Areas in Intervals

ID: 9472

Activity Overview

In this activity, students use several methods to determine the probability of a given normally distributed value being in a given interval. First they use the Integral tool to find areas under the curve and to the left of given values. Then they use subtraction to find the difference of areas. Last, they use the normal Cdf command with the lower and upper bounds to find the area. Students continue the activity to find probabilities for which the corresponding area is greater than a given value, and then they conclude the activity by finding areas between z-scores.

Topic: The Normal Distribution

- *Express a normally distributed random variable X, of mean μ and standard deviation σ, in terms of a normally distributed random variable Z with mean 0 and $\sigma = 1$.*
- *Determine the probability that a normally distributed random variable given its mean and standard deviation.*

Teacher Preparation and Notes

- *Students should already be familiar with the normal distribution and it characteristics.*
- *Problem 3 uses the standard normal distribution and z-scores. This problem can be skipped if you have not or are not ready to discuss z-scores.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9472" in the keyword search box.***

Associated Materials

- *AreasInIntervals_Student.doc*
- *AreasInIntervals.tns*
- *AreasInIntervals_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the Keyword search box.

- *Percentiles and Z-Scores (TI-Nspire Technology) — 11205*
- *Z-Scores (TI-Nspire Technology) — 13318*
- *Normal Curve Application (TI-Nspire Technology) — 13368*

Problem 1 – Finding the Area between Two Given Scores

Step 1: The graph on page 1.3 shows a normal
distribution with a mean of 76 and a standard
deviation of 7. A line is drawn through the *x*-value
of 80. Students are to use what they know about
normal distributions to make a mental estimate
(guess) of the proportion of scores less than 80.
This corresponds to the area under the curve and
to the left of the perpendicular line.

All estimates should be greater than 0.5 since the
total area under the curve is 1 and 80 is greater
than the mean, which occurs at the maximum of
the curve.
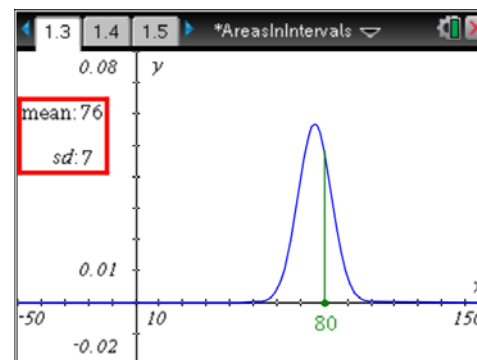
TI-Nspire Navigator Opportunity: *Quick Poll, Screen Capture*

See Note 1 at the end of this lesson.

Step 2: Students can use the **Integral** tool to find this area
on the graph (**MENU > Analyze Graph >
Integral**).

With the **Integral** tool selected, they need to
select the curve (another vertical line then
appears), select the origin, and then select the
point at (80, 0). The area is 0.71615. To display
more digits of the calculated value, students can
hover over the calculated value and press the ⊞
key.

Note that the area under the curve near the origin
is negligible and students could actually select
their first point closer to about (40, 0).
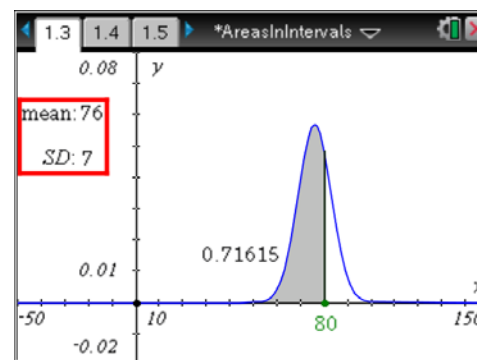
TI-Nspire Navigator Opportunity: *Live Presenter*

See Note 2 at the end of this lesson.

Step 3: On page 1.4, students are to find the area by using the **normCdf** command. They can type the command, select it from the Catalog, 📖, or choose it from **MENU > Statistics > Distributions > Normal Cdf**.

The lower bound is negative infinity, entered as -9ᴇ999. The symbol ∞ can be found in the Symbol Pallete, (⌈ctrl⌉ + 📖). If entering the information in parentheses, the order is lower bound, upper bound, mean, and standard deviation. The proportion found should equal that found on the graph.
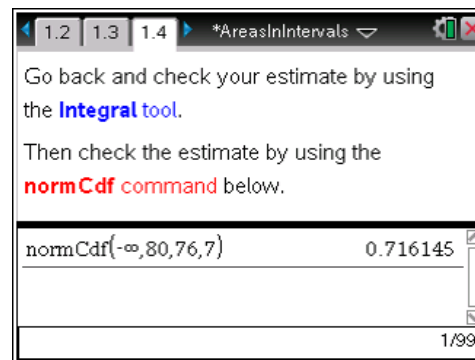
Step 4: Students now move to page 1.6, estimate the area of the curve that is to the left of 60, and then find the area using the **Integral** tool.

