Connecting Factors and Zeros

Math Concepts	Overview
• Linear functions • Factoring • Graphing **Materials** • TI-83 Plus	The student will use the TI-83 Plus to improve factoring skills.

- **Support factoring graphically:**

  Factor $2x^2 + 4x$

  Solution: $2x^2 + 4x = 2x*x + 2x*2 = 2x(x+2)$

- **Connect factoring to solving an equation**. Remember that the zeros of a function are the solutions of an equation.

  Make a connection: Solve $2x^2 + 4x = 0$

  Solution: $2x^2 + 4x = 0$

  $2x*x + 2x*2 = 0$

  $2x(x+2) = 0$

  $2x = 0$ or $x + 2 = 0$

  $x= 0$ or $x = -2$

> • If ab = 0 then a = 0 or b = 0.
>
> (The Zero Product Rule)

If the zeros do not agree, the factoring is not correct.

- **Support by graphing using the TI-83 Plus**

  Enter the function in **Y1** and its factors in **Y2** and **Y3**. (Figure 1)

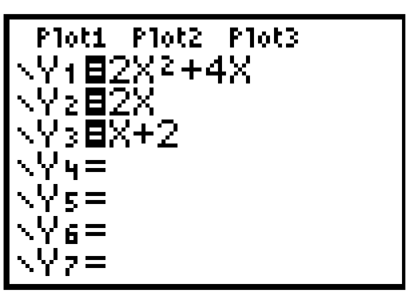


Figure 1

Teacher Notes

- Students **should be** able to determine if a quadratic function is factorable. Then factor the equation, set each factor equal to zero, and solve for x. Many students are unable to perform these algebraic operations with certainty. Learning to use the graphing and table features of the TI-83 Plus will enable students to independently check their work and subsequently become more confident algebra students.
- The graphing features of the TI-83 allow students to make connections between factors, the zeros of functions, and the solutions of an equation.
- If factored correctly, the zeros of the function should also be the solutions of the equation.

- **Press** ZOOM 4 **to graph the functions and its factors in a decimal window.** (Figure 2)

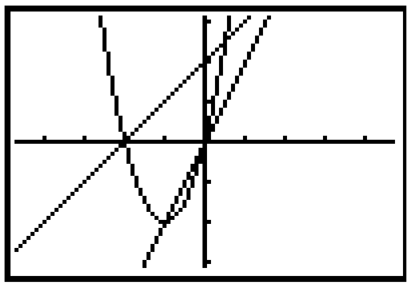


Figure 2

- **Note that the zeros of Y1 (the points where the function Y1 crosses the x-axis) are also the zeros of Y2 and Y3.**

  For example, suppose we had solved incorrectly as follows:

  $2x^2 + 4x = 2x\,(x + 4)$

  $2x = 0$ or $x + 4 = 0$

  $x = 0$ or $x = -4$

  Enter the function in **Y1** and its factors in **Y2** and **Y3**. (Figure 3)

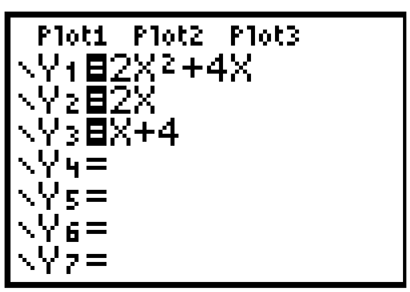


Figure 3

  Graph to show that the zeros of **Y1** and **Y3** do not agree. Graphing clearly shows that the solution is incorrect. (Figure 4)

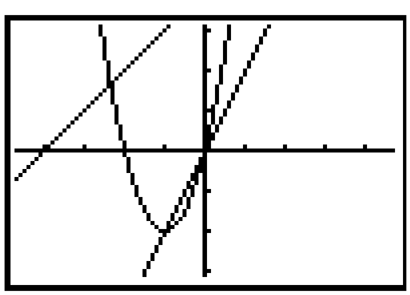


Figure 4

Support factoring logically:

- **Always** begin by storing a decimal value in the variable. Any decimal will work, but a convenient one to use is π. (Figure 5)

  On the home screen, press [2nd] [^] [STO▶] [X,T,Θ,n] [ENTER].

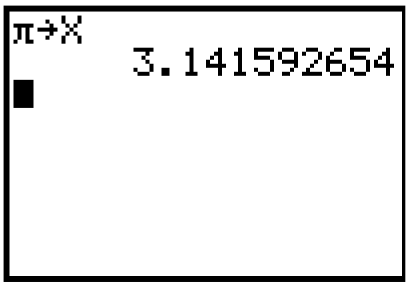


Figure 5

Check a problem such as $2x^2 + 4x = 2x(x+2)$ by entering the following on the home screen:

Press [2] [X,T,Θ,n] [x^2] [+] [4] [X,T,Θ,n] [2nd] [MATH] [1] [2] [X,T,Θ,n] [(] [X,T,Θ,n] [+] [2] [)]. (Figure 6)

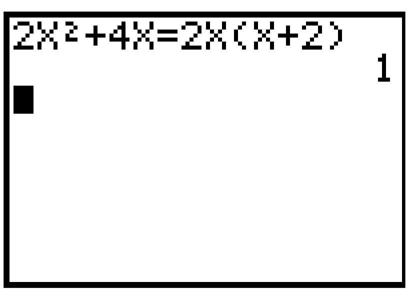


Figure 6

The value "1" shows that the factoring is correct.

