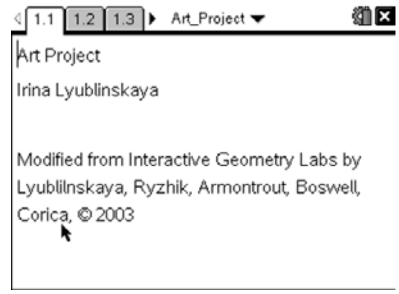
The Art Project

by Dr. Irina Lyublinskaya

Activity overview



In this activity students explore the locus of points in the interior of the right angle such that the sum of the distances to the sides of the angle is constant.

Statement of the Problem: *As part of an art project, a fourth-grade student decorates a rectangular sheet of paper with bent pipe cleaners. She gathers several pipe cleaners, all the same size, and bends each one at a different location to form right angles. She then takes a bent pipe cleaner and staples its tips so they touch two adjacent edges of the paper. Only the tips of the pipe cleaner touch the edges of the paper. The two edges of the pipe cleaner are parallel to the edges of the paper. One by one, the child takes her bent pipe cleaners and overlaps them according to the same rule: the tips of each pipe cleaner touch the same two adjacent edges. A point sits at the bend of each pipe cleaner. When viewed collectively on the paper, what path will these points form?*

Teacher is encouraged to have students work through the following stages of this problem.

- visualize the bent pipe cleaners being placed so that the end are on the rays of a right angle
- describe or draw a diagram of what has been visualized; and describe the construction method, the diagram should <u>not</u> be a sketch of the situation
- investigate the problem with TI-Nspire technology; by this stage students should start to consider how the proof of their conjecture will be approached
- give a convincing argument for the solution to the problem

Concepts

Key property: the sum of the distances from a point on the hypotenuse of a right isosceles triangle, to the legs, is a constant.

Teacher preparation

Before carrying out this activity teacher should review with the students the following concepts: isosceles triangle, parallel lines and corresponding angles, slope. The screenshots on pages 2-4 demonstrate expected student results. Refer to the screenshots on page 5 for a preview of the student TI-Nspire document (.tns file).

Classroom management tips

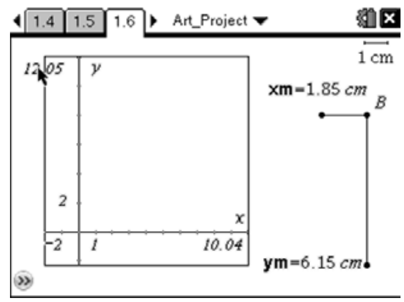
- *This activity is designed to be **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.*
- *Students may answer the questions posed in the .tns file using the Notes application or on a separate sheet of paper.*

- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

TI-Nspire Applications
Graphs, Geometry, Notes.

Step-by-step directions

Problem 1 – Case of Right Angle



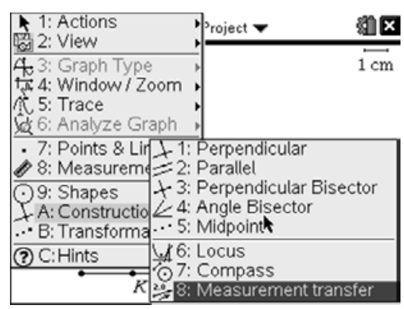
Step 1. Students open file Art_Project.tns, read problem statement on pages 1.2-1.4, and move to page 1.5 where they need to answer first question.

Q1. Close your eyes. Can you visualize what will the overall shape of the filled part of the poster board look like? If so, describe or draw your conjecture(s) about this shape.

A. The correct answer is: the shape of the filled part of the poster board will be a right isosceles triangle.

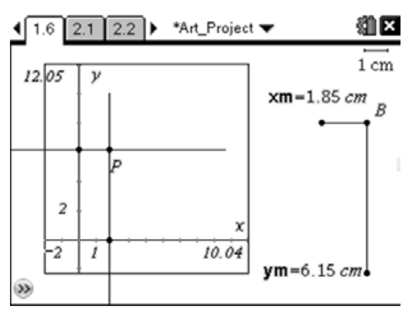
At this initial stage of the problem many students may offer incorrect answers. They should not be corrected, since they can correct themselves after the exploration using TI-Nspire.

Step 2. Students should move to page 1.6 where they will simulate the process of filling the corner of the poster board.



