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SPACE SHUTTLE GUIDANCE, NAVIGATION, AND CONTROL DATA

Background

Since its conception in 1981, NASA has used the space shuttle for human transport, the construction of the International Space Station (ISS), and to research the effects of space on the human body. One of the keys to the success of the Space Shuttle Program is the Space Shuttle Mission Control Center (MCC). The Space Shuttle MCC at NASA Johnson Space Center uses some of the most sophisticated technology and communication equipment in the world to monitor and control the space shuttle flights.

Within the Space Shuttle MCC, teams of highly qualified engineers, scientists, doctors, and technicians, known as flight controllers, monitor the systems and activities aboard the space shuttle. They work together as a powerful team, spending many hours performing critical simulations as they prepare to support preflight, ascent, flight, and reentry of the space shuttle and the crew. The flight controllers provide the knowledge and expertise needed to support normal operations and any unexpected events.

One of the flight controllers in the Space Shuttle MCC is the Guidance, Navigation, and Control (GNC) officer. To understand the roles of the GNC officer, one must first understand the basics of the GNC system. Guidance equipment (gyroscopes and accelerometers) and software first compute the location of the vehicle and the orientation required to satisfy mission requirements. Navigation software then tracks the vehicle's actual location and orientation, allowing the flight controllers to use hardware to transport the space shuttle to the required location and orientation. The job of the GNC officer is to ensure the hardware and software that perform these functions are working correctly. This control portion of the process consists of two modes: automatic and manual. In the automatic mode, the primary avionics software system allows the onboard computers to control the guidance and navigation of the space shuttle. In the manual mode, the crew uses data from the GNC displays and hand controls for guidance and navigation. The GNC officer ensures that the GNC system has the accuracy and capacity necessary to control the space shuttle in both modes and that it is being utilized correctly.

The state vector of the spacecraft is the primary data used to determine the guidance function. The space shuttle's state vector is an estimate of vehicle position in space and velocity at a given time. Beginning with a known initial position, velocity, and orientation (such as on the launch pad just prior to launch), all sensed accelerations from that point can be integrated and incorporated with a physics model to calculate the new position, velocity, and orientation. For accurate control of the spacecraft, the GNC officer must ensure that the state vector is accurate at all times during each mission phase (ascent, orbit operations, and reentry).

To understand how the state vector is calculated, it is helpful to know the history involved in determining it. Throughout time, astronomers have used a three-dimensional Cartesian coordinate system to identify positions in space. The origin of this coordinate system is located at the center of the Earth. The z-axis is defined as the line that runs through the North and South poles of the Earth. The x and y axis both lie on the plane formed by Earth's equator. The x-axis points toward the vernal equinox. Every year there are two equinoxes, one in the spring (the vernal equinox), and one in the fall (the autumnal equinox). An equinox occurs when the sun passes directly over the equator of the Earth causing equal amounts of daylight and night. The direction of the x-axis is always drifting because the



Earth is always moving (rotating about the polar axis and orbiting the sun). For this reason, it is necessary to fix the orientation of the x-axis at a particular moment in time. The M50 coordinate system is based on the orientation of the Earth on January 1, 1950. (Figure 1)

The space shuttle is equipped with three Inertial Measurement Units (IMUs) that are used for attitude and position estimation. These three IMUs are mounted on the navigation base that is a metal beam used to maintain a constant orientation with respect to the rest of the vehicle. (Figure 2)

