EXPLORATIONS

Activity 6

You're So Dense

Objectives

- To investigate the relationship between mass and volume
- To find the *x* value of a function, given the *y* value
- To find the *y* value of a function, given the *x* value
- To use technology to find a linear regression
- To use technology to plot a set of ordered pairs

Materials

- TI-73 graphing device
- 50 ml graduated cylinder
- Balance
- Pennies (25 pre-1982 and 25 post-1984)

Introduction

Have you ever picked up an object that you thought was heavy, and it was light? For example, if you saw a solid metal block that is 10 cm^3 you might expect it to be difficult to lift. If that block were aluminum, its mass would only be 2.7 kg (or a little under 6 pounds). On the other hand, if the metal block were platinum, its mass would be 21.5 kg (or 47 pounds). The reason for this difference is that metals have different densities. The density of an object is a measure of the mass divided by the volume.

Problem

How can you use the density of pennies to predict the dates they were made?

Collecting the data

1. Your teacher will give you pennies. Separate the pennies into two piles: those that were made before 1982 and those that were made after 1984. For steps 2 through 8, use the pennies that were made before 1982. Repeat steps 2 through 8 using the pennies that were made after 1984.

2. Fill a graduated cylinder with water to the 20 ml mark. Using a balance, determine the mass of the graduated cylinder with the water.

3. Add five pennies to the graduated cylinder. Determine the difference in the volume by subtracting the original 20 ml from the new volume. This difference is the volume of the pennies that you have added. Record the volume of the five pennies on the **Data Collection and Analysis** page.

   Note: *It is assumed that 0 pennies has a volume of 0 ml and a mass of 0 grams. These values have already been recorded on the* **Data Collection and Analysis** *page.*

4. Determine the difference in the mass by subtracting the original mass from the new mass. The difference is the mass of the pennies that you have added. Record the mass of the five pennies on the **Data Collection and Analysis** page.

5. Add a second set of five pennies (for a total of 10) to the graduated cylinder. Follow the directions in steps 3 and 4 to determine the *total* volume and *total* mass of the 10 pennies. Record the volume and mass of the 10 pennies on the **Data Collection and Analysis** page.

6. Add another five pennies (for a total of 15) to the graduated cylinder. Follow the directions in steps 3 and 4 to determine the *total* volume and *total* mass of the 15 pennies. Record the volume and mass of the 15 pennies on the **Data Collection and Analysis** page.

7. Add another five pennies (for a total of 20) to the graduated cylinder. Follow the directions in steps 3 and 4 to determine the *total* volume and *total* mass of the 20 pennies. Record the volume and mass of the 20 pennies on the **Data Collection and Analysis** page.

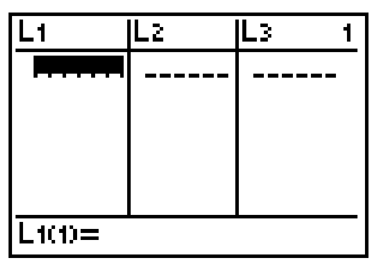
8. Add another 5 pennies (for a total of 25) to the graduated cylinder. Follow the directions in steps 3 and 4 to determine the *total* volume and *total* mass of the 25 pennies. Record the volume and mass of the 25 pennies on the **Data Collection and Analysis** page.

Setting up the TI-73

Before starting your data collection, make sure that the TI-73 has the STAT PLOTS turned OFF, Y= functions turned OFF or cleared, the MODE and FORMAT set to their defaults, and the lists cleared. See the Appendix for a detailed description of the general setup steps.

Entering the data in the TI-73

1. Press [LIST].

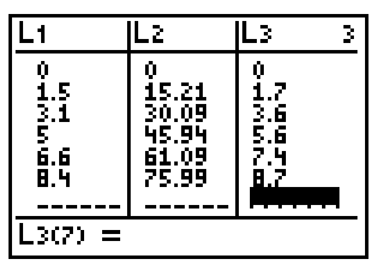


2. Enter the volume of the pre-1982 pennies in **L1**.

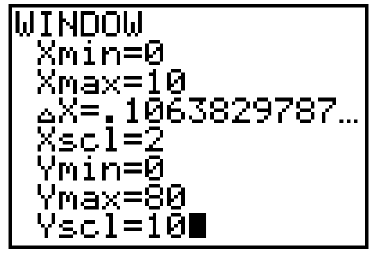
3. Enter the mass of the pre-1982 pennies in **L2**.

