Polynomials—Factors, Roots, and Zeros

Math Objectives

- Students will discover that the zeros of the linear factors are the zeros of the polynomial function.
- Students will discover that the real zeros of a polynomial function are the zeros of its linear factors.
- Students will determine the linear factors of a quadratic function.
- Students will connect the algebraic representation to the geometric representation.
- Students will see the effects of a double and/or triple root on the graph of a cubic function.
- Students will see the effects of the leading coefficient on a cubic function.
- Students will look for and make use of structure (CCSS Mathematical Practice).
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).

Vocabulary

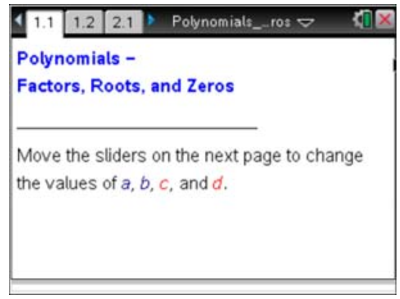
- zeros
- double or triple root
- leading coefficient

About the Lesson

- This lesson merges graphical and algebraic representations of a polynomial function and its linear factors.
- As a result, students will:
  - Manipulate the parameters of the linear functions and observe the resulting changes in the polynomial function.
  - Find the zeros of the polynomial equations by finding the zeros of the linear factors.

TI-Nspire™ Navigator™ System

- Use Screen Capture to examine patterns that emerge.
- Use TI-Nspire Teacher software or Live Presenter to review student documents and discuss examples as a class.
- Use Quick Poll to assess student understanding at the end of the lesson.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- In the *Graphs* application, you can hide the function entry line by pressing ctrl G.

Lesson Materials:
Student Activity

Polynomials_Factors_Roots_and_Zeros_Student.pdf
Polynomials_Factors_Roots_and_Zeros_Student.doc

TI-Nspire document

Polynomials_Factors_Roots_and_Zeros.tns

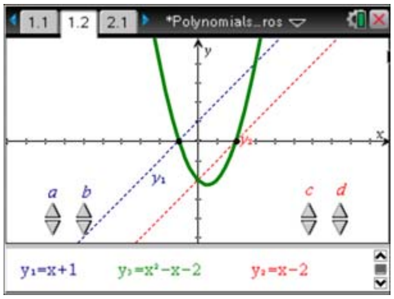
Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty clicking a slider, check to make sure that they have moved the cursor over the slider and have them press [click] to change the value of the slider.

TI-Nspire Navigator Opportunity: ***Live Presenter*** **or** ***Screen Capture***
See Note 1 at the end of this lesson.

Move to page 1.2.



1. Using the sliders, set $y_1 = 1x + 1$ and $y_2 = 1x - 2$. Observe that the graph of $y_1 = 1x + 1$ appears to cross the x-axis at $x = -1$. When $x = -1$, $y_1 = 0$ because $-1 + 1 = 0$. $x = -1$ is called a *zero* or *root* of the function $y_1 = 1x + 1$.

    a. Where does the graph of $y_2 = 1x - 2$ appear to cross the x-axis?

    Answer: $x = 2$

    b. Write a simple equation to verify that this value of x is a zero of y_2.

    Answer: $1(2) - 2 = 0$

    > **Teacher Tip:** A zero of a function is an input value for which the function value is zero. Thus, if $x = 2$ is a zero of the function, then $\mathbf{f}(2) = 0$ and the point (2, 0) is on the graph of the function.

    c. When $y_1 = 1x + 1$ and $y_2 = 1x - 2$, what is the function y_3?

    Answer: $y_3 = x^2 - x - 2$

    d. The graph of y_3 is a parabola. How many times does the graph of y_3 cross the x-axis?

    Answer: The graph crosses the x-axis twice.

e. What are the zeros of y_3?

Answer: $x = -1$ and $x = 2$

f. Factor y_3.

Answer: The factors of $x^2 - x - 2$ are $(x + 1)$ and $(x - 2)$.

