

The Remainder Theorem Using TI-Nspire CAS

by – Paul W. Gosse

Activity overview

This activity uses the CAS `polyRemainder(`, and related commands found in the Calculations-Algebra-Polynomial Tools menu, to explore the relationship between the remainder when $f(x)$ is divided by the linear factor $(x-a)$, and $f(a)$. The approach involves calculating remainders when dividing by $(x - a)$ for captured values of 'a' using CAS and a drag point for 'a', and scatterplotting those values against the values of the function, $f(a)$ in real time. While the symbolism of the relationship expressed in the Remainder Theorem is clear, the visualization of plotting remainders against function values 'live', illustrates the symbolism in a new and concrete way. Problem 1 introduces the basic relationship between divisor, dividend, quotient, and remainder; Problem 2 uses CAS to explore the same relationship with polynomial functions; Problem 3 captures data from a ten-sample polynomial division situation to a spreadsheet and illustrates it as a function in real time; and Problem 4 poses some reflective questions on the various representations in the activity.

Concepts

Remainder Theorem, Synthetic Division, Factor Theorem, Zeros of a Function.

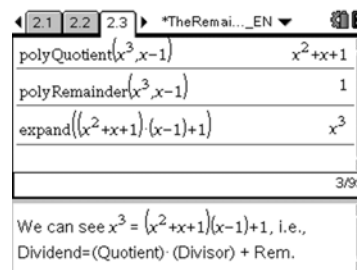
Teacher preparation

Teachers may wish to review the basic relationship between divisor, dividend, quotient, and remainder (although this is also done at the beginning of the activity) and long division or synthetic division for divisor $(x - a)$. Teachers will need to be familiar with the syntax of `polyQuotient(` and `polyRemainder(`. As seen on page 2.3 below, `polyQuotient( $x^3, x-1$ )` returns the quotient when x^3 is divided by $x-1$.

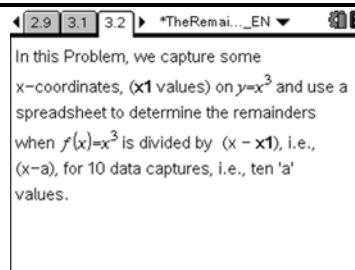
Similarly, `polyRemainder( $x^3, x-1$ )` returns the remainder. Finally, the activity screens are set up for handheld view and may need re-positioning in computer view.

Classroom management tips

Students could be encouraged to confirm the calculations in 2.3 (as shown) through long division or synthetic division. It is important that students identify/locate the divisor, quotient and remainder among calculations on the top of the split screen.



Note: the polyRemainder(command does not seem to easily lend itself to working with lists, multiple inputs of any kind, or automatic data captures. Hence, the ten 'a' values referred to on page 3.2, were inserted in b1 and filled downward to b10 in advance on upcoming page 3.6. If you wish more points, simply continue to fill the related formula downward. There may be a better way. If so, let me know ☺



TI-Nspire Applications

Notes including (Q&A), Calculator, G&G, L&S.

Step-by-step directions

The activity is self-reinforcing and self-directed. Limited preparation and explanation should be needed. Answers are offered below.