4. Enter the volume of the post-1984 pennies in **L3**.

5. Enter the mass of the post-1984 pennies in **L4**.



Setting up the window

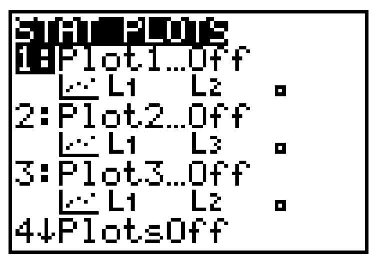


1. Press [WINDOW] to set up the proper scale for the axes.
2. Set the **Xmin** value by identifying the minimum value in **L1**. Choose a number that is less than the minimum.
3. Set the **Xmax** value by identifying the maximum value in each list. Choose a number that is greater than the maximum. **Do Not Change the ΔX Value.** Set the **Xscl** to **2**.
4. Set the **Ymin** value by identifying the minimum value in **L2**. Choose a number that is less than the minimum.
5. Set the **Ymax** value by identifying the maximum value in **L2**. Choose a number that is greater than the maximum. Set the **Yscl** to **10**.

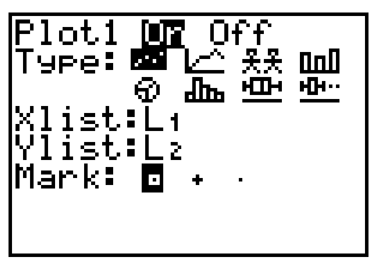
Graphing the data: Setting up the scatter plots

To analyze the data, you will need to set up a scatter plot for each set of data and then model that data by graphing a line of best fit (linear regression). You will then use the data that you collected to compare the pre-1982 to the post-1984 pennies.

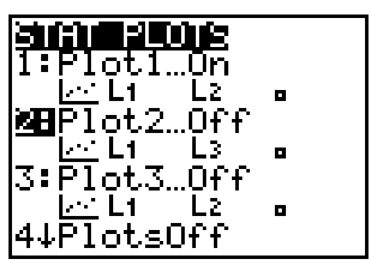
1. Press [2nd] [PLOT]. Select **1:Plot1** by pressing **1** or [ENTER].



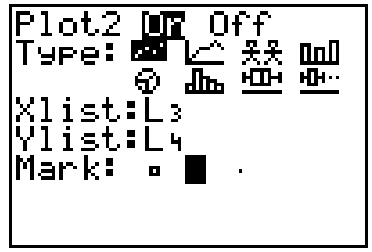
2. Set up the plot as shown by pressing [ENTER] [▼] [ENTER] [▼] [2nd] [STAT] **1:L1** [▼] [2nd] [STAT] **2:L2** [▼] [ENTER].



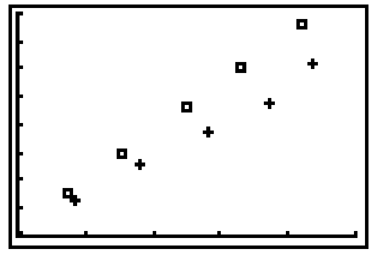
3. Press [2nd] [PLOT]. Select **2:Plot2** by pressing **2**.



4. Set up the plot as shown by pressing [ENTER] [▼] [ENTER] [▼] [2nd] [STAT] **3:L3** [▼] [2nd] [STAT] **4:L4** [▼] [▶] [ENTER].



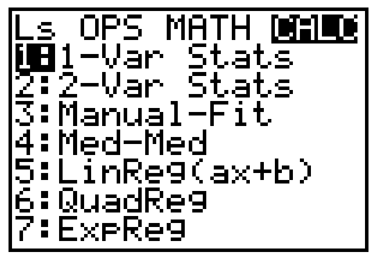
5. Press [GRAPH] to see the plots.



