Parametric Equations

by:

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Typically we express most graphs as a single equation which involves two variables, x and y. By using parametric mode on the calculator you may use three variables to represent a curve. Commonly, that third variable is t, time. This enables you to locate a point on a curve and know when it was there. To begin, go to mode and set your calculator to Par, for parametric. We will be working in radians.
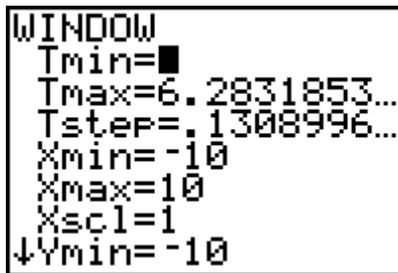
```
           Sci Eng
Float  0123456789
Radian Degree
Func  Par  Pol Seq
Connected Dot
Sequential Simul
Real  a+bi  re^θi
Full  Horiz G-T
```

Investigation

Now enter the graphs as shown. You will notice that your Y= screen is different.

```
 Plot1  Plot2  Plot3
\X₁ᴛ■T+2
 Y₁ᴛ■3T+1■
\X₂ᴛ=
 Y₂ᴛ=
\X₃ᴛ=
 Y₃ᴛ=
\X₄ᴛ=
```
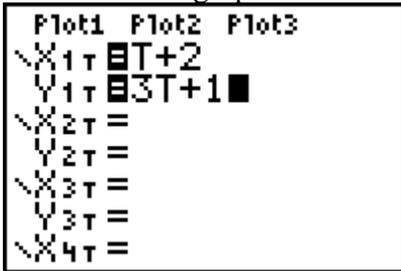
Zoom Standard to see a picture.

```
WINDOW
 Tmin=■
 Tmax=6.2831853…
 Tstep=.1308996…
 Xmin=-10
 Xmax=10
 Xscl=1
↓Ymin=-10
```

Next investigate your window.

You have an initial T value of 0 and a maximum T value of 2π. The steps determine the increments of values. Experiment with these to see how it changes the picture or the speed with which the picture is drawn.

Solve the x equation for T and then sub into the y equation to get an equation in x and y. What is the slope and y intercept of this equation?

```
Plot1  Plot2  Plot3
\X₁ᴛ◻T+2
 Y₁ᴛ◻3T+1
\X₂ᴛ◻2T+2
 Y₂ᴛ◻3T+1■
\X₃ᴛ=
 Y₃ᴛ=
\X₄ᴛ=
```

Now return to Y= and enter [   ] and graph.
Compare the two graphs. Find an equation in x and y for the second set of parametric equations. What is the slope and y intercept?

Experiment with other values of coefficients and of constants in the parametric equations until you can see what effect these have on the slope and y intercept of the equation in x and y.

Activity

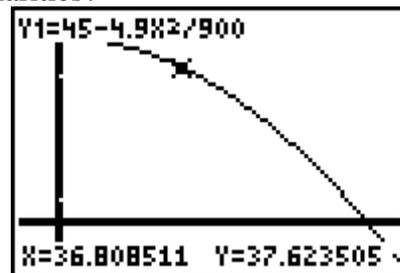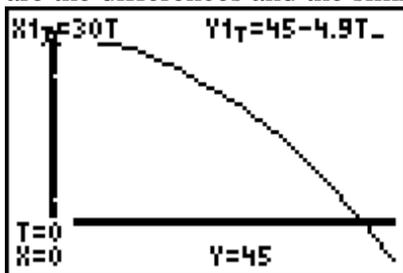To investigate the advantage of the time parameter in parametric mode, enter the following problem.

```
Plot1  Plot2  Plot3
\X₁ᴛ◻30T
 Y₁ᴛ◻45-4.9T²
\X₂ᴛ=■
 Y₂ᴛ=
\X₃ᴛ=
 Y₃ᴛ=
\X₄ᴛ=
```