> **Teacher Tip:** This activity is assuming that factoring of a quadratic has already been completed. Teachers may need to do some reviewing of factoring at this point.

g. Given the information below, use the sliders to fill in the rest of the table:

Answer: Completed table is below. Answers may vary because students may choose to factor completely. It is acceptable for linear factors not to be completely factored.

y_1	y_2	Zeros of y_1	Zeros of y_2	y_3	Zeros of y_3	Factors of y_3
$(x + 4)$	$(x + 3)$	–4	–3	$x^2 + 7x + 12$	–4 and –3	$(x + 4)(x + 3)$
$(2x - 4)$	$(x + 2)$	2	–2	$2x^2 + 0x - 8$	2 and –2	$(2x - 4)(x + 2)$
$(x - 5)$	$(-1x - 2)$	5	–2	$-1x^2 + 3x + 10$	5 and –2	$(x - 5)(-1x - 2)$
$(3x + 3)$	$(x + 4)$	–1	–4	$3x^2 + 15x + 12$	–1 and –4	$(3x + 3)(x + 4)$
$(x + 1)$	$(x - 4)$	–1	4	$x^2 - 3x - 4$	–1 and 4	$(x + 1)(x - 4)$
$(2x + 4)$	$(3x - 3)$	–2	1	$6x^2 + 6x - 12$	–2 and 1	$(2x + 4)(3x - 3)$

h. Write a conjecture about the relationship between the zeros of the linear functions and the zeros of the quadratic function.

Answer: The zeros of the linear functions are the zeros of the quadratic function.

> **Teacher Tip:** Some of the polynomials are not fully factored. This is a topic you might choose to explore with the students.

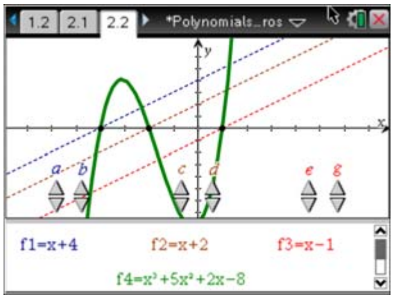
i. How do the factors of the quadratic equation relate to the zeros of the function?

Answer: The factors of the quadratics are the same linear functions that multiply together to make the quadratic and therefore can be solved to find the zeros or x-intercepts of the quadratic function. If a polynomial can be factored, factoring is one strategy for finding the real solutions of a polynomial equation.

> **Teacher Tip:** If students haven't solved quadratics by factoring, this would be a good time to discuss the concept.

Move to page 2.2.

2. Use the sliders to make $f1 = 1x + 4$, $f2 = 1x + 2$, and $f3 = 1x + -1$. Observe that the graphs of each appear to cross the x-axis at –4, –2, and 1, respectively.



a. Verify algebraically that each is a zero of each linear function.

Answer: $1(–4) + 4 = 0$, $1(–2) + 2 = 0$, $1(1) – 1 = 0$

b. When $f1 = 1x + 4$, $f2 = 1x + 2$, and $f3 = x – 1$, what is **f**4?

Answer: $f4 = x^3 + 5x^2 + 2x – 8$

c. How many times does $f4$ cross the x-axis and where?

Answer: Three times: at –4, –2, and 1

d. Show the multiplication of the factors of $f1$, $f2$, and $f3$ to equal $f4$.

Answer: $(x + 4)(x + 2) = x^2 + 6x + 8$, then $(x^2 + 6x + 8)(x – 1) = x^3 + 5x^2 + 2x – 8$

> **Teacher Tip:** If necessary, review multiplication of polynomial expressions by distributing each term in the first parentheses by every term in the second parentheses.

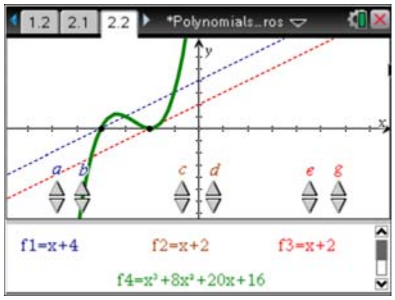
e. Try other slider values and make a conjecture about the relationship between the zeros of the linear equations and the zeros of the cubic function.

Answer: The zeros of the linear functions are the zeros of the cubic function.

3. Use the sliders to make *f1* = $x + 4$, *f2* = $x + 2$, and *f3* = $x + 2$.

   a. How has the graph changed? The value –2 is called a double root.

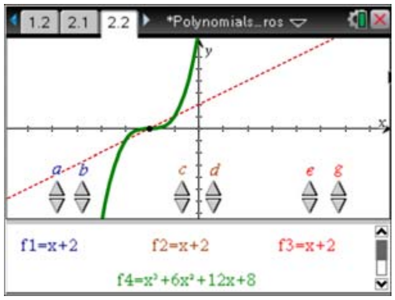
   Sample answer: Answers may vary, but students should say something similar to the fact that the graph no longer crosses the *x*–axis in three places, but appears to "bounce back up" at 2. It still crosses at –4. The new equation is *f4* = $x^3 + 8x^2 + 20x + 16$.