Analyzing the data

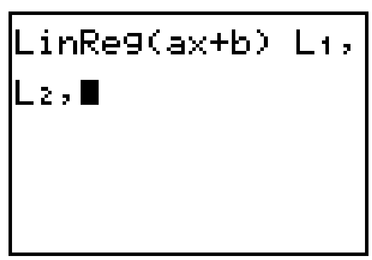
Finding a linear regression

1. Plot a linear regression for the pre-1982 penny data. Press [2nd] [STAT] [◀] to move the cursor to the **CALC** menu.

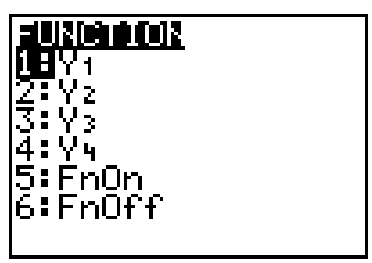


2. Select **5:LinReg(ax+b)** by pressing **5**.

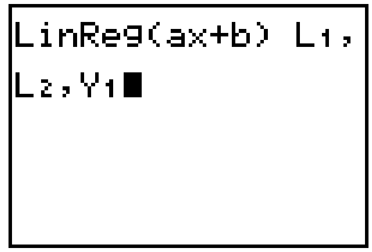
3. Press [2nd] [STAT] **1:L1** [,] [2nd] [STAT] **2:L2** [,].



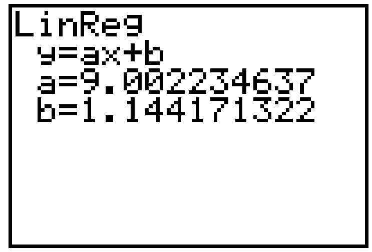
4. Press [2nd] [VARS]. Select **2:Y-Vars** by pressing **2**.



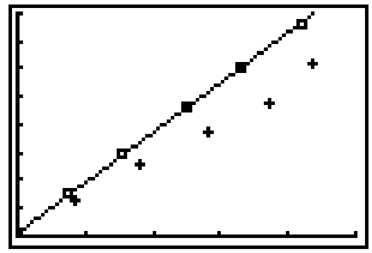
5. Select **1:Y1** by pressing **1** or [ENTER].



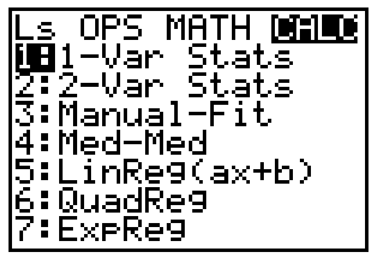
6. Press [ENTER] to calculate the linear regression. The function is pasted in **Y1**.



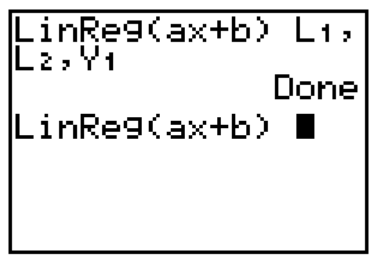
7. Press [GRAPH] to see the linear regression model.



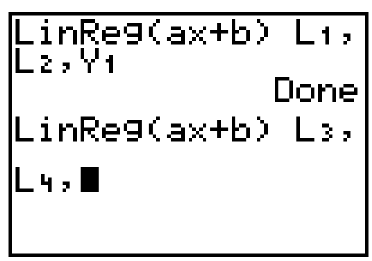
8. Plot a linear regression for the post-1984 penny data. Press [2nd] [STAT] [◄] to move the cursor to the **CALC** menu.



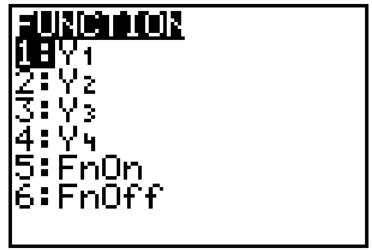
9. Select **5:LinReg(ax+b)** by pressing **5**.



