NUMB3RS Activity: Truth or Dare
Episode: "Money for Nothing"

Topic: Logic Riddles   **Grade Level:** 9 - 12
Objective: Explore conditional statements in math riddles
Time: 20 - 45 minutes

Introduction
In "Money for Nothing," an armored truck carrying cash and medicine for African Relief is hijacked. The FBI team is able to capture two of the robbers but not the hijacked truck or the hostages. Don plans to interrogate the robbers to determine if the truck is in Los Angeles or if it has left the area, but he does not know if the robbers will tell the truth or lie. Charlie tells Don that "there's a classic logic puzzle that kind of applies."

For the Teacher
The first question in this activity is the logic puzzle to which Charlie refers. Following that are several other classic riddles that explore the distinction between "and" and "or." The time needed to explore the questions may vary, depending on how quickly students reach some critical realizations or approaches. You may decide to pick and choose from the questions provided.

Group discussions of the questions can be fun and enlightening. Hints to the solutions are not included on the student pages to allow discussions to progress more naturally. Because guidance is often needed for these problems, each question is discussed in more detail below, including an explanation of the solution. Leading questions should ideally follow class discussions, but suggestions are given for each question. The most important strategy is to allow students time to think about the problems (perhaps over several days).

Students will enjoy the opportunity to explore and conjecture by themselves before getting too much support. More importantly, finding the solutions on their own is more enjoyable than just hearing the solution from the teacher. For most of the situations presented, students will need to accept the impracticality of some conditions (for example, in Question 1, "Why do you only get to ask one question?"). Students may find these conditions frustrating, but encourage them to accept the conditions and focus on the logic. As a final general note, understanding how a solution works can be as challenging as developing a strategy. Acting out the scenarios, even after the solution is known, can be an illuminating endeavor.

Student Page Answers:
1. **Solution**: Ask either guard, "If I ask the other guard which door leads to freedom, what will he say?" The door he indicates leads to the tiger.
    Explanation: Call door A the door to freedom. The liar will tell you that the other guard will say door B, lying about what the other will say. The guard who tells the truth will say door B, accurately relaying the other guard's lie. In both cases, the door named leads to the tiger.
    Leading question: What will each guard say if you ask which door to open? (A two-part question is not allowed but a compound question is. The question involves asking about the correct door.)
    Relationship to episode: Charlie tells Don to "ask each guy where the other one will say the truck is." When Don points out that at least one of them will certainly be lying, Charlie responds "Exactly—one's lying, but one's telling the truth. Whatever either one says, the truck will be in the opposite place."

2. **Answer**: The third native is a liar.
    Explanation: The first native will always say she tells the truth, even if she is a liar lying about who she is. So the second native was telling the truth, and we know the third native is lying.
    Leading questions: How would you answer the first question if you told the truth? If you lied? What does that mean about the second native's answer?

3. **Solution**: One senator is honest.
   Explanation: If there were 2 (or more) honest senators, you could choose a pair with only honest senators.
   Leading question: A common first response is 50 honest and 50 dishonest. For any proposal, ask if EVERY pair you can construct has a dishonest senator.

4a. **Solution**: Open Box B.
   Explanation: If both signs are false, neither box has money, and there is money in box A, which is a contradiction. So both signs are true, which means one box has money and there is a snake in box A.

4b. **Solution**: Open Box A.
   Explanation: If there is money in box B, then there is a snake in A, which means that no box has money. This is a contradiction. If there is a snake in box B, then there is money in A, and at least one box has money.
   Leading questions: Encourage students to try different possible scenarios. For example, what is true if the sign on box A is false? What is true if a snake is in box A? You may need to clarify that both boxes could have money, both could have snakes, or there could be one of each.

5. **Solution**: They leave together at 1:00.
   Explanation: Consider the scenario from the perspective of just one teacher. He sees a red dot on his colleague's hat, which means he may or may not have one. Because he does not know, he does not leave at noon. But his colleague also does not leave at noon. That means he must have gone through the same logic and seen a red dot. Now they know that they both have red dots and can leave the next time the door is unlocked at 1:00.
   Leading questions: What would happen if the situation were different and only one person had a red dot? What would you do if you saw a hat without a red dot?

Name _____ Date _____

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The first question below is the logic puzzle to which Charlie refers. Following that are several other classic riddles that celebrate the distinction between "and" and "or."

1. You are trapped in a room with two doors. One door leads to freedom, and the other door leads to a vicious tiger. Unfortunately, you do not know which door is which. Fortunately, each door has a guard, and you are allowed to ask one question to one of the guards. One guard always lies, but the other guard always tells the truth. Unfortunately, you do not know which guard is which. What one question should you ask, and to which guard, to determine the correct door to open?

2. You visit a new land where every inhabitant either tells the truth or lies. You meet a native and ask if she tells the truth or lies. She responds in her native tongue, which you do not understand. So you ask a second native what the first said. The second replies "She said she tells the truth." Still skeptical, you ask a third native if the second tells the truth or lies. "He is a liar," says the third. Does the third native tell the truth or lie?

3. A house contains 100 senators, each of which is either honest or dishonest. At least one senator is honest. If any two senators are chosen at random, at least one of the two is guaranteed to be dishonest. How many honest senators are in the house?

4. Two boxes, A and B, are presented to you. Each box contains either one million dollars or a deadly snake. Box A has a sign that reads, "At least one box contains money." Box B has a sign that reads, "Box A contains a deadly snake." You must open one of the boxes.

    a. Either both signs are true or both signs are false. Which box do you open, and why?

    b. A box with money has a sign that is true. A box with a snake has a sign that is false. Which box do you open, and why?

5. Two math teachers are captured by an evil logician. The logician puts a hat with a red dot on each teacher's head and tells them, "At least one of you has a red dot on your hat. The door to this room will be unlocked for one minute at the start of every hour, beginning at noon. Anyone may walk through the door during that time. If you have a red dot and walk through the door, you are free. If you walk out the door but do not have a red dot, you will be returned to this prison for eternity. You may look at each other, but you may not remove your hat, and you may not communicate in any manner." How and when do the mathematicians escape?

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Below are two variations on Question 5. The logician has captured people and put hats on all of them, though they do not necessarily all have red dots. The logician tells them that at least one person has a red dot and the door will be open for one minute at the top of every hour. If all of the people with red dots leave the room, everyone will be saved. But if anyone without a red dot leaves, they will all remain captives. The people cannot remove their hats and cannot communicate with each other in any way.

- If three people are in the room, then one, two or three people have red dots. What will happen in each case?

- You are one of six people in the room. You see three people with red dots. What do you do?

Additional Resources
More puzzles like those in the activity can be found at the following Web sites.
- **http://www.rinkworks.com/brainfood/p/practical1.shtml**
- **http://www.mycoted.com/Category:Puzzles**
- **http://www.folj.com/lateral**
- **http://www.lateralpuzzles.com**
- **http://www.pzzls.com**
- **http://www.braingle.com**