TI-nspire™

Grade level: secondary
Subject: mathematics
Time required: 45 to 90 minutes

TI-Nspire Activity: *Derivatives: Applied Maxima and Minima*
By: Tony Duncan

Activity Overview
This activity takes the concept of derivative and applies it to various maximum and minimum problems.

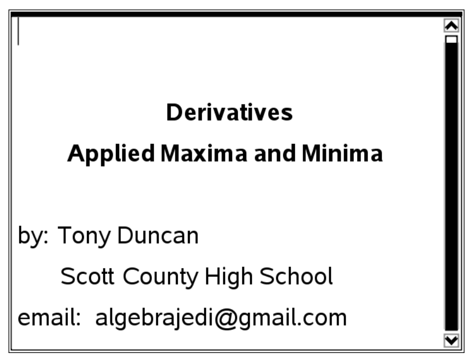
Concepts
This activity employs the first and second derivative of a function. The derivatives are used to help discover minimum and maximum form possible critical points.

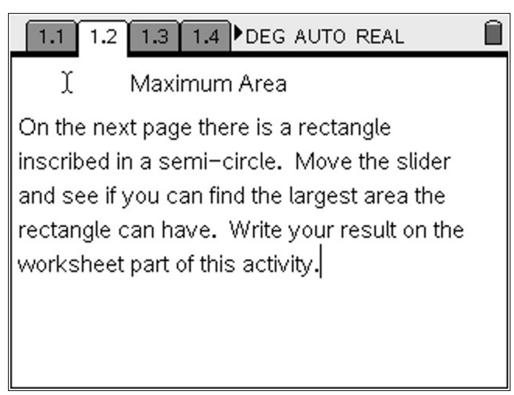
Teacher Preparation
Students should be able to find the first and second derivative of a function.

The Classroom.
This activity could be used as a demonstration model or as an independent activity for a single student or with groups of 2 or 3 students.

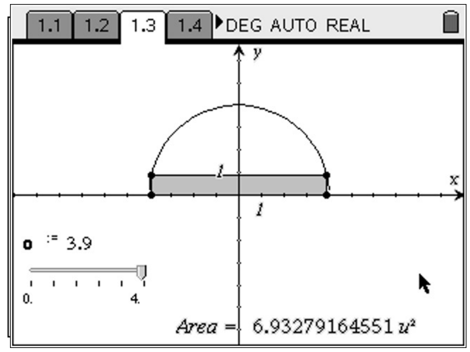
Files Required for This Activity.
Dervmax&min.tns
Derivative Applied Maxima and Minima.pdf
Derivative Applied Maxima and Minima Student Worksheet.pdf

Open the file dervmax&min.tns

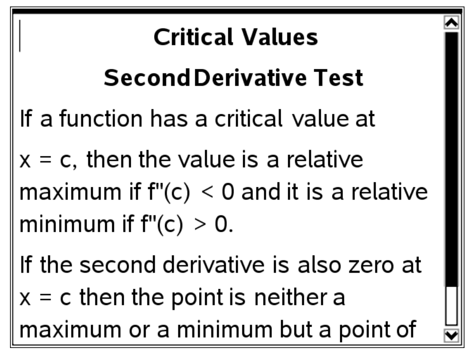




The Document
Go to page 2 (ctrl ▸) and read the instructions.



Look at the figure on page 1.3.
Move the slider and see if you can find the largest area the rectangle can have. Write your result Below.



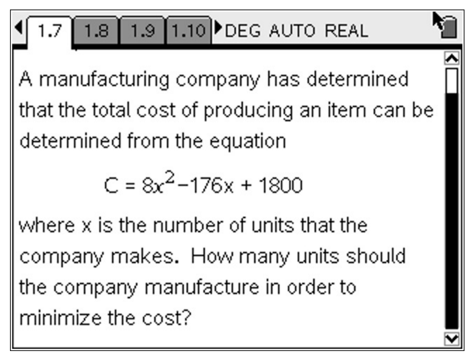
Maximum and Minimum

A maximum or a minimum of a function occurs at a point where the derivative of a function is zero.

f'(x) = 0

((dy)/(dx)) = 0 or ((df)/(dx)) = 0

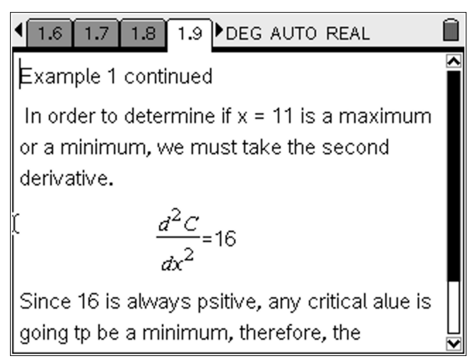
Write the definition of a critical value (point).



Example1: We want to find the minimum value of the function. This minimum value will provide the minimum cost for the manufacturing company.

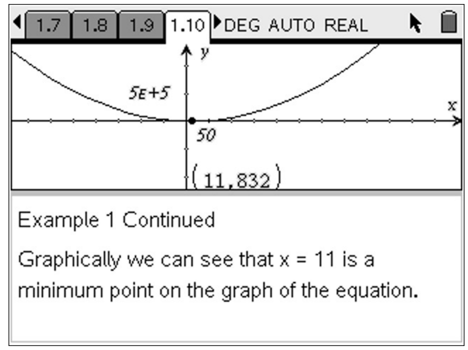
Find the derivative of the function $C = 8x^2 - 176x + 1800$

Check your answer on page 1.8



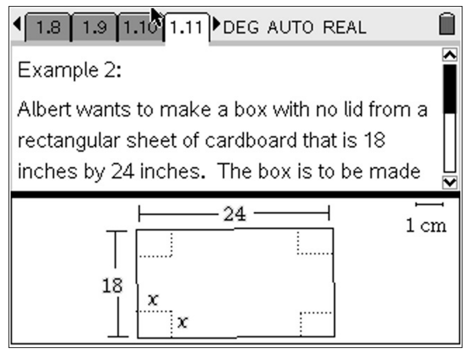
Take the derivative and set it equal to zero and find the critical points. You must then take the second derivative to determine if the critical point is a minimum or a maximum or neither.