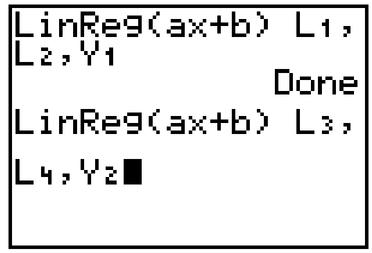
10. Press [2nd] [STAT] **3:L3** [,] [2nd] [STAT] **4:L4** [,].



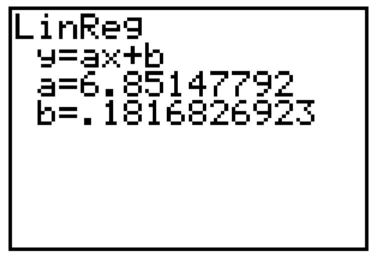
11. Press [2nd] [VARS]. Select **2:Y-Vars** by pressing **2**.



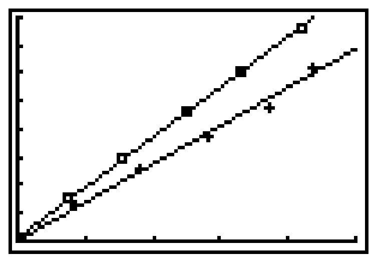
12. Select **2:**$\mathbf{Y_2}$ by pressing **2**.



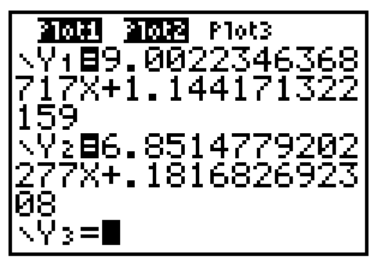
13. Press [ENTER] to calculate the linear regression. The function is pasted in $\mathbf{Y_2}$.



14. Press [GRAPH] to see the linear regression model.



15. Press [Y=] and observe the two equations. $\mathbf{Y_1}$ is the equation that describes the data for the pre-1982 pennies and $\mathbf{Y_2}$ is the equation that describes the data for the post-1984 pennies. The equations are for linear regressions and are therefore in the *Y* = *AX* + *B* format where *A* is the *slope* and *B* is the *y*-intercept.



Answer questions 1 through 5 on the **Data Collection and Analysis** page.

Determining the density of pennies

You can determine the dates that the pennies were minted based on their density.

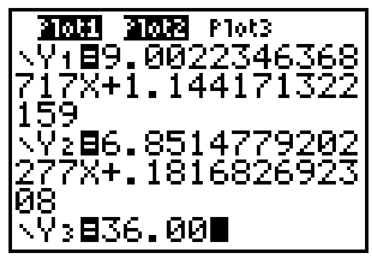
1. Your teacher will give you 15 pennies, but will not tell you whether they are pre-1982 or post-1984. They will all be from one of the two date categories (that is, either all will be pre-1982 or post-1984). Measure their mass and determine their volume by water displacement as you did earlier in the activity.
2. Determine the density by dividing the mass by the volume. Which category of penny do you have? Confirm your findings by checking the dates.

Predicting the volume of pennies

You can predict the volume of pennies if you know their mass.

1. Your teacher will give you 15 pennies and tell you whether they are pre-1982 or post-1984. Using the equation $D = \frac{M}{V}$ and the density determined earlier, measure their mass with a balance and mathematically determine their volume.
2. Verify your answer using the TI-73. Press [Y=] and move the cursor to **Y3** and enter the mass of the pennies.

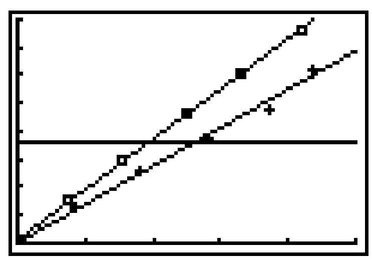
   (In this example, assume that the mass is 36 kg and that the pennies are pre-1982. The teacher may give you a different number of pennies with a different mass.)



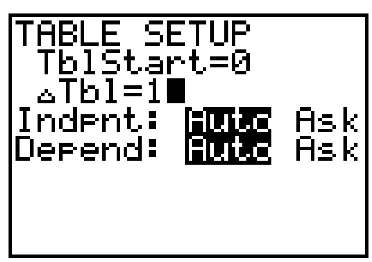
3. Press [GRAPH] to see the intersection of the lines.

