [ENTER] to advance past the introduction screen.



5. Select **APPLICATIONS**.



6. Select either **METERS** or **FEET**.



7. Select **BALL BOUNCE**. Follow the instructions with the **BALL BOUNCE** program.

```

APPLICATIONS
1: DIST MATCH
2: VEL MATCH
3: BALL BOUNCE
4: MAIN MENU
  
```

8. When you have good data appearing on the graphing handheld screen, exit the program by pressing **ENTER** and then choosing **QUIT**. The data you will work with is stored in **L1** and **L2**.

```

OPS MENU
1: DIST-TIME
2: VEL-TIME
3: ACCEL-TIME
4: PLOT TOOLS
5: REPEAT SAMPLE
6: MAIN MENU
7: QUIT
  
```

```

L1=TIME
L2=DIST
L3=VEL
L4=ACCEL
Done
  
```

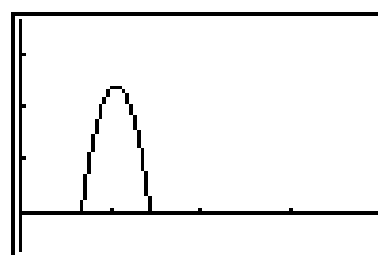
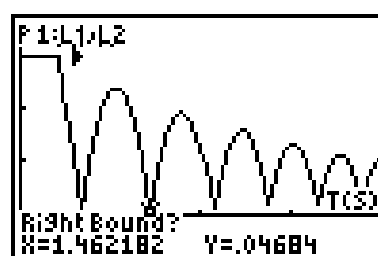
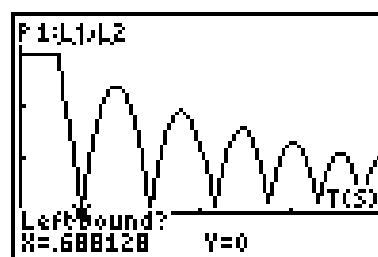
9. With the plot displayed, press **2nd** **[LIST]** and then press **▸** to go to the **OPS Menu**. Choose **8:Select(**, and enter where you want to store the selected data. To use **L3** and **L4**, press **2nd** **[L3]** **,** **2nd** **[L4]** **)** **ENTER**.

```

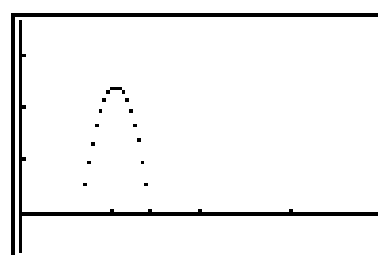
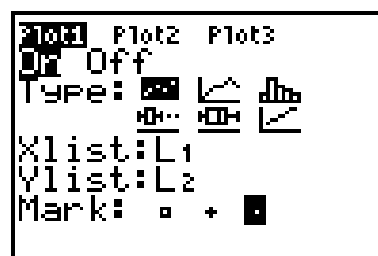
Select(L3,L4)
  
```

10. To actually select a part of the graph you will use, press \leftarrow or \rightarrow to move to the left end of the data you want to keep. Press ENTER . This sets the left bound. Press \rightarrow to move to the right end of the data you want. Press ENTER . The selected data will be placed in **L3** and **L4**, and then this data will be displayed.

Note: You can also find instructions for using **Select()** and setting up Stat Plots in the graphing handheld guidebook.



11. The CBL™/CBR™ application leaves the data connected. Press 2nd [STAT PLOT] 1 to select **1:Plot1**. Select the unconnected scatter plot option.



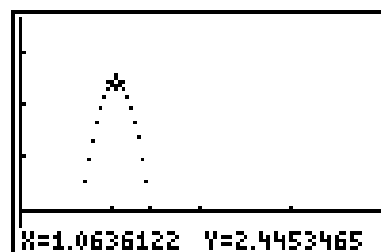
Developing the First Model

1. Try to position the cursor's + at the approximate location of the vertex. Record these coordinates.

Note: To use the free-moving cursor, press the cursor keys (← ↑ → ↓) to move the crosshatch around the screen.