Assessment and evaluation

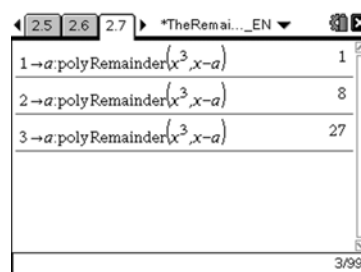
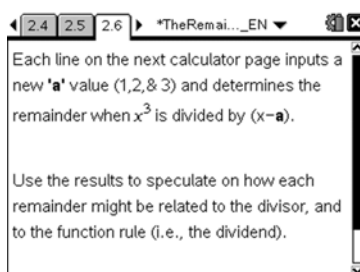
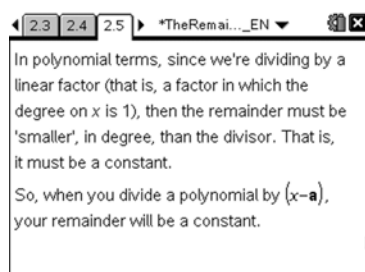
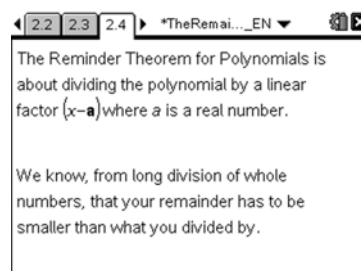
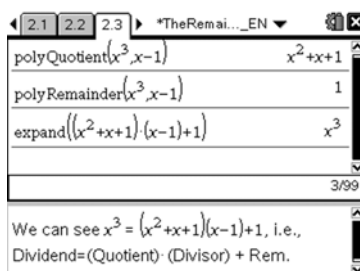
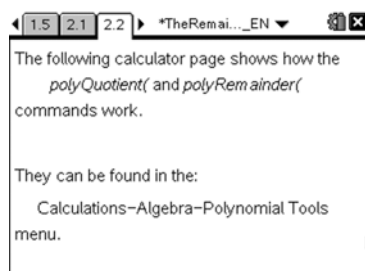
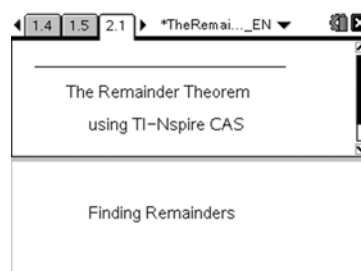
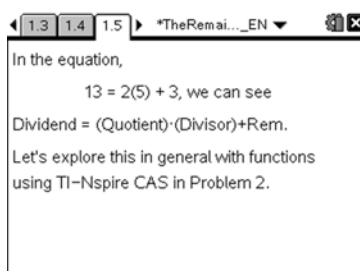
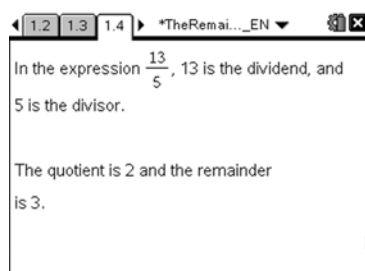
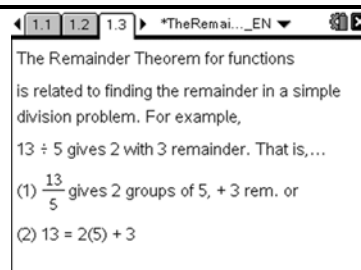
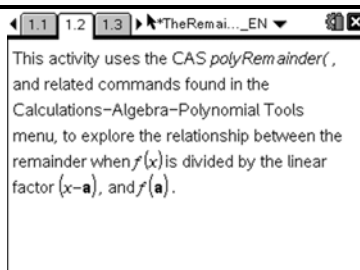
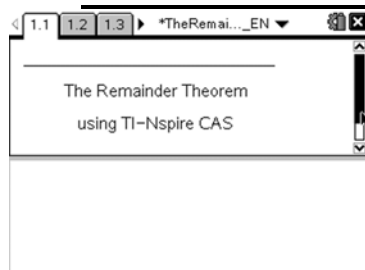
- Various questioning types lend themselves to assessing competence on this content such as multiple choice, short answer, graphical interpretation, etc. It would be encouraging and interesting to direct a question as indicated on pages 4.3 or 4.4. For example, $f(x) \div (x-1)$ is positive. Therefore, which is true about $f(1)$? (A) It is positive (B) It is negative (C) It is zero (D) No conclusion can be drawn.
- Answers: [2.8] $f(a)$. [3.7] The remainders when $f(x)$ is divided by $(x-a)$ appear to be the same as the function values $f(a)$. [3.9] The reminder values coincide with the function values. [4.3] The points on the graph where the remainder as indicated is positive are where $f(x)$ is positive. [4.4] The points on the graph where the remainder as indicated is negative are where $f(x)$ is negative. [4.5] These points lie on the x-axis. That is, neither above nor below the x-axis. [4.6] The points on the graph where the remainder as indicated is zero are the x-intercepts (i.e., where $f(x) = 0$) which, of course, indicates that $(x-a)$ is a factor of $f(x)$.

Activity extensions

- The Factor Theorem is alluded to in the last bullet in **Assessment and evaluation** and is a natural extension of this activity

Student TI-Nspire Document

TheRemainderTheoremUsingTI-NspireCAS_EN.tns



2.6 2.7 2.8 *TheRemai..._EN

Question

What does it appear the remainder would be when x^3 is divided by $(x-a)$?