The answer to the problem can be verified graphically. Grab the point and drag to verify the minimum.



Example 2: Volume of a Box

Albert wants to make a box with no lid from a rectangular sheet of cardboard that is 18 inches by 24 inches. The box is to be made by cutting a square of side x from each corner of the sheet and then folding up the sides. (See the figure at the right.) Find the value of x that maximizes the volume of the box.



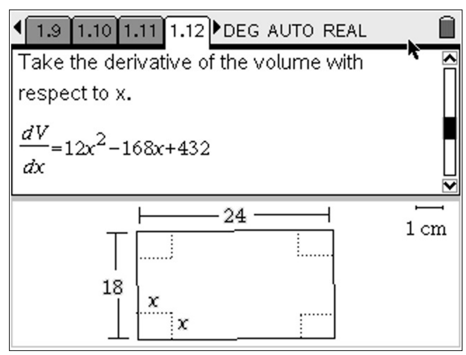
After we cut out the squares of side x and fold up the sides, the dimensions of the box will be:

- width: 18-2x
- length: 24-2x
- height (depth): x

Using the formula for the volume of a rectangular prism we have
V = x(18-2x)(24-2x)
expanded that is

$$V = 4x^3 - 84x^2 + 432x$$

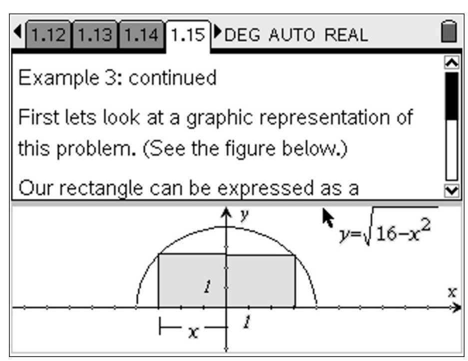
Now the steps are the same as the first example:
Take the derivative of the volume with respect to x.



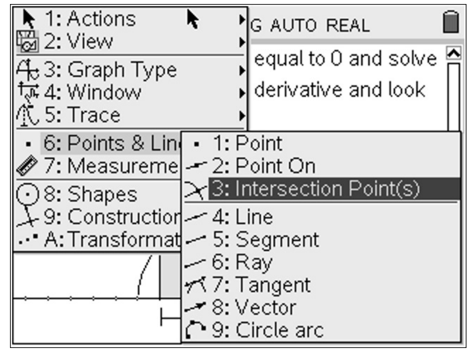
Use the second derivative to test the critical points and find the value for value for x.

Example 3: Inscribed Rectangle in a Semicircle

A rectangle is to be inscribed in a semicircle with radius of 4. with one side on the semicircle's diameter. What is the largest area this rectangle can have?



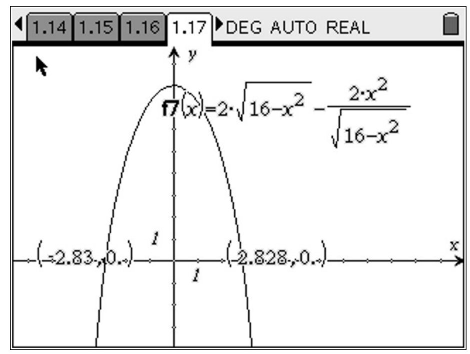
The equation for the semicircle is $y = \sqrt{16 - x^2}$. This equation is also the height of the rectangle and the width is 2x. The area of the rectangle can be expressed as $A = 2x\sqrt{16 - x^2}$



Take the derivative of the area.

$$\frac{dA}{dx} = 2\sqrt{16 - x^2} - \frac{2x^2}{\sqrt{16 - x^2}}$$

We can set this derivative equal to 0 and solve, but it would be easier to graph the function and use the intersection tool to find the x-intercepts.



Note that the domain of the function (not the derivative) is -4 ≤ x ≤4. These numbers serve as endpoints of the interval. We must check the end points and the critical values to find the x value that provides the maximum area.

Go back to first problem and see if our answer here is close to the answer for the slider problem on page 1.3

Solutions to the student practice problems:

1. Area is 32 units squared.

2. x = 1.70

3. 16 X 34

4. Radius is $\sqrt[3]{\frac{256}{\pi}}$ in.

Student Practice Problems

(Remember that a picture is worth a thousand words.)

1. A rectangle has its base on the x-axis and its two upper corners on the parabola $y = 12 - x^2$. What is the largest possible area of the rectangle?

2. An open rectangle box is to be made from a 9 X 12 inch piece of tin by cutting squares of side x inches from the corners and folding up the sides. What should x be to maximize the volume of the box?

3. A 384 square meter plot of land is to be enclosed by a fence and divided into two equal parts by another fence parallel to one pair of sides. What dimensions of the outer rectangle will minimize the amount of fence used?

4. What is the radius of a cylindrical soda can with volume of 512 cubic inches that will use the minimum material?