

by – Paul W. Gosse

Activity overview

This activity uses the CAS polyRemainder(, and related commands found in the Calculations-Algebra-Polynomial Tools menu, to explore the relationship between the remainder when f(x) is divided by the linear factor (x-a), and f(a). The approach involves calculating remainders when dividing by (x - a) for captured values of 'a' using CAS and a drag point for 'a', and scatterplotting those values against the values of the function, f(a) in real time. While the symbolism of the relationship expressed in the Reminder Theorem is clear, the visualization of plotting remainders against function values 'live', illustrates the symbolism in a new and concrete way. Problem 1 introduces the basic relationship between divisor, dividend, quotient, and remainder; Problem 2 uses CAS to explore the same relationship with polynomial functions; Problem 3 captures data from a ten-sample polynomial division situation to a spreadsheet and illustrates it as a function in real time; and Problem 4 poses some reflective questions on the various representations in the activity.

Concepts

Remainder Theorem, Synthetic Division, Factor Theorem, Zeros of a Function.

Teacher preparation

Teachers may wish to review the basic relationship between divisor, dividend, quotient, and remainder (although this is also done at the beginning of the activity) and long division or synthetic division for divisor (x - a). Teachers will need to be familiar with the syntax of polyQuotient(and polyRemainder(. As seen on page 2.3 below, polyQuotient(x^3 , x - 1) returns the quotient when x^3 is divided by x - 1.

Similarly, polyRemainder(x^3 , x-1) returns the remainder. Finally, the activity screens are set up for handheld view and may need re-positioning in computer view.

Classroom management tips

Students could be encouraged to confirm the calculations in 2.3 (as shown) through long division or synthetic division. It is important that students identify/locate the divisor, quotient and remainder among calculations on the top of the split screen.

4 2.1 2.2 2.3 ▶ *TheRemaiEN ▼	· (1) D
$polyQuotient(x^3, x-1)$	x ² +x+1
polyRemainder $(x^{3}, x-1)$	1
$expand((x^2+x+1)\cdot(x-1)+1)$	x ³
	3/99
We can see $x^3 = (x^2+x+1)(x-1)+1$, i. Dividend=(Quotient)·(Divisor) + Rem	e.,



Materials: TI-Nspire CAS

Note: the polyRemainder(command does not seem to easily lend itself to working with lists, multiple inputs of any kind, or automatic data captures. Hence, the ten 'a' values referred to on page 3.2, were inserted in b1 and filled downward to b10 in advance on upcoming page 3.6. If you wish more points, simply continue to fill the related formula downward. There may be a better way. If so, let me know © ↓ 2.9 3.1 3.2 → "TheRemai..._EN ▼ In this Problem, we capture some x-coordinates, (x1 values) on $y=x^3$ and use a spreadsheet to determine the remainders when $f'(x)=x^3$ is divided by (x - x1), i.e., (x-a), for 10 data captures, i.e., ten 'a'

values.

TI-Nspire Applications

Notes including (Q&A), Calculator, G&G, L&S.

EXAS

NSTRUMENTS

Step-by-step directions

The activity is self-reinforcing and self-directed. Limited preparation and explanation should be needed. Answers are offered below.

Assessment and evaluation

- Various questioning types lend themselves to assessing competence on this content such as multiple choice, short answer, graphical interpretation, etc. It would be encouraging and interesting to direct a question as indicated on pages 4.3 or 4.4. For example, f(x) ÷ (x 1) is positive. Therefore, which is true about f(1) ? (A) It is positive (B) It is negative (C) It is zero (D) No conclusion can be drawn.
- Answers: [2.8] f(a). [3.7] The remainders when f(x) is divided by (x a) appear to be the same as the function values f(a). [3.9] The reminder values coincide with the function values. [4.3] The points on the graph where the remainder as indicated is positive are where f(x) is positive. [4.4] The points on the graph where the remainder as indicated is negative are where f(x) is negative. [4.5] These points lie on the x-axis. That is, neither above nor below the x-axis. [4.6] The points on the graph where the remainder as indicated is recepts (i.e., where f(x) = 0) which, of course, indicates that (x a) is a factor of f(x).