Adjust your window appropriately.
This problem represents the path of a ball thrown from a tower. How high was the tower? How far from the tower will the ball land? When will it land?
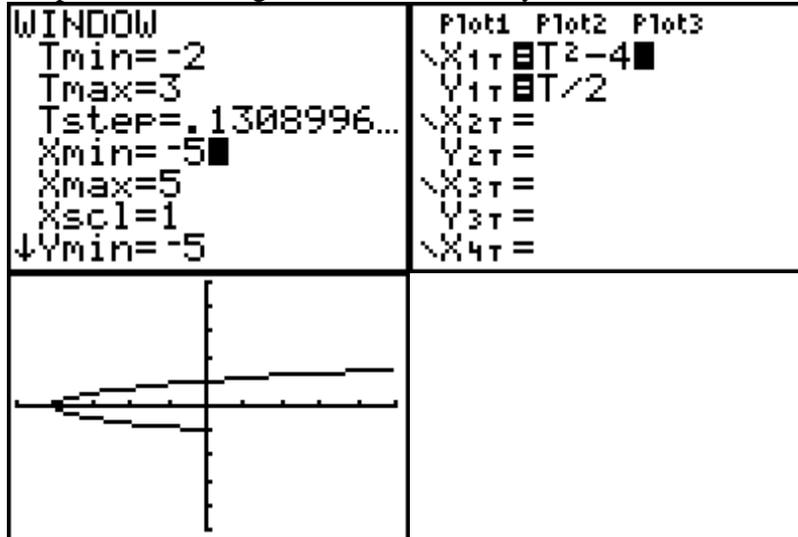
Now write an equation for this problem using only x and y. Compare the graphs. What are the differences and the similarities?

```
X1ᴛ=30T        Y1ᴛ=45-4.9T_
                                 Y1=45-4.9X²/900



T=0
X=0            Y=45


X=36.808511  Y=37.623505
```

Activity

Because the parametric mode gives not only a picture but also an orientation, sometimes the results when tracing can be surprising.
Graph the following and then trace. Set your window as follows.

```
WINDOW
 Tmin=-2
 Tmax=3
 Tstep=.1308996…
 Xmin=-5▪
 Xmax=5
 Xscl=1
↓Ymin=-5
```
```
Plot1  Plot2  Plot3
\X₁ᴛ⊟T²-4▪
 Y₁ᴛ⊟T/2
\X₂ᴛ=
 Y₂ᴛ=
\X₃ᴛ=
 Y₃ᴛ=
\X₄ᴛ=
```

Sample Problems
1. $x = 1/\sqrt{t} + 1$
    $y = t/(t + 1)$

2. $x = 3\cos 6$
    $y = 4\sin 6$

3. $x = e-t$
    $y = 2e-t + 1$

Activity

An example of a vector-valued function:
The center field fence in a ballpark is 12 feet high and 450 feet from home plate.
If a ball is hit from 3 feet above ground at 110 miles per hour, it may or may not
clear the fence. The angle at which the ball leaves the bat will determine whether
it clears the fence. First write a vector valued-function for the path of the ball.
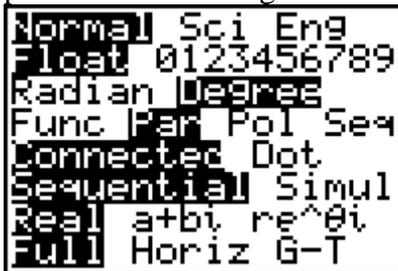
$$r(t) = \left\langle v_0 \cos(\theta)t, -16t^2 + v_0 \sin(\theta)t + h \right\rangle$$

$$v_0 = \left(110\frac{mi}{hr}\right)\left(5280\frac{ft}{mi}\right)\left(\frac{1hr}{3600\sec}\right) = 161.3\frac{ft}{\sec}$$

$$h = 3$$

$$r(t) = \left\langle 161.3\cos(\theta)t, -16t^2 + 161.3\sin(\theta)t + 3 \right\rangle$$

The first component of the vector represents the horizontal distance covered by the
ball and the second component represents the height of the ball. Set the calculator to
parametric mode/degrees.

```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T
```