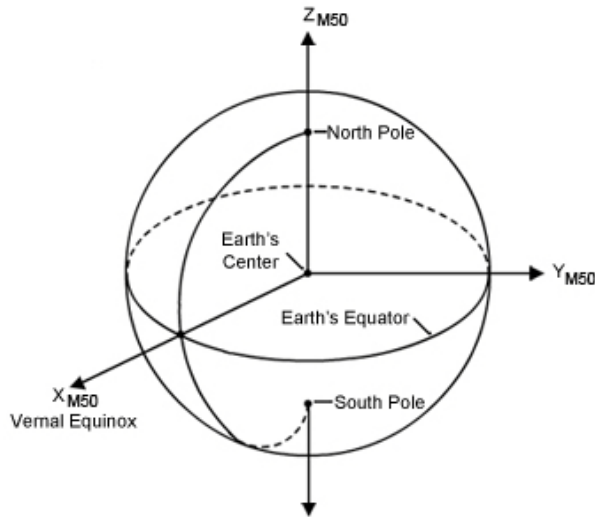


Figure 1: M50 (Aries-Mean-of-1950) coordinate system

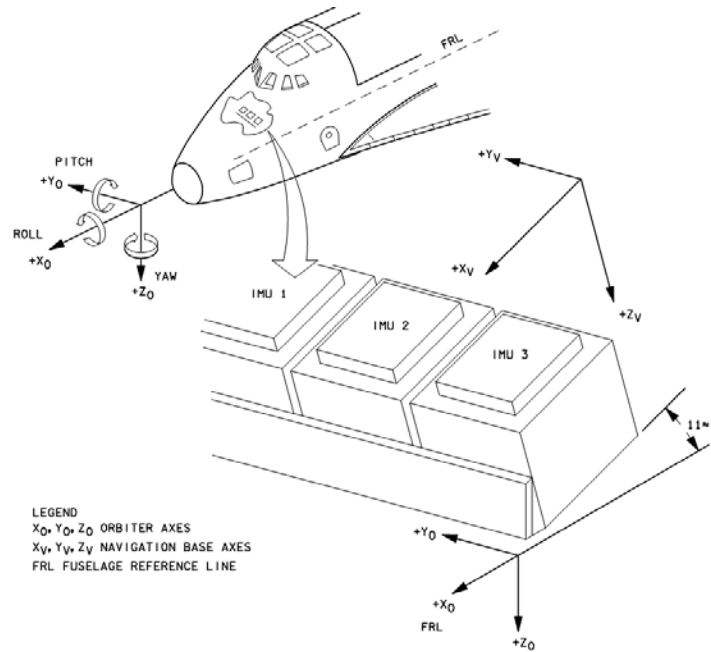


Figure 2: IMU position

At launch, the space shuttle's position in M50 coordinates is known. The IMUs have accelerometers that measure acceleration in the x , y , and z directions, as defined by the space shuttle's frame of reference. By integrating the acceleration data, the IMUs can determine the space shuttle's velocity and position. This can then be used to determine the change in the initial M50 launch coordinates. The accuracy of this information is monitored by the GNC officer to ensure that the space shuttle arrives at its pre-determined destination as outlined by mission objectives.



Problem

Open the TI-Nspire™ document, *ShuttleGNC*, read through the problem set-up, and complete the questions in the document.

The table below (also provided in the TI-Nspire document) gives M50 positions and velocity data from the space shuttle accelerometers for approximately one orbit around the Earth. As you answer questions A – C, justify your answers by showing all work.

Time (min)	Position (x) (m)	Position (y) (m)	Position (z) (m)	Velocity (x) $\left(\frac{\text{m}}{\text{s}}\right)$	Velocity (y) $\left(\frac{\text{m}}{\text{s}}\right)$	Velocity (z) $\left(\frac{\text{m}}{\text{s}}\right)$
0	6521479	-734565	1458505	-732	5020	5798
5	5924596	784838	3078097	-3207	5010	4893
10	4635033	2212521	4336801	-5305	4415	3416
15	2804233	3381908	5087694	-6780	3305	1541
20	646291	4156853	5243442	-7464	1811	-513
25	-1587017	4447125	4786055	-7278	105	-2507
30	-3635191	4218696	3768833	-6242	-1613	-4208
35	-5258953	3497785	2310392	-4476	-3146	-5419
40	-6268174	2368178	581241	-2186	-4311	-5995
45	-6544580	961726	-1216040	361	-4973	-5868
50	-6055983	-557150	-2870787	2864	-5053	-5054
55	-4860024	-2010971	-4189288	5030	-4544	-3649
60	-3096963	-3230170	-5017592	6608	-3505	-1819
65	-972801	-4072773	-5259205	7414	-2058	224
70	1264767	-4440582	-4885923	7356	-370	2241
75	3354753	-4290364	-3941001	6440	1362	3997
80	5052847	-3639037	-2534456	4769	2938	5287
85	6159973	-2562141	-830686	2539	4171	5959
90	6546101	-1185374	970565	10	4917	5931



- A. With the data provided, use sinusoidal regression to write a set of parametric equations which approximate the position of the space shuttle at any time t , where t is in minutes and position is in meters. Overlay each of your parametric equations on the corresponding scatter plot.

Note: An orbital motion should have x , y , and z coordinates which oscillate between minimum and maximum values. This suggests sinusoidal functions. For simplicity, ignore the drag and precessional forces caused by the Sun and Moon and use simple sinusoidal functions of the form:

$$f(t) = a\sin(bt + c) + d$$

- I. Find the position of the space shuttle in M50 coordinates (in meters) according to your equations when $t = 15$.
 - II. How does your answer compare to the accelerometer data for $t = 15$? Can you explain any difference in your predicted position and accelerometer data?
- B. Differentiate the position equations to obtain a set of parametric equations that will give the space shuttle's velocity at any time t , in meters per second (m/s). Graph the velocity equations with the corresponding scatter plots.

Note: Time is given in minutes and position in meters. However, velocity is given in m/s.

- I. Use your equations to find the velocity vector of the space shuttle in m/s when $t = 10$ and compare it to the table.
 - II. Find the speed of the space shuttle in m/s when $t = 10$.
- C. Differentiate the velocity equations to obtain a set of parametric equations that will give the space shuttle's acceleration at any time t , in meters per second per second (m/s^2).

Note: The units on the y -axis of the velocity graphs were meters per second, but the units on the x -axis were minutes, so you will need to make an adjustment to your derivatives in order to obtain acceleration equations in terms of m/s^2 .

- I. Use your equations to find the acceleration vector of the space shuttle in m/s^2 when $t = 10$.
- II. Find the magnitude of the space shuttle's acceleration in m/s^2 when $t = 10$.
- III. Do you notice anything about your answer in part II? (Hint: Why do astronauts experience weightlessness when they are still within the gravitational field of the Earth?)