EXPLORATIONS™

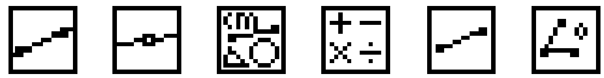
Activity 1

Betweenness and the Sum of Parts

Objectives

- To investigate the concept of betweenness for lines and angles
- To extend the concept of betweenness to the idea of the sum of parts

Cabri® Jr. Tools



Introduction

Basic geometric ideas like *betweenness* and the *sum of parts* are important elements of geometric thinking. In this activity, you will explore these ideas visually, geometrically, and numerically for segments and angles using the Cabri Jr. application. Though betweenness may be obvious visually, it is not always obvious geometrically.

Part I: Segments

Construction

Draw and label a segment and a point on the segment.

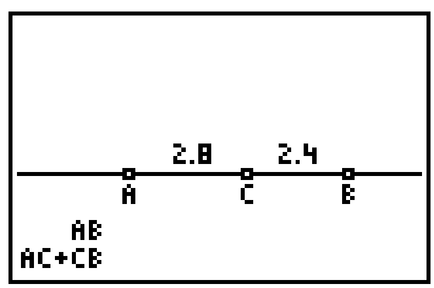
Draw a horizontal line $\overleftrightarrow{AB}$ near the center of the screen.

Construct a point *C* on the line between *A* and *B*.

Measure the lengths of segments $\overline{AC}$ and $\overline{CB}$. Place these measurements above the segments.

Measure the length of segment $\overline{AB}$. Label the measurement and place it near the bottom of the screen.

Calculate the sum of the lengths of segments $\overline{AC}$ and $\overline{CB}$. Label the calculation and place it near the bottom of the screen.



Note: Not all measurements are shown.

Exploration

Observe the changes in the measures and note how they are related when you change the location of *C* by:

- dragging it closer to *A*.
- dragging it closer to *B*.
- dragging it to the other side of *A*.
- dragging it to the other side of *B*.

Questions and Conjectures

1. Make a conjecture about the lengths of segments $\overline{AC}$, $\overline{CB}$, and $\overline{AB}$, when *C* is between *A* and *B*. Explain your reasoning.
2. Make a conjecture about the lengths of segments $\overline{AC}$, $\overline{CB}$, and $\overline{AB}$, when *C* is not between *A* and *B*. Explain your reasoning.

Part II: Angles

Construction

Draw segments to form adjacent angles.

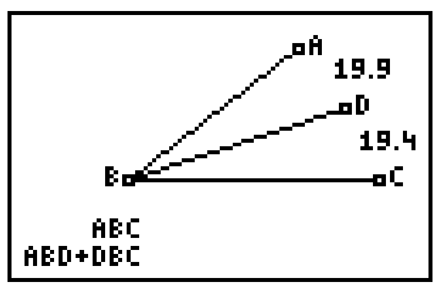
Draw and label segments $\overline{AB}$ and $\overline{BC}$ to form $\angle ABC$ having vertex *B*.

Construct $\overline{BD}$ so that point *D* is in the interior of $\angle ABC$.

Measure $\angle ABD$ and $\angle DBC$. Place these measures in the interior of each angle.

Measure $\angle ABC$. Label the measure and place it near the bottom of the screen.

Calculate $m\angle ABD + m\angle DBC$. Label the calculation and place it near the bottom of the screen.



Note: *Not all measurements are shown.*

Exploration

Observe the changes in the measures and note how they are related when you change the position of $\overline{BD}$ by:

- dragging *D* closer to $\overline{AB}$.
- dragging *D* closer to $\overline{BC}$.
- dragging *D* to the other side of $\overline{AB}$.
- dragging *D* to the other side of $\overline{BC}$.

Questions and Conjectures

1. Make a conjecture about the measures of $\angle ABD$, $\angle DBC$, and $\angle ABC$ when D is in the interior of $\angle ABC$. Explain your reasoning.
2. Make a conjecture about the measures of $\angle ABD$, $\angle DBC$, and $\angle ABC$ when D is not in the interior of $\angle ABC$. Explain your reasoning.

Teacher Notes

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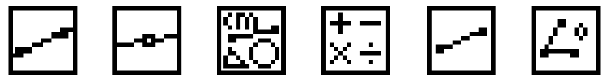

Additional Information

Students should conclude that the sum of the parts equals the whole. It is the interactive nature of the Cabri Jr. application that makes it such a powerful tool for investigations. The student can now investigate many examples to gain insights into geometry.

Activity 5, Shortest Distance Between Points and Lines, is an extension to Part I of this activity. In Activity 5, students investigate the concept of betweenness with a point *not* on the line.

Part I: Segments

Answers to Questions and Conjectures

1. Make a conjecture about the lengths of segments $\overline{AC}$, $\overline{CB}$, and $\overline{AB}$, when C is between A and B. Explain your reasoning.

   Students should see that $AC + CB = AB$. The sum of the parts equals the whole.

2. Make a conjecture about the lengths of segments $\overline{AC}$, $\overline{CB}$, and $\overline{AB}$ when C is not between A and B. Explain your reasoning.

   When point C is dragged so that it is no longer between points A and B, $AC + CB \neq AB$. When C is to the right of B, $AC - CB = AB$. When C is to the left of A, $CB - AC = AB$.

   When point C is dragged so that it is on point A or B, one of the parts will equal the whole because the other part equals zero.

Part II: Angles

Answers to Questions and Conjectures

1. Make a conjecture about the measures of $\angle ABD$, $\angle DBC$, and $\angle ABC$ when D is in the interior of $\angle ABC$. Explain your reasoning.

   $m\angle ABD + m\angle DBC = m\angle ABC$. The sum of the parts equals the whole. If $\overline{BD}$ is the angle bisector, then $m\angle ABD = m\angle DBC = \frac{1}{2}m\angle ABC$.

2. Make a conjecture about the measures of $\angle ABD$, $\angle DBC$, and $\angle ABC$ when D is not in the interior of $\angle ABC$. Explain your reasoning.

   When D is in the exterior of $\angle ABC$, $m\angle ABD + m\angle DBC \neq m\angle ABC$. In this case, either $m\angle ABD - m\angle DBC = m\angle ABC$ or $m\angle DBC - m\angle ABD = m\angle ABC$.

   When D is on $\overline{BA}$ or $\overline{BC}$, the measure of one of the parts will equal the whole because the other part equals zero.