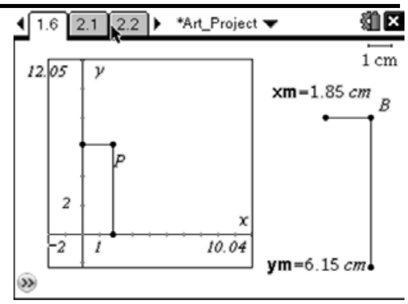
On this page a pipe cleaner is shown to the right of the graph. Point B is point of bending that can be changed. Point B divides the pipe cleaner into two segments with lengths xB and yB. Students can move point B to simulate different bents of the pipe cleaner. The following are the steps that will allow students to model the process described in the statement of the problem

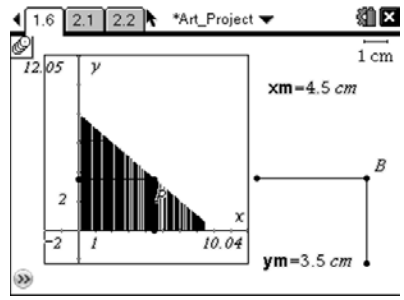
Step 3. Click on (menu), 9: Construction, 8: Measurement Transfer. Click on xB and then on the x-axis, the point will appear on the x-axis that corresponds to the length xB. Transfer yB to the y-axis.

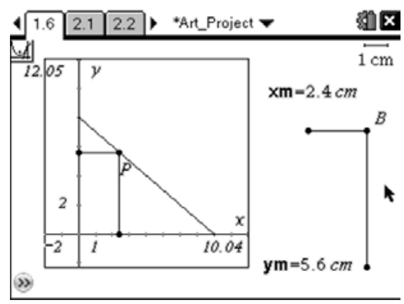


Step 4. Construct perpendicular lines to the axes through the points by clicking (menu), 9: Construction, 1: Perpendicular. Then click on the coordinate axis, then on the point on the axis. Repeat the same for the 2nd point.

Step 5. Mark point of intersection between these two perpendiculars with the point tool: (menu), 6: Points & Lines, 1: Point. Label this point P.







Q2. What is geometrical meaning of the point of intersection?

A. The point P represents the corner of the bent pipe cleaner after it is placed on the board.

Step 6. Construct segments connecting point B with the coordinate axes and hide the perpendiculars, by doing the following: Click (menu), 6: Points & Lines, 5: Segment. Construct the segments.

Step 7. Click on (menu), 1: Actions, 2: Hide/Show and select each perpendicular. Then press (esc). The perpendicular lines will disappear.

Q3. Drag point B on the pipe cleaner. Can you determine now what will the overall shape of the filled part of the poster board look like? Is this different from what you predicted earlier?

A. By moving the point B students should see that the point P moves along a straight line forming a triangle shape along with the sides along the coordinate axes.

Q4. Trace both segments to confirm your prediction.

Step 9. Students can trace point P and both segments by clicking (menu), 5: Trace, 3: Geometry Trace. While in Trace menu, click on point P and both segments. Drag point B and observe the traces left by these two segments and the point.

Q5. Formulate your final conjecture and prove it.

A. The overall shape of the filled part of the poster board is a right isosceles triangle with legs equal to the length of the pipe cleaner. The possible geometric proof for this statement is following:

Let *O* is the origin and points M and N are vertices of the triangle formed by the pipe cleaners. Given $\angle$MON = 90°. By construction, *OM* = *ON* = *C* > 0. Let *P* is such point that the sum of distances from *P* to the lines *OM* and *ON* remains constant = *C* > 0. Let prove that point *P* moves along segment *MN*. Construct *PK* $\perp$ *OM* and *PL* $\perp$ *ON*. From the rectangle *OKPL*, we get *PK* = *OL* and *PL* = *OK*, so *PK* + *PL* = *PK* + *OK*. Since *OM* = *ON*, then $\angle$OMN = $\angle$ONM = 45°. Then, ΔMKP is isosceles right triangle and *PK* = *MK*. Then, *PK* + *PL* = *MK* + *OK* = *OM* , which proves that segment *MN* is a locus of point *P* and the region OMN is a triangle.

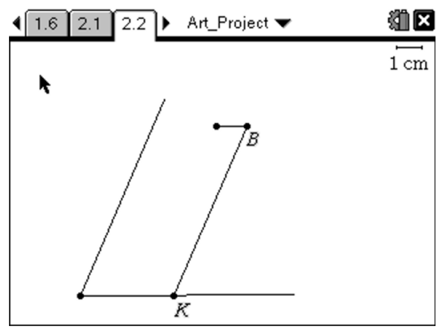
The possible algebraic proof for this problem is following:

Use coordinate method. Let *O* is the origin with coordinates (0, 0), ray *OM* is *y* – axis and ray *ON* is *x* – axis. Consider any point *P*(*x*, *y*) such that $PK + PL = C$. Since, $PK + PL = OK + OL = x + y$, then the equation of the locus is $x + y = C$ or $y = C – x$, which is an equation of a line. Additional restrictions follow from the condition that point P is inside the right angle $\angle MON \Rightarrow x \geq 0$ and $y \geq 0$. Then, the locus is a segment of the line $y = C – x$ in the first quadrant, which is the segment MN.

Teacher may also request students to prove uniqueness of the solution.

