TI-nspire™

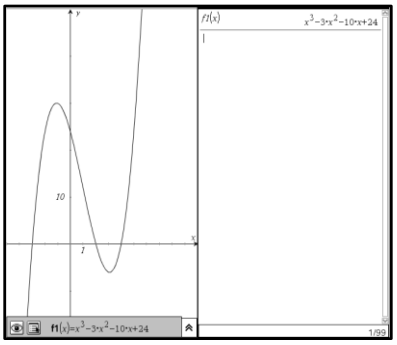
Grade level: secondary
Subject: mathematics (calculus)
Time required: 90-135 minutes

Two Investigations of Cubic Functions: Student Worksheet

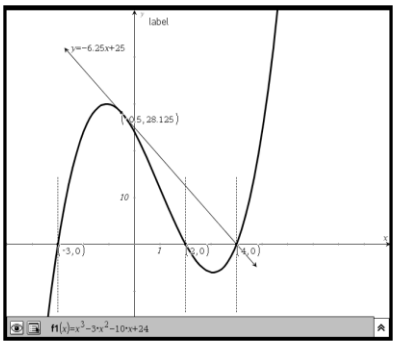
Name: _____ **Date**_____

In this activity, you will explore two interesting features of cubic functions which have three real roots. To get started, open the file labeled "CubicInvestigation.tns" and follow instructions provided in the document as well as on this worksheet.

The first couple pages are simply introductory. The first investigation begins as a problem in page 2.1 with a consideration of the function $f(x) = x^3 - 3x^2 - 10x + 24$. Examine the zeros on the graph given on page 2.2 and use the calculator in the split-screen to factor f(x) and solve f(x)=0. The FACTOR and SOLVE commands are available in the ALGEBRA menu.



1. What are the three roots of f(x)?



2. Consider the example given on pages 2.4 and 2.5 where, beginning with roots -3 and 2, which average to ***n*** = -0.5, the equation of the tangent at -0.5 is y = (-25/4)x + 25. What is the root of the equation of this tangent line? How does this root compare to the roots of f(x)?

3. Pick any other pair of roots of f(x) and find their average (labeled ***n*** for "***n***ew x-value").

4. Find the equation of the tangent line to f(x) at your value for ***n***.

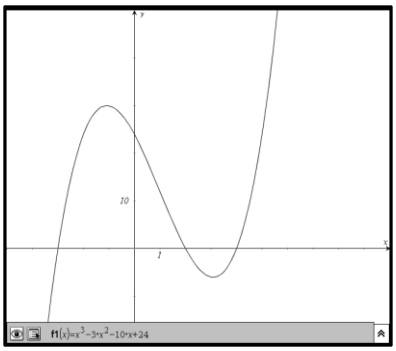
5. What is the root of your tangent line equation? How does this root compare to the roots of f(x)?

6. Pick another pair of roots of f(x) and repeat problems #3 – 5.

7. Consider the general case g(x)=(x-a)(x-b)(x-c), where g(x) is a cubic function with roots a, b, and c. Beginning with roots a and b, which average to ***n***=(a+b)/2, use CAS capabilities to determine the equation of the tangent at ***n***. Then find the root of the equation of the tangent and compare it to the roots of g(x).

8. The second investigation begins as a problem in page 3.1 with a consideration of the function $f(x) = x^3 - 3x^2 - 10x + 24$. Start by finding the relative minimum and relative maximum points of f(x).

9. The graph of the function is provided on page 3.3.
(a) Plot your relative minimum and relative maximum points with the POINT ON command from the POINTS & LINES menu.
(b) Create a line segment that connects your relative minimum and relative maximum points with the SEGMENT command from the POINTS & LINES menu.
(c) Find the midpoint of your segment with the MIDPOINT command from the CONSTRUCTION menu.
(d) Label your midpoint with the COORDINATES AND EQUATIONS command from the ACTIONS menu.



What is your midpoint?

10. Find the inflection point of f(x). How does the inflection point compare to your midpoint?

11. Consider the general case g(x)=(x-a)(x-b)(x-c), where g(x) is a cubic function with roots a, b, and c. Use CAS capabilities to determine the relative minimum and relative maximum points of g(x).

12. Use the midpoint formula to find the x- and y-coordinates of the midpoint between your relative minimum and relative maximum points (with CAS capabilities).

13. Use CAS capabilities to determine the inflection point of g(x). How does the inflection point compare to your midpoint?