The Remainder Theorem Using TI-Nspire CAS

by – Paul W. Gosse

Activity overview

This activity uses the CAS polyRemainder(, and related commands found in the Calculations-Algebra-Polynomial Tools menu, to explore the relationship between the remainder when f(x) is divided by the linear factor (x-a), and f(a). The approach involves calculating remainders when dividing by (x – a) for captured values of 'a' using CAS and a drag point for 'a', and scatterplotting those values against the values of the function, f(a) in real time. While the symbolism of the relationship expressed in the Reminder Theorem is clear, the visualization of plotting remainders against function values 'live', illustrates the symbolism in a new and concrete way. Problem 1 introduces the basic relationship between divisor, dividend, quotient, and remainder; Problem 2 uses CAS to explore the same relationship with polynomial functions; Problem 3 captures data from a ten-sample polynomial division situation to a spreadsheet and illustrates it as a function in real time; and Problem 4 poses some reflective questions on the various representations in the activity.

Concepts

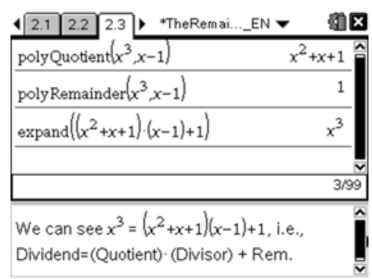
Remainder Theorem, Synthetic Division, Factor Theorem, Zeros of a Function.

Teacher preparation

Teachers may wish to review the basic relationship between divisor, dividend, quotient, and remainder (although this is also done at the beginning of the activity) and long division or synthetic division for divisor (x – a). Teachers will need to be familiar with the syntax of polyQuotient(and polyRemainder(. As seen on page 2.3 below, polyQuotient($x^3, x-1$ *) returns the quotient when* x^3 *is divided by* $x-1$*. Similarly, polyRemainder(* $x^3, x-1$ *) returns the remainder. Finally, the activity screens are set up for handheld view and may need re-positioning in computer view.*

Classroom management tips

Students could be encouraged to confirm the calculations in 2.3 (as shown) through long division or synthetic division. It is important that students identify/locate the divisor, quotient and remainder among calculations on the top of the split screen.



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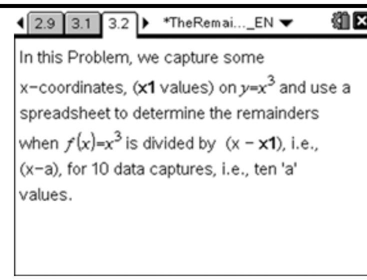
Grade level: secondary
Subject: mathematics
Time required: 45 minutes

Materials: TI-Nspire CAS

Note: the polyRemainder(command does not seem to easily lend itself to working with lists, multiple inputs of any kind, or automatic data captures. Hence, the ten 'a' values referred to on page 3.2, were inserted in b1 and filled downward to b10 in advance on upcoming page 3.6. If you wish more points, simply continue to fill the related formula downward. There may be a better way. If so, let me know ☺



TI-Nspire Applications
Notes including (Q&A), Calculator, G&G, L&S.

Step-by-step directions
The activity is self-reinforcing and self-directed. Limited preparation and explanation should be needed. Answers are offered below.

Assessment and evaluation
- *Various questioning types lend themselves to assessing competence on this content such as multiple choice, short answer, graphical interpretation, etc. It would be encouraging and interesting to direct a question as indicated on pages 4.3 or 4.4. For example, $f(x) \div (x-1)$ is positive. Therefore, which is true about $f(1)$? (A) It is positive (B) It is negative (C) It is zero (D) No conclusion can be drawn.*
- *Answers: [2.8] $f(a)$. [3.7] The remainders when f(x) is divided by (x – a) appear to be the same as the function values f(a). [3.9] The reminder values coincide with the function values. [4.3] The points on the graph where the remainder as indicated is positive are where f(x) is positive. [4.4] The points on the graph where the remainder as indicated is negative are where f(x) is negative. [4.5] These points lie on the x-axis. That is, neither above nor below the x-axis. [4.6] The points on the graph where the remainder as indicated is zero are the x-intercepts (i.e., where f(x) = 0) which, of course, indicates that (x – a) is a factor of f(x).*

