EXPLORATIONS™

Objectives

- Graph systems of linear inequalities
- Investigate the concepts of constraints and feasible polygons

Activity 3

Winning Inequalities (Part 2)

Introduction

The graph of a system of linear inequalities can create a region defined by a polygon, such as a triangle or rectangle. In this activity, you will use skills learned in Activities **1** and **2** to create a polygon that models the solution of the basketball problem.

In Activity **1**, you wrote equations and inequalities that represented various game situations involving the Shooting Stars basketball team. For example, the equation $2x + 3y = 60$ indicated that the Shooting Stars scored *exactly* 60 points from field goals when x represented the number of 2-point field goals, and y represented the number of 3-point field goals. The inequality $2x + 3y \le 60$ indicated that the Shooting Stars scored *no more than* 60 points from field goals.

Many ordered pairs satisfy $2x + 3y \le 60$. However, some of these ordered pairs do not make sense in the context of the problem. For example, it is not possible to score $^-20$ 2-point field goals and 30 3-point field goals even though the ordered pair ($^-20$, 30) satisfies the inequality $2x + 3y \le 60$. In addition, it would not be possible to score only one 2-point field goal and $\frac{58}{3}$ 3-point field goals even though the ordered pair $(1, \frac{58}{3})$ satisfies the inequality.

Problem

What linear inequalities could you use to guarantee that only integers equal to or greater than zero, but less than or equal to 60 are included in the solution of the Shooting Stars basketball problem?

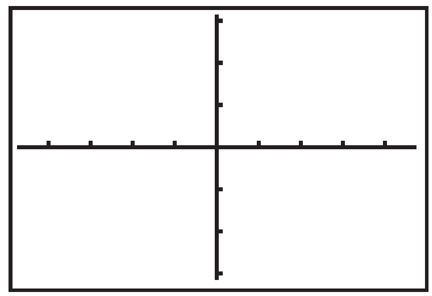
Exploration

1. Because it is impossible to make a negative number of field goals, you need to write inequalities that exclude negative values. Write two inequalities in terms of *x* and *y* that produce non-negative values.

   ________________________ ________________________

   These two inequalities, combined with the inequality $2x + 3y \leq 60$, create a system of linear inequalities that models the Shooting Stars game situation. Let's look first at the solution set of these three inequalities.

2. Before graphing the system of three inequalities using your graphing handheld, think about what the graph will look like. Discuss your ideas with a classmate. Then, sketch your prediction of the graph on the grid provided.

   Write a mathematical sentence that describes your graph.

To check the accuracy of your graph, use the Inequality Graphing application to graph this system of inequalities. (Check that the Inequality Graphing App is running.)

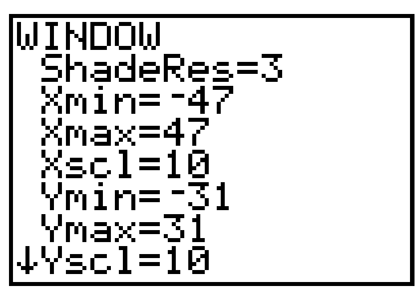
3. **a.** Enter the slope-intercept form of two of your inequalities in **Y1=** and **Y2=**. (Remember to clear any equations from the **Y=** editor first.)

   Hint: *Press* [ALPHA] [F5] *to select* $\geq$ *and* [ALPHA] [F3] *to select* $\leq$.

   b. Enter the third inequality in the **X=** editor. Press [▲] to highlight **X=** and press [ENTER] to access the **X=** editor.

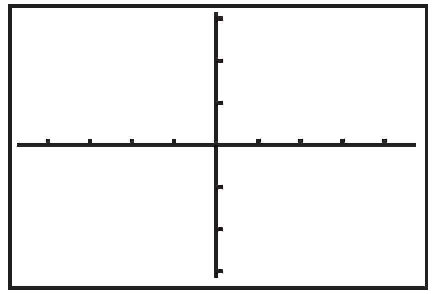
4. The **ZInteger** viewing window is used to view the graph of this system because it eliminates fractions when you trace. Press [ZOOM] **8** to select **8:ZInteger**. Check that *x* = 0 and *y* = 0 so that your new window is centered at the origin, and then press [ENTER].

5. Check the window settings for your graph. Press [WINDOW] and input the settings shown.



Draw the graph you see in the viewing window on the grid provided.

The graph of the three inequalities is difficult to read because of the different shaded regions.



6. Use **Shades** to see only the common region that shows the solution set of this system. Press [ALPHA] [F1], and then select **Ineq Intersection**.