   The x values of the points where the lines intersect relate to the volumes for 15 pennies.

   Use the **Table** function of the TI-73 to determine the coordinates of the points of intersection.

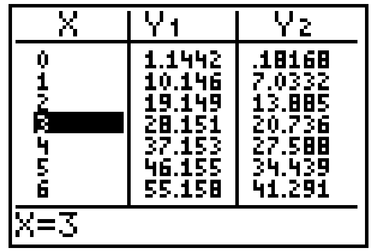


4. Press [2nd] [TBLSET]. Type **0** for **TblStart**. Press [▼] **1** to set the **ΔTbl** value.

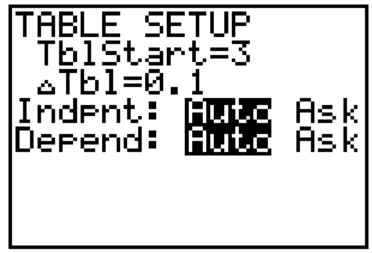


5. Press [2nd] [TABLE]. If necessary, use [▼] or [▲] to scroll the table.

   Note: *For this example, in the **Y1** column, 36 g falls between 28.151 and 37.153 which corresponds to 3 and 4 mL. Based on this information, the table will be readjusted.*

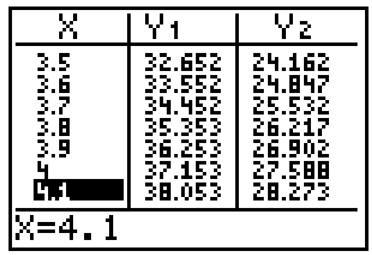


6. Press [2nd] [TBLSET]. Enter the results from Step 5 for the **TblStart** value. Press [▼] **0.1** to set the **ΔTbl** value.

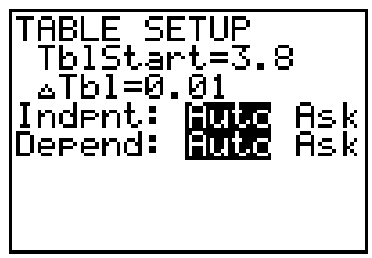


7. Press [2nd] [TABLE]. If necessary, use [▼] or [▲] to scroll the table.

   Note: *For this example, in the **Y1** column, 36 g falls between 35.353 and 36.253 which corresponds to 3.8 and 3.9 mL. Based on this information, the table will be readjusted.*

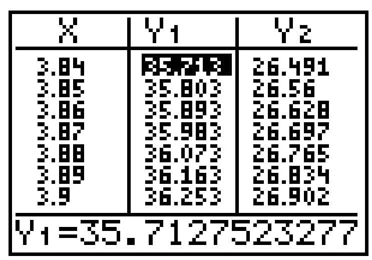


8. Press [2nd] [TBLSET]. Enter the results from Step 7 for the **TblStart** value. Press [▼] **0.01** to set the **ΔTbl** value.

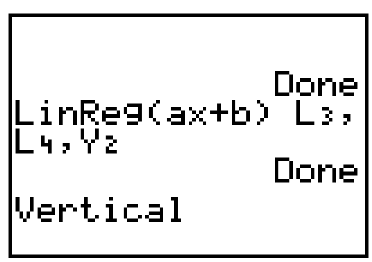


9. Press [2nd] [TABLE]. If necessary, use [▼] or [▲] to scroll the table.

   Note: *The data used to construct the linear model had volumes measured to the nearest tenth of a mL. Therefore, the volume of the pennies is reported to the same level of precision. From the table, 36 g falls between 35.983 and 36.073 which corresponds to 3.87 and 3.88 mL. Rounding to the nearest tenth of a mL, the intersection point is (3.9, 36).*