- a. Do you think the vertex will always be one of the points that is collected by the CBR™ unit? Why or why not?

- b. How does the fact that the parabola has an axis of symmetry help you estimate the location of the vertex?



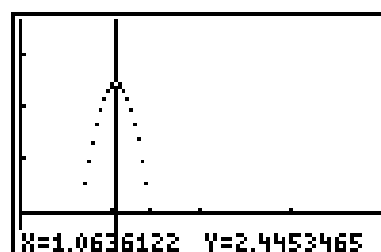
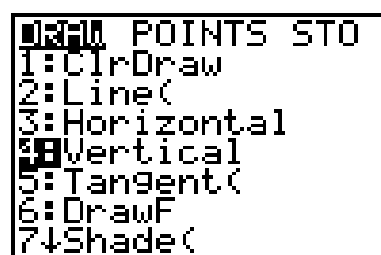
2. Next, you will sketch the axis of symmetry through the point you have selected as the vertex. Press [2nd] [DRAW] 4 to select **4:Vertical**. A vertical line will be drawn by the graphing handheld at the same location as the cursor.

Does the line appear to be the axis of symmetry? If not, move the line left or right with the cursor keys until it appears to be the axis of symmetry.

If you need to move your axis of symmetry, be sure to update the starting location of the vertex accordingly.

Record the approximate coordinates of the vertex.

$x = \underline{\hspace{2cm}}$ $y = \underline{\hspace{2cm}}$



3. Press **[APPS]** and select **Transfrm**.



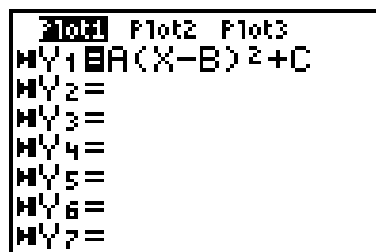
Note: If you do not see the screen shown, select **Continue**.



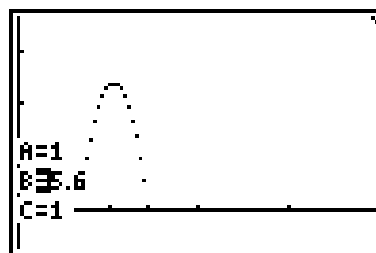
4. Display the **Y=** editor and enter the general vertex form of the quadratic function, $y = a(x - b)^2 + c$.

Note: Clear any equations that are listed in the **Y=** editor and turn off any plots. Make sure you are in the **Function mode (Func)**.

If **Play-Pause** mode (**>||**) is not selected at the left of **Y1**, press **[◀]** until the cursor is over the symbol, and then press **[ENTER]** until the correct symbol is selected.



5. Press **[GRAPH]** to display the plot and the graph. The graph will start with the most recent values of A , B , and C . These values do not necessarily relate to the problem you are working on, so the graph might not appear on the screen.



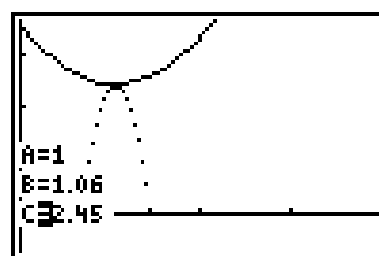
- a. Why do you think you recorded the approximate coordinates of the vertex? How will you use them in this activity?

- b. The “eyeball” method to find a model for quadratic data works most quickly when you are working with the vertex form of the parabola. You have already recorded a starting point for the vertex and thus have starting values for B and C . Which value is B and which is C ? Why?