Since 60 is more than two standard deviations below the mean, estimates should be less than 0.025. (95% of scores are within two standard deviations, leaving 5% for both tails, or 2.5% in each tail.)
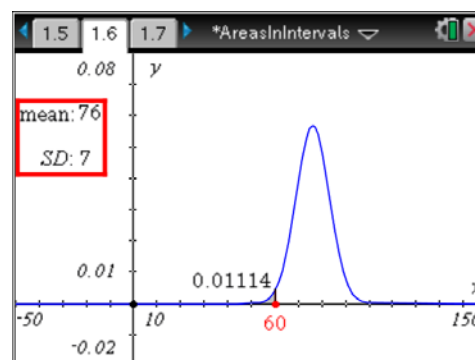
Step 5: Students can confirm the area found on the graph by using the **normCdf** command.
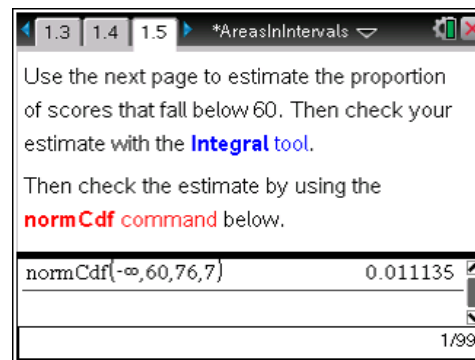
Step 6: When students advance to page 1.8, explain that the area between the two perpendicular lines equals the difference of the area to the left of the rightmost line and the area to the left of the leftmost line. Students are to use the *Scratch Pad* to subtract the areas found on the previous pages. (Press ⌈esc⌉ to exit the *Scratch Pad.*) Then, they can use the **Integral** tool to find the area between 60 and 80. This corresponds to the proportion of scores between 60 and 80.
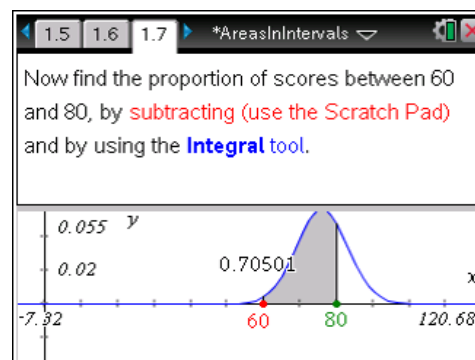
Step 7: Show students they can also find this area with the **normCdf** command by using 60 for the lower bound and 80 for the upper bound.

Discuss that since the total area is equal to 1, the proportions can also be used as probabilities. The probability that a randomly chosen score is between 60 and 80 is about 70.5%.
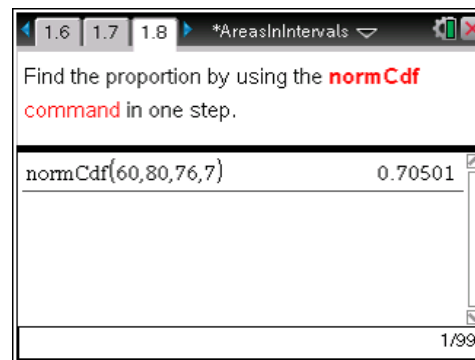
Problem 2 – Finding the Area Greater than a Given Value

Step 1: In Problem 2, the weights of game tokens are normally distributed with a mean of 4.85 grams and a standard deviation of 0.08 gram.

