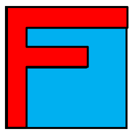
Name ______________________

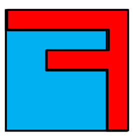
Class ______________________

Problem 1 – Symmetry group for a square

Identity

Sketch	Description	Inverse
		

Reflections

Sketch	Description	Inverse
	reflect over $x = 0$	reflect over $x = 0$
	reflect over $y =$ ____	
	reflect over $y =$ ____	
	reflect over $y =$ ____	

Rotations

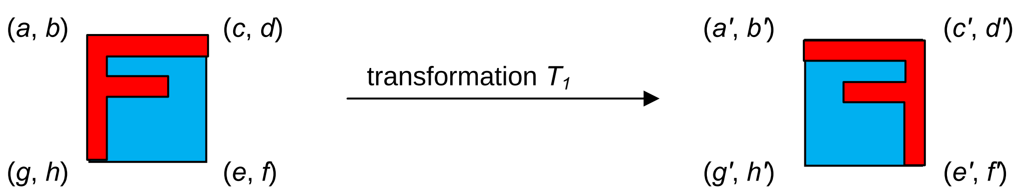
Sketch	Description	Inverse
	rotate around origin ____ °	
	rotate around origin ____ °	
	rotate around origin ____ °	

- How many different transformations are in the symmetry group of a square? Include the identity.

- What do you notice about the inverse transformations? Describe them.

Problem 2 – Transformer matrices

original square *S* **image square *S′***



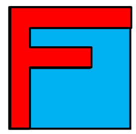
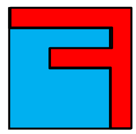
$$\boldsymbol{S} \times \boldsymbol{T_1} = \boldsymbol{S'}$$

$$\begin{bmatrix} a & b \\ c & d \\ e & f \\ g & h \end{bmatrix} \times \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -a & b \\ -c & d \\ -e & f \\ -g & h \end{bmatrix}$$

- Find ***S* · *T2***. (***T2*** is given in the table on the next page).

- What transformations could ***T2*** correspond to?

Complete the table.

Transformer Matrix	Sketch	Description
$T_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$		no change
$T_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$		reflect over $x = 0$
$T_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$		
$T_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$		
$T_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$		
$T_5 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$		
$T_6 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$		
$T_7 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$		

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Use the description columns to match the transformer matrices with their inverses. For example, T_1 is its own inverse.

Transformer Matrix	Inverse	Transformer Matrix	Inverse
$T_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$		$T_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$T_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
$T_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$		$T_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	
$T_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$		$T_5 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	
$T_6 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$		$T_7 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$	

- Multiply each transformer matrix in the table above by its inverse.
  What do you notice?

Use matrix multiplication to answer each question.

- What is the effect of applying T_3 followed by T_5?

- What is the effect of applying T_2 followed by T_3?

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Problem 3 – Symmetry group for an equilateral triangle

Use these transformer matrices.

$$T_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad T_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \quad T_3 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

Sketch	Description	Inverse	Transformer Matrix
			$T_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$