Suppose you factor $x^2 + 4 = (x+2)^2$. Check on the TI-83 Plus by entering the statement on the home screen. Remember that pressing [2nd] [MATH] will access the **TEST** menu and allow you to paste the "=" on the home screen. (Figure 7)

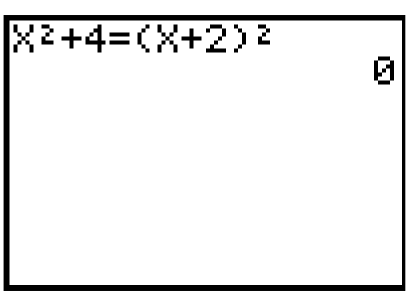


Figure 7

- Students can use the Boolean Algebra features of the TI-83 Plus to support factoring numerically. Remind students that a true statement returns a result of "1" and a false statement returns a value of "0".
- When evaluating a Boolean Algebra statement such as $x^2 + 4 = (x+2)^2$ the TI 83 Plus uses the last value stored in x. In this instance, if the last value stored in "x" was "0" the statement x^2 4 = $(x+2)^2$ will appear to be true. (Figure 13)

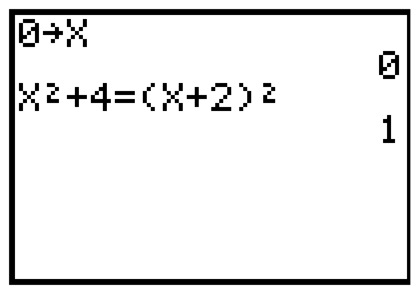


Figure 13

If students are to successfully use Boolean logic to check their work, they must make sure that the last value stored in the variable is not "0" or "1". Any decimal value should work well. Perhaps it would be wise to always store "π" in the variable.

The value "0" tells us that the factoring is incorrect.

Further graphical consideration leads us to conclude that $x^2 + 4$ is not factorable over the Reals. Use the Standard Window [ZOOM][6]. The graph of $y = x^2 + 4$ does not cross the x-axis. **For a function to be factorable, its graph must cross or be tangent to the x-axis**. (Figure 8)

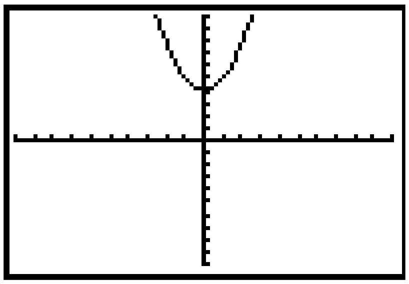


Figure 8

Clearly this function has no Real zeros, and therefore is not factorable.

- **Support factoring numerically**

Suppose you factor $2x^2 + 4x = 2x(x+4)$. To check numerically, enter the left side of the statement in **Y1** and the right side in **Y2**. (Figure 9)

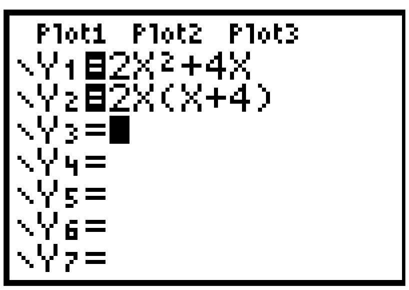
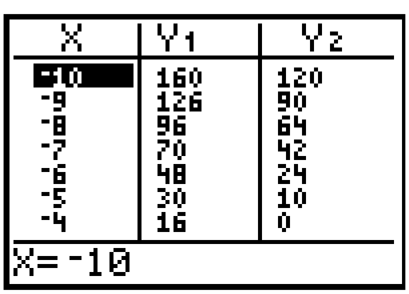


Figure 9

Press [2nd] [WINDOW] [(-)] [1] [0] [ENTER] [1] [▼] [ENTER] [▼] [ENTER] to set up a table. Press [2nd] [GRAPH] to generate a table of values. (Figure 10) If the values in the table differ for any x value then the factoring is not correct.



X	Y1	Y2
-10	160	120
-9	126	90
-8	96	64
-7	70	42
-6	48	24
-5	30	10
-4	16	0

X= -10

Figure 10

Since the values in **Y1** and **Y2** are different, the factors are not correct.

Suppose you factor $2x^2+4x=2x(x+2)$. Check numerically.

Enter Y1 = $2x^2 + 4x$ and Y2 = $2x(x+2)$. Press [2nd] [GRAPH] to generate the table. (Figures 11 and 12)

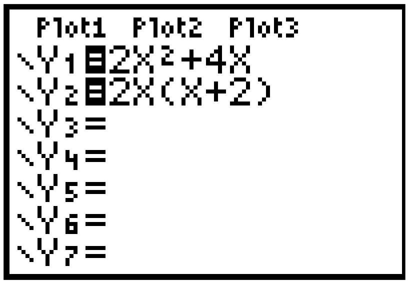


Figure 11

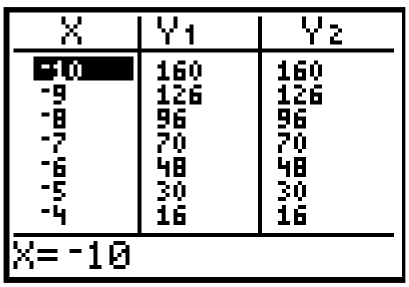


Figure 12

Since the values in **Y1** and **Y2** are equal for all x-values, the factoring must be correct.

- **The TI-83 Plus will not factor algebraic expressions, but it will provide graphical, logical, and numerical support for immediate feedback.**