> **Teacher Tip:** By changing one of the factors to 0*x* and then getting a parabola, students might see that this point is the vertex of the quadratic, as seen previously.

   b. Change *f1* = $1x + 2$. How has the graph changed?

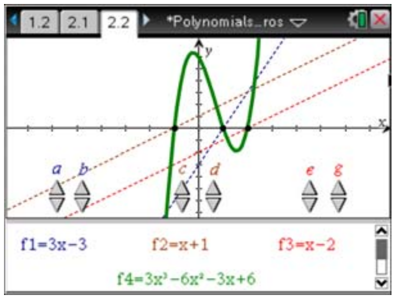
   Sample answer: Answers may vary, but the graph appears to flatten out at –2.The value –2 is a triple root of *f4* = $x^3 + 6x^2 + 12x + 8$.



4. Use the sliders to make *f1* = $3x – 3$, *f2* = $x + 1$, and *f3* = $x – 2$.

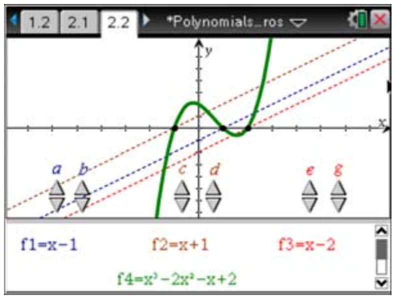
   a. Observe the graph and identify the zeros. What is f4?

   Answer: The zeros are 1, –1, and 2, and *f4* = $3x^3 – 6x^2 – 3x + 6$.



   b. Now change the sliders to make *f1* = $x – 1$, *f2* = $x + 1$, and *f3* = $x – 2$. Observe the graph. What are the zeros? What is *f4*?

   Answer: The zeros are 1, –1, and 2, and *f4* = $x^3 – 2x^2 – 1x + 2$.



c. Identify similarities and differences between the sets of equations in part a and part b.

Answer: The zeros of both functions are the same. The graph rises or falls differently between the two graphs. The second function is the first function multiplied by a factor of 3. The leading coefficient causes a vertical dilation (factor) of 3. Each factor can be used to find the zeros or *x*-intercepts of the functions or to find the roots of the corresponding equations.

Teacher Tip: By changing one of the factors to 0*x* and then getting a parabola, students might see that this point is the vertex of the quadratic, as seen previously.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to use a graph to find possible linear factors of a quadratic function.
- The connections between the algebraic and graphical representations of a quadratic and cubic function and its factors.
- That the zeros of the linear factors of a polynomial function and the zeros of the polynomial function are the same.
- That the zeros of a polynomial function are the same as the zeros of its linear factors.
- How a double or triple root of a polynomial function affects the graph.
- The effects of the leading coefficient on a cubic function.

Assessment

1. Given zeros of –4.5, –1, and 2, find a possible cubic equation. Is your answer unique? Explain.

2. Given that (x + 5) and (2x – 1) are the only factors of a cubic polynomial, find a possible cubic equation. Is your answer unique? Explain.

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Note 1

Entire Lesson, *Live Presenter* or *Screen Capture:* If students experience difficulty with the sliders, use *Screen Capture* or *Live Presenter* with TI-Nspire Navigator to demonstrate to the entire class.

Note 2

End of Lesson, *Quick Polls:* The following *Quick Poll* questions can be given at the conclusion of the lesson. You can save the results and show a class analysis at the start of the next class to discuss possible misunderstandings students may have.

1. Given zeros of –5, 1, and 3, a possible cubic equation is:
   a. $y = (x - 5)(x + 1)(x + 3)$
   b. $y = (x + 5)(x - 1)(x - 3)$
   c. $y = (x - 5)(x - 1)(x + 3)$
   d. $y = (x + 5)(x - 1)(x + 3)$

   Answer: b

2. The zeros of $y = x(x + 4)(x - 2)$ are:
   a. 1, –4, 2
   b. 0, 4, –2
   c. 1, 4, –2
   d. 0, –4, 2

   Answer: d

3. A cubic equation has a root at –6 and a double root at 4. The factors of the equation are:
   a. $(x + 6)$, $(x + 4)$, and $(x - 4)$
   b. $(x - 4)$, $(x + 6)$, and $2(x - 4)$
   c. $(x + 6)$, $(x - 4)$, and $(x - 4)$
   d. $(x - 6)$, $(x + 4)$, and $(x + 4)$

   Answer: c