Activity extensions
- *The Factor Theorem is alluded to in the last bullet in* **Assessment and evaluation** *and is a natural extension of this activity*

Student TI-Nspire Document
TheRemainderTheoremUsingTI-NspireCAS_EN.tns

TEXAS INSTRUMENTS

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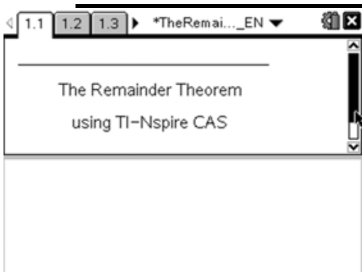

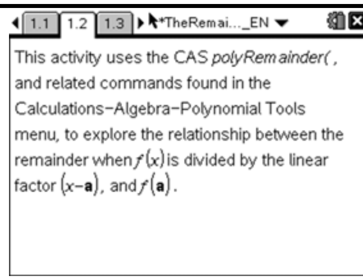

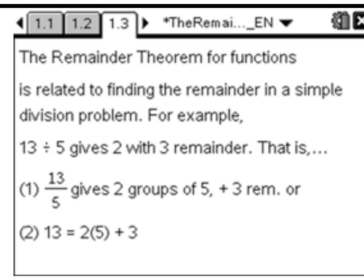

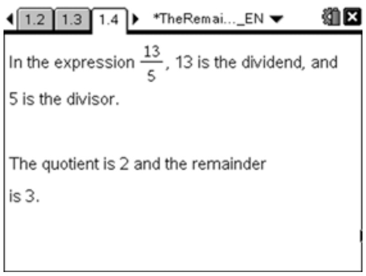

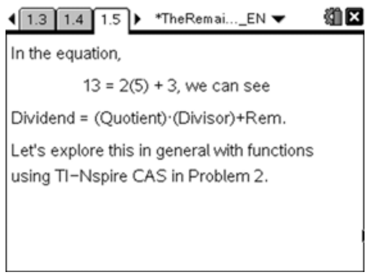

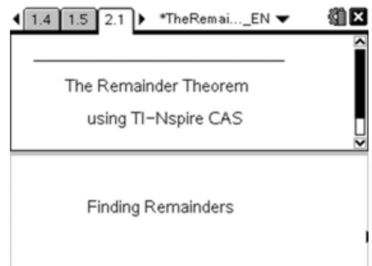