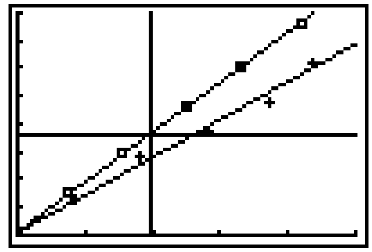


10. To verify this graphically, use the **DRAW** function. Press [DRAW]. Select **4:Vertical** by pressing **4**.



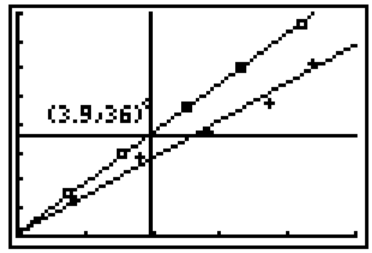
11. Type your results from Step 9 (in this example, **3.9)**, and press [ENTER].

    Note: *The coordinates of the point on the linear model for pre-1982 pennies where all of the lines intersect is defined, for this example, by the vertical drawn at* x=3.9 *and the horizontal at* y=36.



12. The coordinates of the intersection can be added onto the screen by pressing [DRAW] **7:Text**, moving the cursor near the point of intersect and typing your results.

    Note: *Text appears below and to the right of the cursor.*

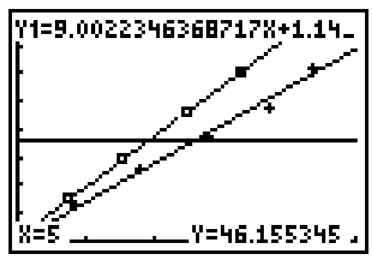


Predicting the mass of pennies

You can predict the mass of pennies if you know their volume.

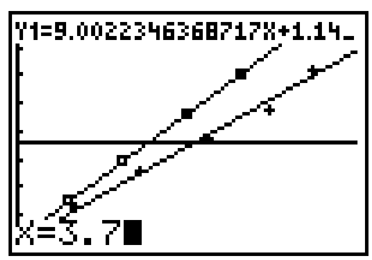
1. Your teacher will give you 15 pennies and tell you whether they are pre-1982 or post-1984. Using the equation $D = \frac{M}{V}$ and the density determined earlier, measure their volume by water displacement and mathematically determine their mass.

2. Verify your answer using the TI-73. Press [TRACE] and press [▼] until you see the equation for **Y1** (if the pennies are pre-1982) or **Y2** (if the pennies are post-1984).

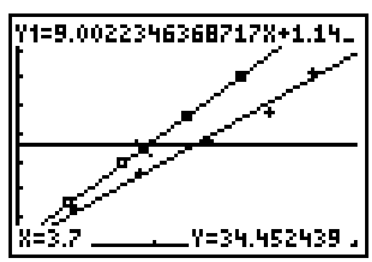


3. Type the volume of the pennies, and press [ENTER].

    Note: *3.7 was used for the value in this example.*



4. The *y* value in the lower right hand corner of the screen is the mass of the 15 pennies.



Data Collection and Analysis

Name___________________________

Date ___________________________

Activity 6: You're So Dense

Collecting the data

Number of pennies	Pre-1982		Post-1984	
	Volume (cm^3)	Mass (g)	Volume (cm^3)	Mass (g)
0	0	0	0	0
5				
10				
15				
20				
25				

Analyzing the data

1. The *slope* of the linear regression line for pre-1982 pennies is:

 ____

2. The *slope* of the linear regression line for post-1984 pennies is:

 ____

3. Explain what the *slope* represents. For a linear model, is the slope the same along the entire model? Explain.

 ____

 ____

4. In theory, the equations should go through the origin. In other words, the *y-intercept* should be 0. The *slope* of a linear model is given by the equation shown here. The first point may be the origin, (0, 0). Therefore, the equation for the slope of *this* particular model can also be written as

$$A = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$A = \frac{Y}{X}$ when Y and X are greater than 0.

Observe that Y is the mass and X is the volume, therefore

$\text{Slope} = \frac{\textit{mass}}{\textit{volume}}$.

What is another term for $\frac{\textit{mass}}{\textit{volume}}$?

____

5. Using information from questions 1 and 2, what is the density of the pre-1982 pennies and what is the density of the post-1984 pennies?