6. Highlight **B=**. Enter your estimate. In this example, you would enter **1.06** and press **ENTER**.

7. Highlight **C=** and enter your estimate. For this example, enter **2.45** and press **ENTER**.

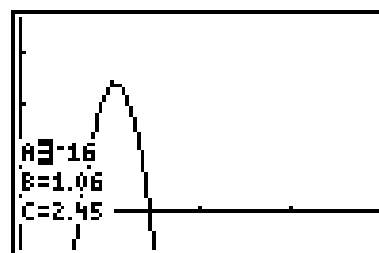
You now have a parabola with its vertex matching the vertex of your data. In the example, this occurs at (1.06, 2.45). Now you need a value for A . What would be a good starting value for A ? Why?



If you have taken a physical science course, you may recall that the coefficient of the quadratic term for a free-falling body is half the acceleration due to gravity. The acceleration due to gravity is approximately 32 ft/sec^2 or 9.8 m/sec^2 . This may give you an idea for a starting value for A .

8. Highlight **A=** and enter your estimate. In the example, enter **-16** and press **ENTER**. How closely does your model fit the data?

If your model does not fit the data as well as the one displayed, you should refine your model by revising the values for A , B , or C .



Internet Research: After you finish this activity, search for the terms *free-fall* or *projectile motion* on the Internet, and discuss how this activity relates to what you find.

Student Worksheet

Name _____

Date _____

Stamp Prices

In 2002, the U.S. Post Office raised the price of a first-class stamp from \$0.34 to \$0.37. The table shows cost increases for a first-class stamp from 1958 to 2002. Enter the data into the stat list editor of your graphing handheld, and then use the Transformation Graphing App to develop a quadratic model for the relationship between year and price.

Use your model to predict the price of a first-class stamp in 2010.

Year	Cost of a First-Class Stamp (in cents)
1958	4
1963	5
1968	6
1971	8
1974	10
1975	13
1978	15
1981	18
1982	20
1985	22
1988	25
1991	29
1995	32
1999	33
2001	34
2002	37

Source: U.S Postal Service data at <http://www.usps.gov> and National Association of Letter Carriers at <http://www.nalc.org/postal.html>

1. Which variable is the independent variable? Which variable is the dependent variable?

Independent variable: _____

Dependent variable: _____

2. Enter the data in the stat list editor of your graphing handheld.

- a. Press $\boxed{\text{STAT}}$ 1 to select **1:Edit**. Enter the values from the table into an empty list. Clear two lists if no lists are empty.

Note: The example shows the year data without the first two digits. (1974 is entered as 74.) 2001 becomes 101.

L1	L2	L3	Z
88	25		
91	29		
95	32		
99	33		
101	34		
102	37		

L2(17) =			

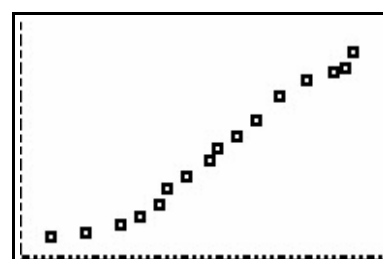
- b. Press $\boxed{\text{Y=}}$, and deselect any functions that have been turned on.

Plot1	Plot2	Plot3
MY_1	$\text{A(X-B)}^2+\text{C}$	
MY_2	=	
MY_3	=	
MY_4	=	
MY_5	=	
MY_6	=	
MY_7	=	

- c. To display the plot, press $\boxed{2\text{nd}}$ $\boxed{\text{STAT PLOT}}$ 1 to select **1:Plot1**. Turn the plot on, and set the plot menu as shown.

Plot1	Plot2	Plot3
On	Off	
Type:	$\text{A(X-B)}^2+\text{C}$	
Xlist:	L1	
Ylist:	L2	
Mark:	\square + \square	

- d. Press $\boxed{\text{ZOOM}}$ 9 to select **9:ZoomStat** and display the plot.