Activity extensions

• The Factor Theorem is alluded to in the last bullet in **Assessment and evaluation** and is a natural extension of this activity

Student TI-Nspire Document

TheRemainderTheoremUsingTI-NspireCAS_EN.tns

TEXAS INSTRUMENTS

The Remainder Theorem Using TI-Nspire CAS

by: Paul W. Gosse Grade level: secondary Subject: mathematics Time required: 45 minutes

Materials: TI-Nspire CAS

 1.1 1.2 1.3 *TheRemaiEN ▼ The Remainder Theorem using TI-Nspire CAS 	1.1 1.2 1.3 ▶ *TheRemaiEN ▼ () () () () () () () ()	 I.1 I.2 I.3 ▶ *TheRemaiEN ▼ I ≥ The Remainder Theorem for functions is related to finding the remainder in a simple division problem. For example, 13 ÷ 5 gives 2 with 3 remainder. That is, (1) 13/5 gives 2 groups of 5, + 3 rem. or (2) 13 = 2(5) + 3
 1.2 1.3 1.4 ▶ *TheRemaiEN ▼ ⁽¹⁾/₅ № In the expression ¹³/₅, 13 is the dividend, and 5 is the divisor. The quotient is 2 and the remainder is 3. 	 I.3 1.4 1.5 ► *TheRemaiEN ▼ IN ► In the equation, 13 = 2(5) + 3, we can see Dividend = (Quotient) · (Divisor) + Rem. Let's explore this in general with functions using TI-Nspire CAS in Problem 2. 	I.4 1.5 2.1 ▶ *TheRemaiEN ▼ ⁽¹⁾ X The Remainder Theorem using TI-Nspire CAS Finding Remainders
I.5 2.1 2.2 ► *TheRemaiEN ▼ I ≥ The following calculator page shows how the polyQuotient(and polyRemainder(commands work.	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	4 2.2 2.3 2.4 ► *TheRemaiEN (a) ► The Reminder Theorem for Polynomials is about dividing the polynomial by a linear factor $(x-a)$ where <i>a</i> is a real number.

They can be found in the:

Calculations-Algebra-Polynomial Tools menu.

4 2.3 2.4 2.5 ▶ *TheRemai..._EN ▼ (1)

In polynomial terms, since we're dividing by a linear factor (that is, a factor in which the degree on x is 1), then the remainder must be 'smaller', in degree, than the divisor. That is, it must be a constant.

So, when you divide a polynomial by $(x-\mathbf{a})$, your remainder will be a constant.

3/99 We can see $x^3 = (x^2 + x + 1)(x - 1) + 1$, i.e., Dividend=(Quotient) (Divisor) + Rem.

-

4 2.4 2.5 2.6 ▶ *TheRemaiEN ▼ 4 4 1
Each line on the next calculator page inputs a new 'a' value (1,2,& 3) and determines the
remainder when x^3 is divided by $(x-a)$.

Use the results to speculate on how each remainder might be related to the divisor, and to the function rule (i.e., the dividend).

4 2.5 2.6 2.7 ▶ *TheRemaiEN ▼	81 ×
$1 \rightarrow a: \text{polyRemainder}(x^3, x-a)$	1
$2 \rightarrow a$:polyRemainder $(x^3, x-a)$	8
$3 \rightarrow a$:polyRemainder $(x^3, x-a)$	27
	3/99

We know, from long division of whole

smaller than what you divided by.

numbers, that your remainder has to be

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by: Paul W. Gosse Grade level: secondary Subject: mathematics Time required: 45 minutes