Activity extension:

Problem 2 – Case of Acute Angle

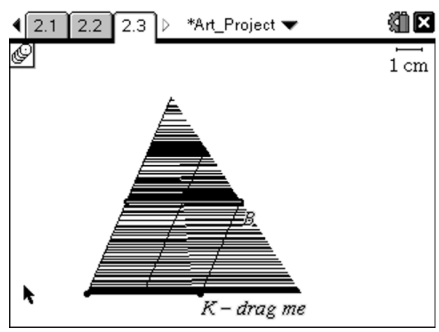


Step 1. Students read the problem statement on page 2.1, make a conjecture and then move to page 2.2 to explore this situation.

Q6. What is the shape of the bounded region if the poster board is a parallelogram and students fill the acute angle of the poster board following the same method?

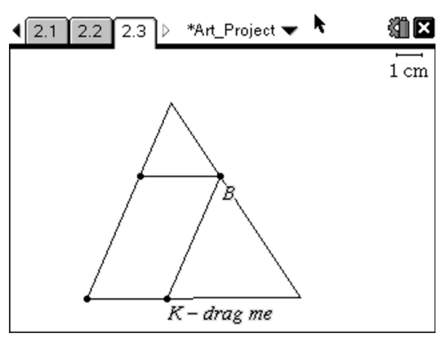
Step 2. Students can move point B to vary the bend of the pipe cleaner and then move point K to adjust the position of the pipe cleaner. They can make conjecture about the shape of the region.

A. The shape will be an isosceles triangle. The locus of the points is a segment.



Step 3. Students should move to page 2.3 to confirm their prediction. Here the construction is adjusted for exploration of the region with the Geometry Trace.

Step 4. Choose (menu), 5: Trace, 3: Geometry Trace. While in Trace menu, click on point B and both segments. Move point K and observe the traces left by these two segments and the point.



Proof:

Let $\angle MON$ is an acute angle, $BK \parallel ON$ and $BL \parallel OM$ and $BK + BL = C$. Choose $M_1 \in OM$ so that $OM_1 = C$ and $N_1 \in ON$ so that $ON_1 = C$. Since $BKOL$ is a parallelogram, $BL = OK$. ΔM_1ON_1 is isosceles, so $\angle BM_1O = \angle BN_1O$. Since $BK \parallel ON$ then $\angle N_1BL = \angle BM_1O$, then so $\angle BN_1O = \angle N_1BL$ and ΔN_1LB is isosceles $\Rightarrow BL = LN_1$, so $BK + BL = KN_1 + OK = ON_1 = C$.
When $\angle MON$ is an obtuse angle, the proof is the same. Teacher may also request students to prove uniqueness of the solution.

The Art Project
by: Dr. Irina Lyublinskaya
Grade level: secondary
Subject: mathematics
Time required: 45 to 90 minutes

Materials: TI-Nspire

Student TI-Nspire Document

Art_Project.tns

1.1 | 1.2 | 1.3 Art_Project

Art Project

Irina Lyublinskaya

Modified from Interactive Geometry Labs by Lyublilnskaya, Ryzhik, Armontrout, Boswell, Corica, © 2003

1.1 | 1.2 | 1.3 Art_Project

An elementary school teacher is doing an art project with his students. Each student receives a rectangular piece of poster board and a set of pipe cleaners of the same length. Each student fills one corner of the poster board by attaching pipe cleaners to the poster board in the following way:

1.1 | 1.2 | 1.3 Art_Project

the ends of each pipe cleaner must be on different sides of the corner, any part of the pipe cleaner must be parallel or perpendicular to the sides of the corner, and each pipe cleaner can be bent only once. What will be the overall shape of the filled part of the poster board?

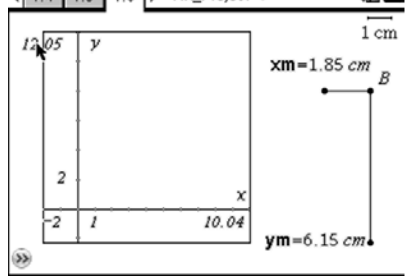
1.2 | 1.3 | 1.4 Art_Project

The problem can be modeled geometrically by considering the region bounded by the sides of a right angle and by the locus of points in the interior of the right angle such that the sum of the distances to the sides of the angle is constant.

1.3 | 1.4 | 1.5 Art_Project

1. Close your eyes. Can you visualize what will the overall shape of the filled part of the poster board look like? If so, describe or draw your conjecture(s) about this shape.

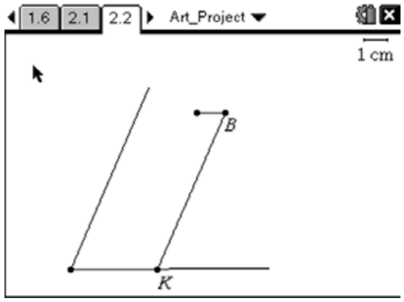
1.4 | 1.5 | 1.6 Art_Project



1.5 | 1.6 | 2.1 Art_Project

What is the shape of the bounded region if the poster board is a parallelogram and students fill the acute angle of the poster board following the same method?

1.6 | 2.1 | 2.2 Art_Project



2.1 | 2.2 | 2.3 Art_Project