Go to Y= and type in the x and y components.
Try letting ∠=10°and investigate whether the ball will clear the fence.

```
Plot1 Plot2 Plot3
\X₁ᴛ◼161.3cos(10
)T
 Y₁ᴛ◼-16T²+161.3
sin(10)T+3
\X₂ᴛ=
 Y₂ᴛ=
\X₃ᴛ=
```
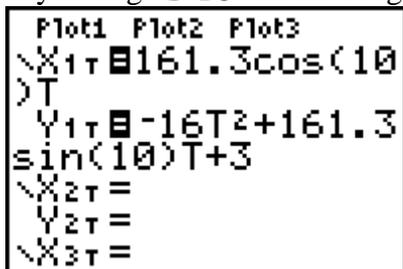
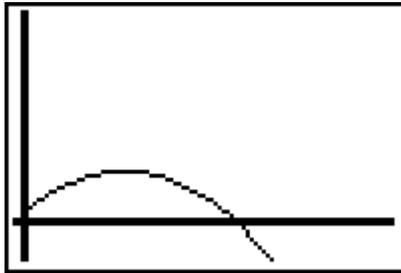Set window as shown

```
WINDOW
 Tmin=█
 Tmax=20
 Tstep=.2
 Xmin=-10
 Xmax=500
 Xscl=1
↓Ymin=-10
```
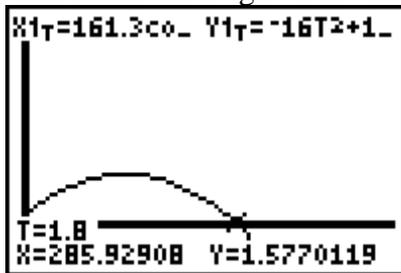```
WINDOW
↑Tstep=.2
 Xmin=-10
 Xmax=500
 Xscl=1
 Ymin=-10
 Ymax=60
 Yscl=█
```

Now graph.

If you will trace to find height of ball after traveling horizontally 450 feet you can then check the height to see if it clears the fence.

```
X1ᴛ=161.3co_ Y1ᴛ=-16T²+1_

T=1.8
X=285.92908 Y=1.5770119
```
Notice that the ball is only 1.577 feet high at a horizontal distance of 285.9 feet.

Students can then try different angles until they find an angle that will cause the ball to clear the fence.  Notice that the desired angle appears to be approximately 18°.

```
X1ᴛ=161.3co_ Y1ᴛ=-16T²+1_




T=3
X=460.21625  Y=8.5333236
```
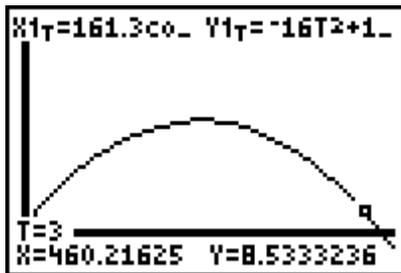
Students can then do the actual calculations and compare.

This problem can be further examined using the derivative features of the calculator.

Here are the equations as they appear in the equation editor.

```
Plot1  Plot2  Plot3
\X₁ᴛ◘■61.3cos(18
)T
 Y₁ᴛ◘-16T²+161.3
sin(18)T+3
\X₂ᴛ=
 Y₂ᴛ=
\X₃ᴛ=
```
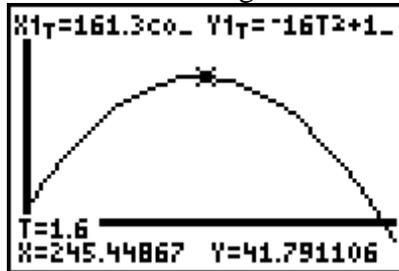
Have students trace to estimate that maximum height and when it occurs.

```
WINDOW
 Tmin=■
 Tmax=20
 Tstep=.2
 Xmin=-10
 Xmax=500
 Xscl=1
↓Ymin=-10
```
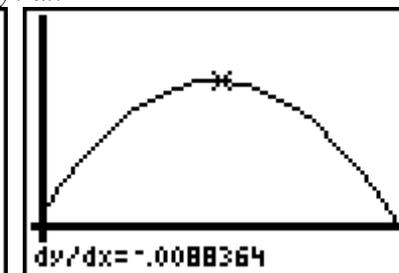
```
X1ᴛ=161.3co_  Y1ᴛ=-16T²+1_



T=1.6
X=245.44867  Y=41.791106
```

Now go to 2ⁿᵈ Trace and find dy/dx

```
CALCULATE
1:value
2↑dy/dx
3:dy/dt
4:dx/dt
```

```



dy/dx=-.0088364
```

Note that dy/dx is about 0. Next have students look at dy/dt and dx/dt to emphasize that at the maximum the vertical component of velocity is 0.

```

dy/dt=-.0755588
```

```

dx/dt=153.40542
```