Materials: TI-Nspire CAS

4 2.8 2.9 3.1 ▶ *TheRemai..._EN ▼

Interpreting the Remainder Theorem Graphically

Graphing Remainders & Functions

4 2.6 2.7 2.8 ▶ *TheRemaiEN ▼	X
Question	
What does it appear the remainder would be when x^3 is divided by (x-a)?	
Answer 😤	1

4 2.9 3.1 3.2 ▶ *TheRemai..._EN ▼

In this Problem, we capture some x-coordinates, (**x1** values) on $y=x^3$ and use a spreadsheet to determine the remainders when $f'(x)=x^3$ is divided by (x - **x1**), i.e., (x-a), for 10 data captures, i.e., ten 'a' values.



	۹×
Next we view a scatterplot of x1 versus remainders when x^3 is divided by (x- x1).	
See page 3.9 for a scatterplot that was created 'live' when the ten captures were made.	

12.7 2.8 2.9 ► *TheRemai..._EN ▼ It certainly appears that the remainder when dividing $f(x) = x^3$ by (x-a) is f(a).

We explore graphing the same function rule, as well as its remainders when divided by (x-a), in Problem 3.

Page 3.5 shows the graph of $y=x^3$ with a drag point.

Page 3.6 shows a spreadsheet with columns **x1**, rem, and function.

In a moment, we will drag the point and manually capture ten **x1** values.

4 3.4 3.5 3.€	> • *TheRem	iaiEN ▼	(1)
A x1	^B rem	function	
 =capture('xc, 		=f1('x1)	
1	_		
2	_		
3	_		
4	_		
5			T,
A x1:=captur	e(' xc ,0)		< >

4 3.7 3.8 3.9 ▶	*TheRemaiEN 🔻	<u>ଣ୍</u> 🗵
Here is a	29.87	$f(x)=x^3$
scatterplot of	11	
x1 versus		
rem. from	5	~
page 3.6	-20 / 5	20
overlaid on		
$f(x)=x^{3}$.		
What do you	[]	
notice?	-29.87	



◀ 3.5 3.6 3.7 ► *TheRemaiEN ▼ 🛛	Î×
Question	
How do the remainders from the polynomial division appear to be related to the function values?	
Answer 😤	ĺ

	A 10
Think about how	
$f(x) = q(x) \cdot (x-a) + rem$	¢
would evoluate at f(a)	
would evaluate at (a).	
We bring the activity to an algebraic close	
with Problem 4.	×



by: Paul W. Gosse Grade level: secondary Subject: mathematics Time required: 45 minutes

Materials: TI-Nspire CAS

(3.9 3.10 4.1) *TheRemaiEN ▼ 4 2 2	Image: State S	(4.1 4.2 4.3) *TheRemaiEN ▼ 4 Question
Interpreting the	function at $x=a$ is equivalent to the remainder	(1)
Remainder Theorem	when that function is divided by $(x-a)$.	Given: $f(x)=q(x)\cdot(x-a)$ +rem when $f(x)$ is divided by $(x-a)$,
Ken.		where are the points on the graph of the
Bringing things together.		function where the remainder is positive?
		Answer 😤
4.2 4.3 4.4 ▶ *TheRemaiEN ▼ 🚳 🛛	 4.3 4.4 4.5 ▶ *TheRemaiEN ▼ 	4 4.4 4.5 4.6 ▷ *TheRemaiEN ▼ ()
Question	Question	Question
Given: $f(x)=q(x)\cdot(x-a)$ + rem when $f(x)$ is	Given: $f(x) = q(x) \cdot (x-a)$ + rem when $f(x)$ is	At what points on $y=f(x)$ is the reminder
divided by $(x-a)$,	divided by $(x-a)$,	when $\frac{f(x)}{(x)} = 0$? What does this mean in
where are the points on the graph of the	where are the points on the graph of the	(x-a)

function where the remainder is negative?

*

Answer

function where the remainder is zero?

*

Answer

terms of f(x)? Explain.

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Answer