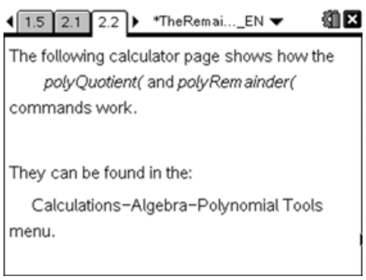

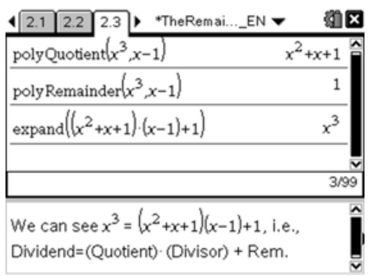

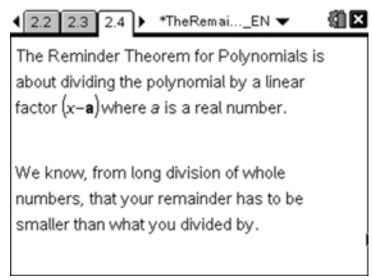

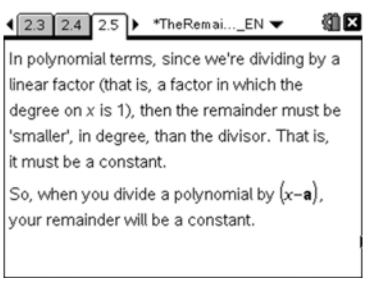

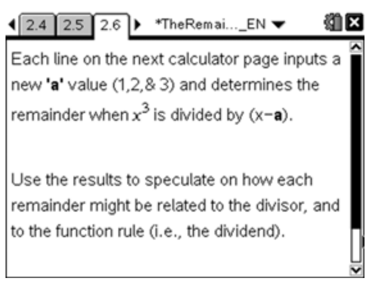

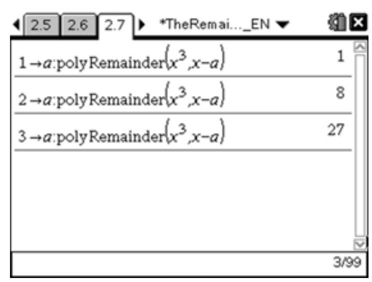

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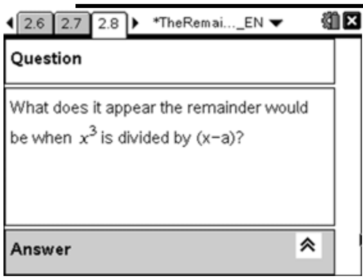
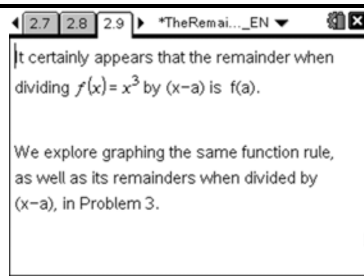
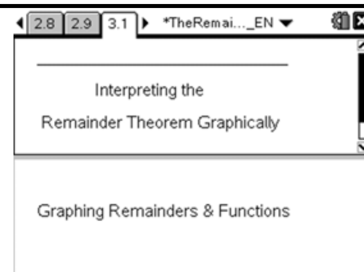
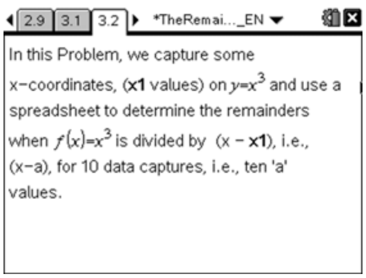
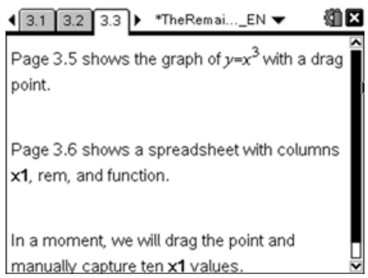
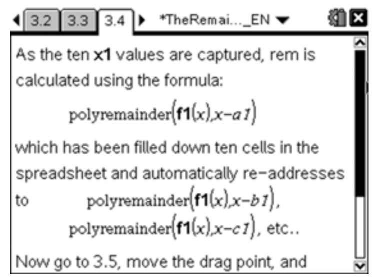