Density of pre-1982 pennies ______________

Density of post-1984 pennies ______________

Extensions

- Construct mass versus volume linear models as you did with the pre-1982 and post-1984 pennies, but use different liquids. Compare alcohol and water or compare water and oil.

  Determine the slope (density) for each liquid studied.

- Construct mass versus volume linear models as you did with the pre-1982 and post-1984 pennies, but compare bird bones and mammalian bones. Measure volume by water displacement as you did with the pennies.

  Determine the slope (density) for each bone type studied.

Teacher Notes

Activity 6

You're So Dense

Objectives

- To investigate the relationship between mass and volume
- To find the *x* value of a function, given the *y* value
- To find the *y* value of a function, given the *x* value
- To use technology to find a linear regression
- To use technology to plot a set of ordered pairs

Materials

- TI-73 graphing device
- 50 ml graduated cylinder
- Balance
- Pennies (25 pre-1982 and 25 post-1984)

Preparation

- The density of an object is an *intensive* property, which means that it is independent of the size of the sample. Any part of the object is representative of the whole, hence the linearity of the regression. Mass and volume by themselves are *extensive* properties, which means that they depend on size (that is, the larger an object, the larger its mass or volume).
- The density of an object changes if the temperature of the object changes. Generally, as an object cools, its density increases, but there are some important exceptions. One exception is water. The density of water increases as its temperature decreases to 4°C. As the temperature drops from 4°C to 0°C (the freezing point of water), the density decreases. Therefore, ice floats – a fact that has important biological significance.
- In 1982, the U.S. government changed the way pennies were minted. Up until that time, pennies were minted from a predominantly copper alloy. Now pennies are minted from a zinc alloy and finished with a thin copper coating. As a result, the two types of pennies have distinctly different densities.

  The change in metal composition took place over a couple of years, depending upon where the pennies were minted. To avoid confusion in this activity, pennies minted in the years 1982-1984 are not to be used.
- The **Extensions** section provides other interesting examples of density that could be used in this activity. Density of increasing amounts of liquids could be examined. Water is denser than cooking oil. Hence, cooking oil will form a

separate layer on top of water. Cooking oil is denser than alcohol and thus will form a separate layer below the alcohol.

- Another interesting comparison could be made using cleaned bones from a bird and a mammal. Bird bones are less dense to allow for flight. Use pork or beef bones and compare to similar bones in a chicken or turkey. Scrub the bones in a bleach solution, rinse, and let dry.

Answers to Data Collection and Analysis questions

Collecting the data

Sample data:

Number of pennies	Pre-1982		Post-1984	
	Volume (cm^3)	Mass (g)	Volume (cm^3)	Mass (g)
0	0	0	0	0
5	1.5	15.21	1.7	12.64
10	3.1	30.09	3.6	25.18
15	5.0	45.94	5.6	37.89
20	6.6	61.09	7.4	48.00
25	8.4	75.99	8.7	62.37

Analyzing the data

1. The *slope* of the linear regression line for pre-1982 pennies is:

Slope (pre-1982) = 9.00.

2. The *slope* of the linear regression line for post-1984 pennies is:

Slope (post-1984) = 6.85.

3. Explain what the *slope* represents. For a linear model, is the slope the same along the entire regression? Explain.

The slope is a ratio of the change in mass to the change in volume. The slope remains the same, regardless of the size of the object.

4. In theory, the equations should go through the origin. In other words, the *y*-intercept should be 0. The *slope* of a linear model is given by the equation shown here. The first point may be the origin, (0, 0). Therefore, the equation for the slope of *this* particular model can also be written as

$$A = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$A = \frac{Y}{X}$ when Y and X are greater than 0.

Observe that *Y* is the mass and *X* is the volume, therefore

$$\text{Slope} = \frac{mass}{volume}.$$

What is another term for $\frac{mass}{volume}$?

The slope is equal to the mass divided by the volume; therefore, the slope is the density of the pennies.

5. Using information from questions 1 and 2, what is the density of the pre-1982 pennies and what is the density of the post-1984 pennies?

Density of pre-1982 pennies = 9.00 grams per cubic centimeter.

Density of post-1984 pennies = 6.85 grams per cubic centimeter.