Answer

2.7 2.8 2.9 *TheRemai..._EN

It certainly appears that the remainder when dividing $f(x) = x^3$ by $(x-a)$ is $f(a)$.

We explore graphing the same function rule, as well as its remainders when divided by $(x-a)$, in Problem 3.

2.8 2.9 3.1 *TheRemai..._EN

Interpreting the Remainder Theorem Graphically

Graphing Remainders & Functions

2.9 3.1 3.2 *TheRemai..._EN

In this Problem, we capture some x -coordinates, ($x1$ values) on $y=x^3$ and use a spreadsheet to determine the remainders when $f(x)=x^3$ is divided by $(x-x1)$, i.e., $(x-a)$, for 10 data captures, i.e., ten 'a' values.

3.1 3.2 3.3 *TheRemai..._EN

Page 3.5 shows the graph of $y=x^3$ with a drag point.

Page 3.6 shows a spreadsheet with columns $x1$, rem, and function.

In a moment, we will drag the point and manually capture ten $x1$ values.

3.2 3.3 3.4 *TheRemai..._EN

As the ten $x1$ values are captured, rem is calculated using the formula:

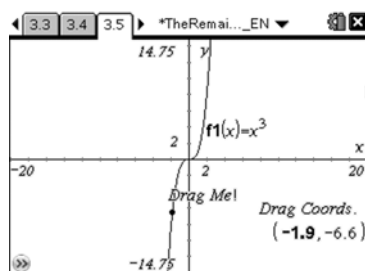
$$\text{polyremainder}(f1(x), x-a1)$$

which has been filled down ten cells in the spreadsheet and automatically re-addresses to

$$\text{polyremainder}(f1(x), x-b1),$$

$$\text{polyremainder}(f1(x), x-c1), \text{ etc..}$$

Now go to 3.5, move the drag point, and



3.4 3.5 3.6 *TheRemai..._EN

A	B	C
x1	rem	function
=capture('xc,		=f1('x1)
1	-	
2	-	
3	-	
4	-	
5	-	

A x1:=capture('xc,0)

3.5 3.6 3.7 *TheRemai..._EN

Question

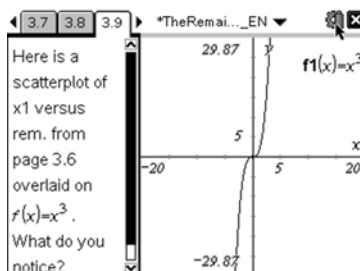
How do the remainders from the polynomial division appear to be related to the function values?

Answer

3.6 3.7 3.8 *TheRemai..._EN

Next we view a scatterplot of $x1$ versus remainders when x^3 is divided by $(x-x1)$.

See page 3.9 for a scatterplot that was created 'live' when the ten captures were made.



3.8 3.9 3.10 *TheRemai..._EN

Think about how

$$f(x) = q(x) \cdot (x-a) + \text{rem}$$

would evaluate at $f(a)$.

We bring the activity to an algebraic close with Problem 4.

3.9 3.10 4.1 *TheRemai..._EN

Interpreting the
Remainder Theorem

Bringing things together.

3.10 4.1 4.2 *TheRemai..._EN

Now we know that the value of a polynomial function at $x=a$ is equivalent to the remainder when that function is divided by $(x-a)$.

4.1 4.2 4.3 *TheRemai..._EN

Question

Given: $f(x) = q(x) \cdot (x-a) + \text{rem}$ when $f(x)$ is divided by $(x-a)$,
where are the points on the graph of the function where the remainder is positive?

Answer

4.2 4.3 4.4 *TheRemai..._EN

Question

Given: $f(x) = q(x) \cdot (x-a) + \text{rem}$ when $f(x)$ is divided by $(x-a)$,
where are the points on the graph of the function where the remainder is negative?

Answer

4.3 4.4 4.5 *TheRemai..._EN

Question

Given: $f(x) = q(x) \cdot (x-a) + \text{rem}$ when $f(x)$ is divided by $(x-a)$,
where are the points on the graph of the function where the remainder is zero?

Answer

4.4 4.5 4.6 *TheRemai..._EN

Question

At what points on $y=f(x)$ is the remainder when $\frac{f(x)}{(x-a)} = 0$? What does this mean in terms of $f(x)$? Explain.

Answer