7. Draw the resulting graph showing only the solution set on the grid provided.

   How would you describe the region that represents the solution set of this system?

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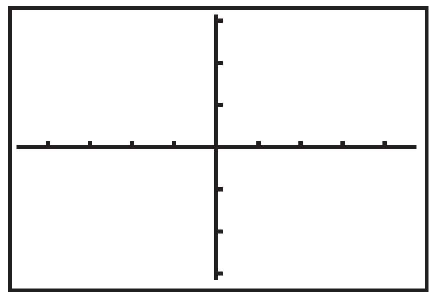


8. Locate the point whose coordinates are (3, 6). Is this point part of the solution set for this system of inequalities? Check algebraically to determine if the point is part of the solution set. Complete the table below as you check.

	Ordered Pair	$x \geq 0$	**True/ False**	$y \geq 0$	**True/ False**	$2x + 3y \leq 60$	**True/ False**
1	(3, 6)	$3 \geq 0$	True				
2							
3							
4							

   Choose two other points that lie in the solution set of this system and verify algebraically that they are part of the solution set. Record your results in rows 2 and 3 in the table above.

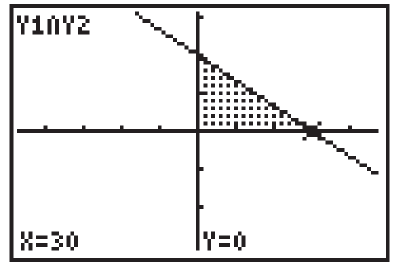
9. Now, choose a point that lies *outside* the solution set and verify algebraically that it is *not* part of the solution set.

   Based on the findings in the table, complete the following statement.

   A point lies in the solution set of a system of inequalities if and only if

   ____.

10. The Point-of-Intersection Trace (**PoI-Trace**) feature allows you to locate the coordinates of the three vertices of the triangle that forms the solution set. Press [ALPHA] [F3] to use **PoI-Trace**. The trace coordinates of $x = 30$ and $y = 0$ appear on the screen, as shown. What two *equations* share this point of intersection?



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Press [▶] to locate the second vertex. What two *equations* share this point of intersection?

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Press [▼] [▶] to locate the third vertex. ([▼] switches the trace from points of intersection involving **Y1** to points of intersection involving **Y2**.) What two *equations* share this point of intersection?

____________________ ____________________

11. The solution set of this system includes all points that lie on the triangle whose vertices are ______________, ______________, and ______________, as well as all points ____.

12. What is the difference between the set of numbers that make up the solution set of the basketball problem and the set of numbers that make up the solution graphed in the Inequality Graphing App?

____

Student Worksheet

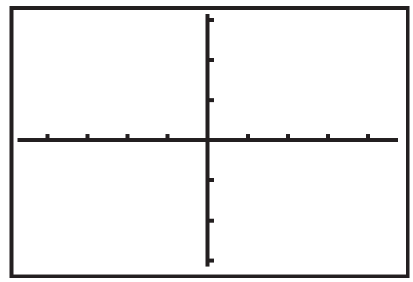
Name ______________________

Date ______________________

1. Use **Shades**, the Shading feature of the Inequality Graphing application, to graph this system of inequalities:
$x \geq {}^{-}15$; $y \geq 0$; $x + 2y \leq 40$; $x \leq 20$

Sketch your graph in the grid provided.

Use **PoI-Trace**, the Point-of-Intersection Trace feature of the Inequality Graphing App, to find the vertices of the polygon that defines the solution set.

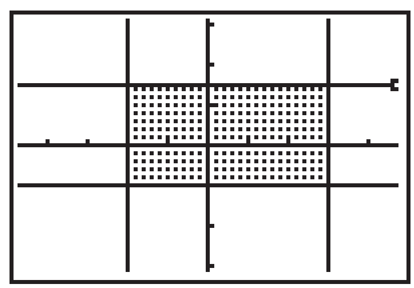


Vertices: __________, __________, __________, __________

For Questions **2** and **3**, determine the systems of linear inequalities that have been used to create each shaded polygonal region. In each case, the vertices are given to assist you.