Ask students how they can use the **normCdf** command to find the probability that a randomly chosen token weighs more than 4.9 grams if **normCdf** gives the area to the left of a value, rather than to the right of a value.
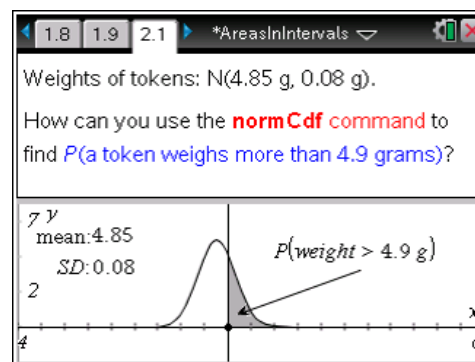
Step 2: Students should say to subtract from 1 since the total area is always 1. They are to find the indicated probability on page 2.2.
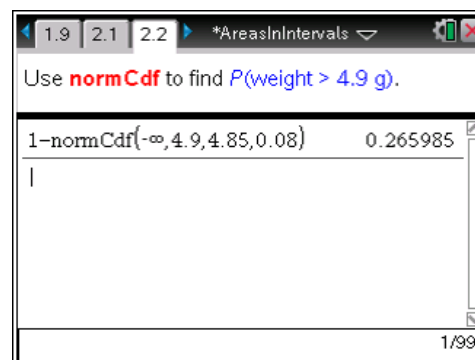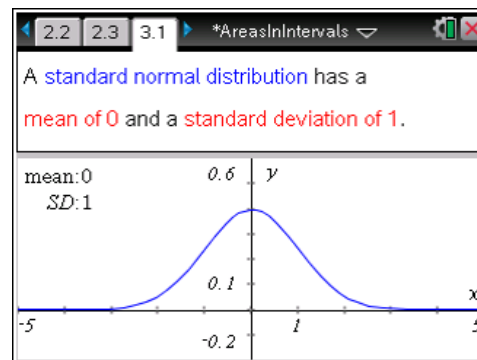
Step 3: Students need to estimate an answer to the problem on page 2.3 and then find the actual answer. Encourage them to explain the reasoning behind their estimates.
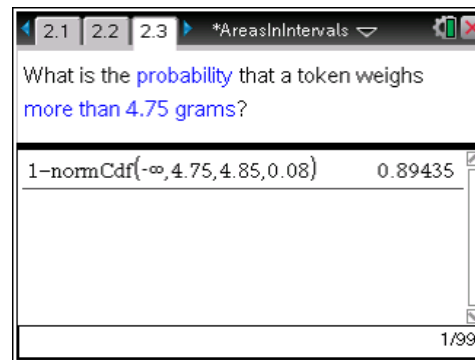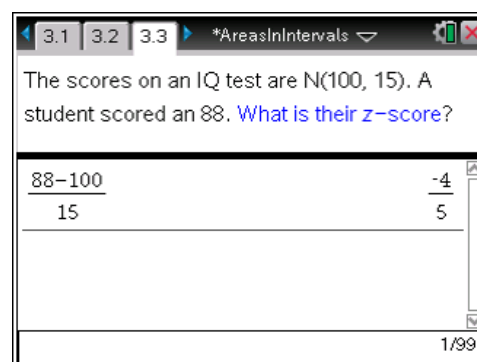
Problem 3 – Using the Standard Normal Distribution

Step 1: Define and discuss the standard normal distribution and *z*-scores as needed.
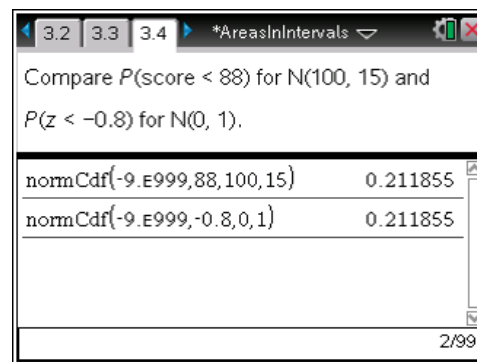
Step 2: Students need to use the formula for converting values to *z*-scores to answer the problem on page 3.3.

The *z*-score is –0.8. Students should discuss why it is negative and why its absolute value is less than 1.

Step 3: On page 3.4, students can see that their *z*-score is correct by finding the area to the left of –0.8 for N(0, 1) and the area to the left of 88 for N(100, 15).
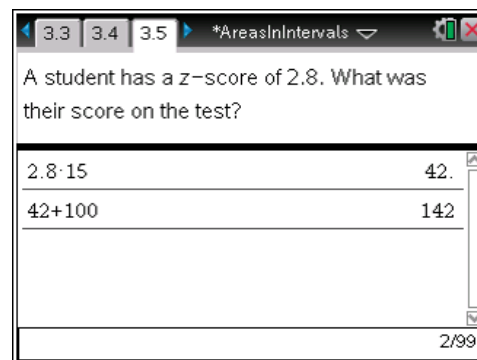
Step 4: Students are to advance to page 3.5 and find the test score that has a *z*-score of 2.8.

They can find this value by substituting 2.8 for *z* in the formula and solving for *x*.

$$2.8 = \frac{x-100}{15}$$
$$142 = x$$

Step 5: Students can check their work on page 3.6.

Step 6: Let students work though pages 3.7 and 3.8 on their own. Discuss the results when everyone is finished and answer any remaining questions.

The probability that a randomly chosen test taker had a score between 77.5 and 134.5 is about 92.2%.

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Quick Poll, Screen Capture*

Send out a *Quick Poll* to display the class' estimate for the proportion of scores that fall below 80. This will point out the students who are finding the area to the right of 80 instead of left. This will allow students to make this correction now rather than latter.

Screen Capture should be used throughout this activity to monitor student progress.

Note 2
Problem 1, *Live Presenter*

Use *Live Presenter* when finding the area using the Integral tool as well as the Cdf Wizard for the first time as students will be using these two tools quite a lot in this activity.