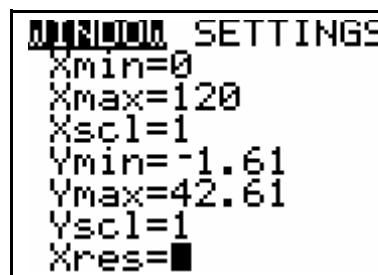
3. What do you notice about the plot of the data? What is the nature of the relationship between the year and the cost of a first-class stamp?

4. Which point(s) seem(s) the most "out of line?"

5. Do you think a quadratic model is a reasonable one? Explain.

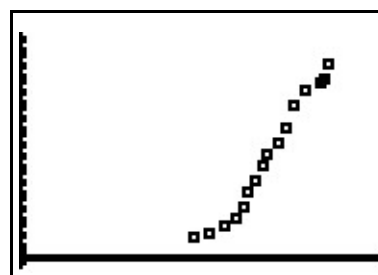
You will use the Transformation Graphing App and the general vertex form of a parabola to find an appropriate and reasonable model for the data.

6. Press **WINDOW**, and set **Xmin=0**, and **Xmax=120**. This will allow a better view of the data points and the coefficients in the graph.



```
WINDOW SETTINGS
Xmin=0
Xmax=120
Xscl=1
Ymin=-1.61
Ymax=42.61
Yscl=1
Xres=1
```

As a starting point, pick one of the given points as the vertex, and then start refining your model.



After you have found a reasonable model, answer the following questions.

7. What kind of model do you have? Why do you think it is an appropriate model?

8. What does your model predict for the price of a first-class stamp in 2010? To have your graphing handheld help answer this question, with the graph of your model showing, press `TRACE` `▲` `110` `ENTER`.

9. Given your model, do you think the cost increase in 2002 was reasonable? Explain mathematically.

10. Some people would say that, in theory, an exponential model would be better. Why do you think they would make this claim?

11. The table listed the years as 58, 63, ... instead of 1958, 1963, Why do you think this was done?

12. Would you use your model to predict the cost of the first stamp issued? You might want to do an Internet search to learn the history of the U.S. Postal Service.

Hint: The first stamp was issued well before 1950.

Teacher Notes



Activity 4

Exploring Quadratic Data

Objectives

- Use translation and dilation to change the general parabola
- Develop models using the parabola to fit approximately a data set

Materials

- TI-84 Plus/TI-83 Plus
- CBL™/CBR™ application
- CBR unit
- Racquetball or basketball
- BOUNCE program (optional)

Time

- 60 minutes

This activity is a follow-up to the previous activity. In Activity 3, students explore the effect of the various parameters on the graph of a parabola. In this activity, they will use what they have just learned to find a quadratic model using the Transformation Graphing application.

The main intent of this activity is to give students practice with the parameters of the vertex form of the parabola and their effect on the shape of the curve. Finding appropriate models through “eyeballing” helps reinforce these relationships and helps students discover some of the properties of the particular curve being studied.

The methods used in this activity should be used before any more formal model development takes place. Doing “eyeball” fits or finding appropriate and reasonable fits also helps develop a sense of when a model is reasonable.

Be careful not to look for only one correct answer. Any reasonable model can be justified and students should be encouraged to do so.

There are two data sets. Set 1 is collected by the students in the classroom.

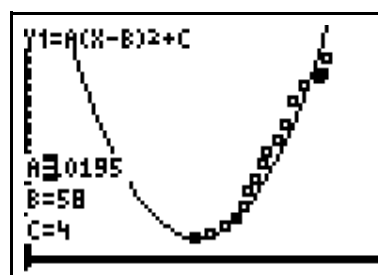
The instructions tell students to use the free-moving cursor to approximate the vertex. It would probably be easier to go to the **DRAW Menu** and use the vertical line to try to visualize the axis of symmetry.

For the Future: You might want to save the ball bounce data and revisit them when you study the exponential function. The data set can be modeled with an exponential function.