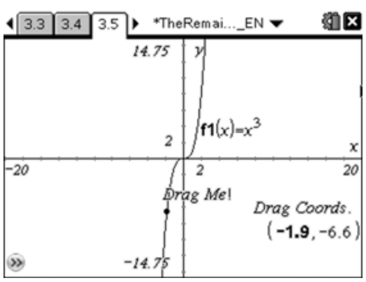


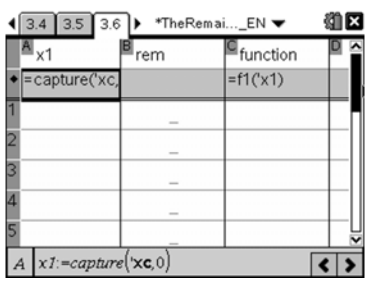


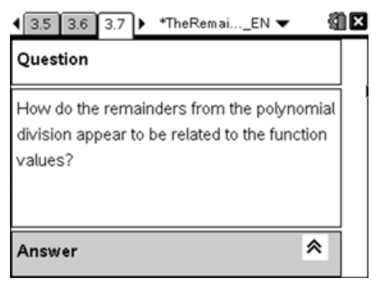


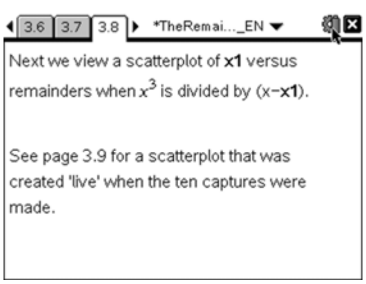


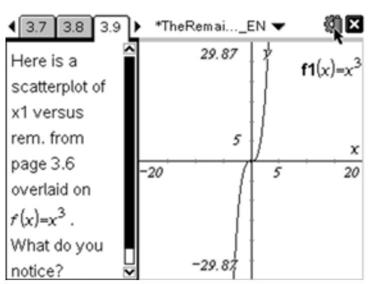


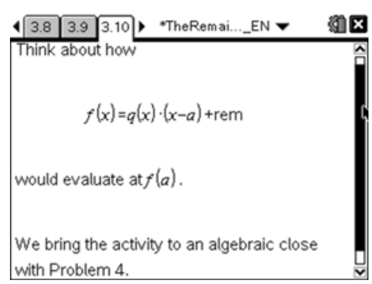


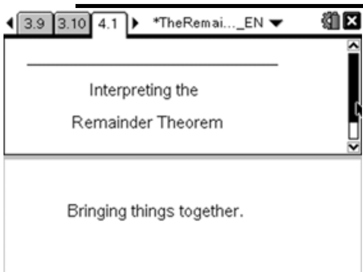


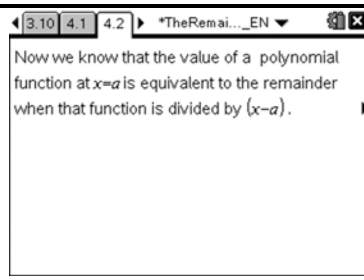


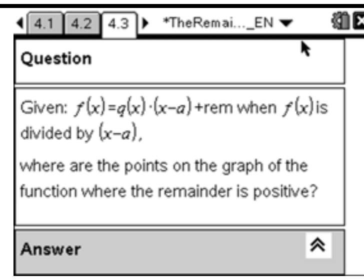


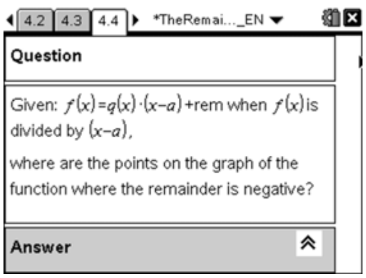


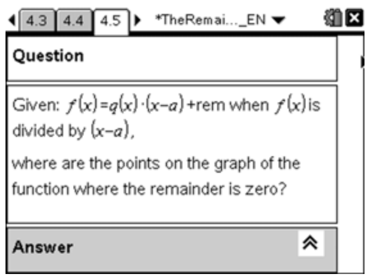


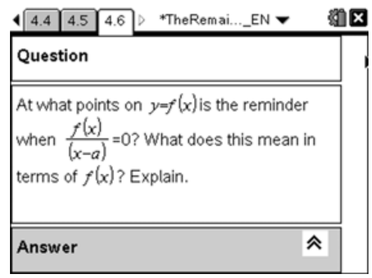

3.9 3.10 4.1 *TheRemai..._EN
Interpreting the
Remainder Theorem
Bringing things together.


3.10 4.1 4.2 *TheRemai..._EN
Now we know that the value of a polynomial function at $x=a$ is equivalent to the remainder when that function is divided by $(x-a)$.


4.1 4.2 4.3 *TheRemai..._EN
Question
Given: $f(x)=q(x)\cdot(x-a)+\text{rem}$ when $f(x)$ is divided by $(x-a)$,
where are the points on the graph of the function where the remainder is positive?
Answer


4.2 4.3 4.4 *TheRemai..._EN
Question
Given: $f(x)=q(x)\cdot(x-a)+\text{rem}$ when $f(x)$ is divided by $(x-a)$,
where are the points on the graph of the function where the remainder is negative?
Answer


4.3 4.4 4.5 *TheRemai..._EN
Question
Given: $f(x)=q(x)\cdot(x-a)+\text{rem}$ when $f(x)$ is divided by $(x-a)$,
where are the points on the graph of the function where the remainder is zero?
Answer


4.4 4.5 4.6 *TheRemai..._EN
Question
At what points on $y=f(x)$ is the reminder when $\frac{f(x)}{(x-a)}=0$? What does this mean in terms of $f(x)$? Explain.
Answer