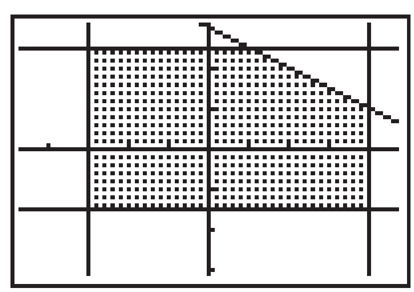
2. **Vertices:** (⁻20, ⁻10), (30, ⁻10),

(⁻20, 15), (30, 15)



3. **Vertices:** (⁻30, ⁻15), (⁻30, 25),

(40, 10), (40, ⁻15), (10, 25)



Teacher Notes

Activity 3

Winning Inequalities (Part 2)

Objectives

- Graph systems of linear inequalities
- Investigate the concepts of constraints and feasible polygons

Materials

- TI-84 Plus/TI-83 Plus
- Inequality Graphing application

Teaching Time

- 60 minutes

Prerequisite Skills

- Solving for one variable in terms of another
- Graphing systems of linear inequalities in the coordinate plane

Management

Students can work individually or in pairs for this activity.

Notes about Exploration

Question **1** provides an opportunity to review all real number sets. Students often confuse natural, whole, and integer sets.

Question **2** asks students to predict the appearance of the system's graph before using their graphing handheld. Time permitting, have students share their sketched graphs and explain their reasoning for the sketch.

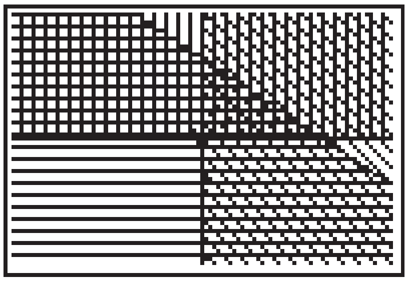
Remind students to solve for *y* before they can enter the inequality $2x + 3y \leq 60$ in the **Y=** editor.

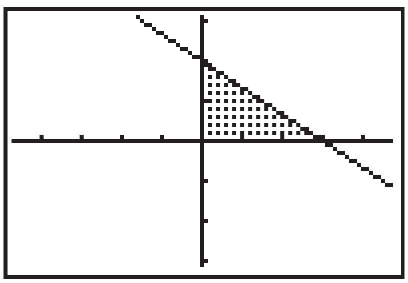
A teacher-led demonstration is recommended for Question **10** because it introduces the Point-of-Intersection Trace feature of the Inequality Graphing App. This feature is used to determine the vertices of a feasible region.

Answers to the Exploration Questions

1. $x \geq 0$; $y \geq 0$

2. Students' sketches and mathematical sentences will vary. They are predictions only. Do not correct. Students should notice, however, that for all real values of *x* and *y*, only first quadrant values, along with points on the positive *x*- and *y*-axes and on the boundary line, are possible solutions.

4. 

6. 

7. The region that represents the solution set is triangular. The graph of the solution of the real-valued system is the set of points that make up the triangle whose vertices are (30, 0), (0, 20) and (0, 0), along with the set of points that lie in the interior of the triangle.

8. Answers will vary. Sample correct responses:

	Ordered Pair	$x \geq 0$	**True/ False**	$y \geq 0$	**True/ False**	$2x + 3y \leq 60$	**True/ False**
1	(3, 6)	$3 \geq 0$	True	$6 \geq 0$	True	$6 + 18 \leq 60$	True
2	Answers will vary.		True		True		True
3	Answers will vary.		True		True		True
4	Answers will vary.	At least one of the inequalities should yield a false statement.					

9. A point lies in the solution set of a system of inequatities if and only if its x- *and* y-*coordinates satisfy each inequality in the system*.

10. $y = 0$; $2x + 3y = 60$

$x = 0$; $2x + 3y = 60$

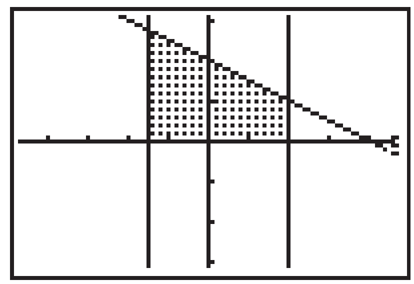
$x = 0$; $y = 0$

11. (30, 0), (0, 20), (0, 0), as well as all points in the interior region of the triangle

12. The solution set to the basketball problem is a subset of the solution set of the three inequalities graphed in the Graphing App. The subset of solutions to the basketball problem include only positive integers or 0.

Answers to Student Worksheet

1. Vertices: ($^-$15, 27.5), (20, 10), (20, 0), ($^-$15, 0)



2. $y \geq {}^-10$; $y \leq 15$; $x \geq {}^-20$; $x \leq 30$

3. $y \geq {}^-15$, $y \leq 25$, $y \leq {}^-0.5x + 30$, $x \geq {}^-30$, $x \leq 40$