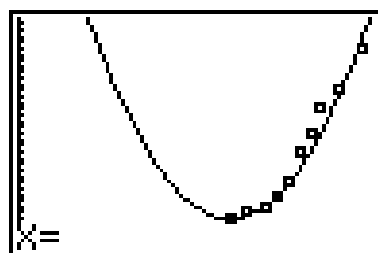


There are any number of reasonable models. Do not expect all students to get the same model for predicted cost of a stamp in 1999. Any reasonable model should be accepted.

Using the first data point (58, 4) as the vertex will be a very popular method. A reasonable model can be found using this approach. Both models shown here will predict a stamp cost of \$0.56 for 2010.

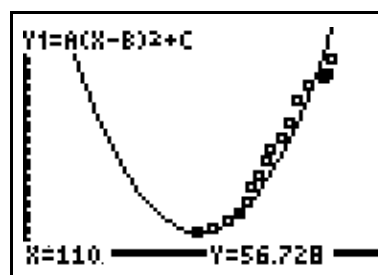


You might want to show your students how to use the **value** option under the **CALC Menu** if they are not already familiar with this option.



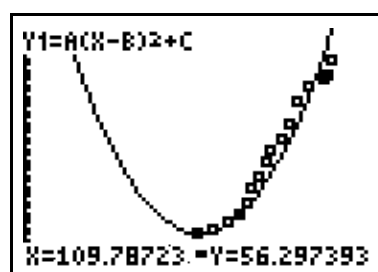
To use the **value** option:

- With the graph showing, press $\boxed{2nd}$ $\boxed{[CALC]}$ $\boxed{1}$ to select **1:value**.
- Enter the number you want to use to evaluate the function (in this example, $X=110$).
- Press \boxed{ENTER} .



In the activity, the same result was found using the **TRACE** option.

Remember, there are many equally reasonable models. This model gives a predicted 2010 cost of \$0.56.



Studying the quadratic model provides an opportunity to discuss domain and range issues. If no restriction is placed on the domain, any quadratic model will show that many years before those listed on the table, the price of a stamp was much greater and that in fact the cost had dropped going into the 1940s and 1950s.

With unrestricted domains, the logistic or exponential models would make more sense for years previous to those in the table.

Two Internet searches are suggested during the activity. As a third Internet activity, you might have your students research the cost of stamps before 1958, add those data to their plot, and then determine a new model. If you use this option, you might want to discuss the modeling concept of maintaining, updating, and revising a model.

Answers

Developing the First Model

1.
 - a. There is no reason to believe that any particular point would be collected by the CBR™ unit, which takes readings as set intervals. The ball may or may not be at the vertex when the CBR takes a reading.
 - b. Because the vertex is on the axis of symmetry, if you visualize the axis of symmetry you can more easily find the x -coordinate of the vertex. It is often easier to start looking for your model by visualizing the axis of symmetry because you can use all the collected points.
2. Students' coordinates of the vertex will vary. Both should nevertheless be positive (in quadrant I).
5.
 - a. You can use these coordinates for B and C .
 - b. The parameter B is the x -coordinate of the vertex and C is the y -coordinate.
7. Any negative value that is appropriate because the parabola opens down.

Student Worksheet

1. Date is the independent variable; cost is the dependent variable.
3. There seems to be a relationship: as x (the year) increases, y (the cost) increases at an increasing rate.
4. Looking at the total picture, the price increases in 1999 or 2001 seem the most out of line.
7. Answers will vary. Students should explain their model.
8. Answers will vary. Have students explain their predictions.
9. Answers will vary. For probably any model students might have, the model will predict a price greater than \$0.37.
10. Most people have heard that the cost of living grows exponentially. Thus, they may expect the cost of any particular item to grow exponentially over time also.

- 11.** This performs a horizontal translation of the data to keep the numbers small. The quadratic relationship would hold before or after this translation.
- 12.** Answers will vary, but the model is working with a restricted domain. Many students will use the first data point (or something close to it) as the vertex, which means that any year before the vertex will have a higher cost. It is safe to assume that the cost of a stamp did not drop in 1958.