NUMB3RS Activity: Settling the Score
Episode: "Guns and Roses"

Topic: Multiple-scale scoring systems **Grade Level:** 9 - 12
Objective: To use matrix multiplication to understand how scoring systems with multiple scales work
Materials: Calculator for matrix multiplication
Time: 10 - 15 minutes

Introduction

In "Guns and Roses," Charlie learns of the alleged suicide of Nikki Davis, a woman from Don's past. He questions the acceptance of the suicide conclusion and decides to investigate further with what he calls a "modified Holmes-Rahe" stress test. This test "scores" stress through values put on certain situations in one's life and produces a number. The higher the number, the more stress in a person's life. Concluding that Nikki had skills to cope with her level of stress, he suggests they investigate the case as a homicide. He explains the mathematics by comparing the stress test to how figure skaters are scored based on the difficulty of the jump and the judges' evaluation of the performance. This activity looks at various aspects of such a scoring system.

Discuss with Students

If students are not familiar with matrix multiplication, this is an excellent opportunity to teach it. Some students may have done matrix multiplication with a calculator, but this could be an opportunity to remind them of the actual process by reviewing the multiplication of a row vector and a larger matrix.

Depending on the type of grading system used in your class, you might point out that in a weighted grading system, each quiz grade is multiplied by a factor of x, each test grade by y, each homework grade by z, etc. and thus create a question regarding their own grades that applies to the objective of this activity.

Student Page Answers: 1. Contestant A: 30.56; Contestant B: 31.89. Contestant B wins

2. $[2.3 \;\; 3.1 \;\; 3.9 \;\; 3.6 \;\; 2.8 \;\; 3.2] \begin{bmatrix} 5.1 & 5.2 & 4.9 \\ 4.2 & 3.8 & 4.0 \\ 3.9 & 4.0 & 4.2 \\ 5.0 & 4.8 & 4.9 \\ 5.8 & 5.7 & 5.6 \\ 5.2 & 5.4 & 5.6 \end{bmatrix} = [90.84 \;\; 89.86 \;\; 91.29]$ (total score by judge) .

3. Answers will vary, but generally the 2.6 degree of difficulty would be ruled out because even with the highest possible score from each judge, the score would not be enough to win. One does not need all the points in the risky 4.2 degree of difficulty. Thus the 3.8 has enough "cushion" to allow the win even if the judges average the scores as low as 4.2.

4. $[0.01 \;\; 6 \;\; 0.1 \;\; 1 \;\; 0.5] \begin{bmatrix} 1890 & 2140 \\ 3.8 & 3.3 \\ 85 & 90 \\ 5 & 4 \\ 5 & 6 \end{bmatrix} = [57.7 \;\; 57.2]$; Candidate X would be accepted.

5. It would be better to work on the grades because 6(0.2) = 1.2 while (0.1)(100) = 1.
6. The total impact each can have is the same. The most either can contribute is 24 points.

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Name: _____ Date: _____

NUMB3RS Activity: Settling the Score

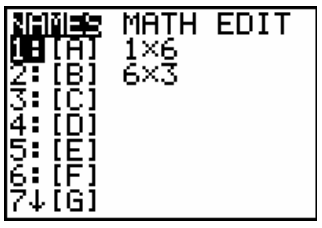
When Charlie learns of a reported suicide, he decides to look into it with what he calls a "modified Holmes-Rahe" stress test. He explains how the test works by comparing it to how figure skaters are scored based on both the difficulty of the jump and the judges' evaluation of the performance. This type of scoring is used with slight variations in many forms of competition, such as diving and gymnastics. (This activity uses a generic event. The Extensions page discusses specific sports.)

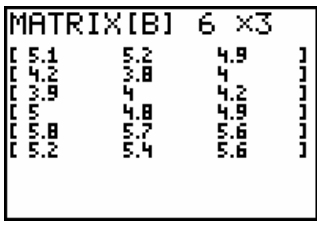
Suppose a contestant attempts a jump with a difficulty rating of 3.2, and the judges give him a score of 5.4 (out of 6). His total score would be (3.2)(5.4) = 17.28 points. His opponent chooses to attempt a jump with a difficulty level of 3.9 but the judges only gave her 5.1. She would still win the competition because (3.9)(5.1) = 19.89 points, which is a higher total. In general, the degree of difficulty multiplied by the judge's score is the score assigned to the athlete for that jump or event.

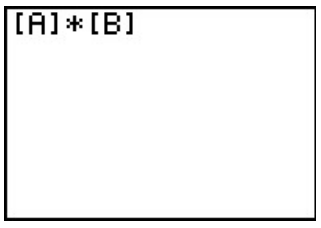
1. Suppose Contestant A attempts two feats in a competition. They have difficulty ratings of 3.2 and 2.7. The judge scores the first one 5.5 and the second 4.8. Contestant B's feats have difficulty ratings of 3.9 and 2.4 and the judge gives a 5.1 to the first and 5.0 to the second. What is the total score for each contestant? Who wins?

2. In competitions like the Olympic Games, there are many judges. Suppose an athlete's six efforts have the difficulty ratings (in order attempted) of 2.3, 3.1, 3.9, 3.6, 2.8, and 3.2. Each of three judges scores these in the same order. Judge X rates them 5.1, 4.2, 3.9, 5.0, 5.8, and 5.2. Judge Y awards 5.2, 3.8, 4.0, 4.8, 5.7, and 5.4. Judge Z awards 4.9, 4.0, 4.2, 4.9, 5.6, and 5.6. The total score for each judge can be displayed as a sum of the product of the degree of difficulty and the score for each effort. So, Judge X would give the athlete (2.3)(5.1) + (3.1)(4.2) + (3.9)(3.9) + (3.6)(5.0) + (2.8)(5.8) + (3.2)(5.2). This is tedious to calculate in this manner. It is much easier to use matrix multiplication, with the difficulty factors for each feat represented in a 1 × 6 matrix and the judging scores for the feat represented in a 6 × 3 matrix (each judge is a column in the matrix).

On the TI-84 Plus, you can enter matrices under the **MATRIX** menu (2nd [MATRIX]). Press ◄ to access the **EDIT** menu. Make matrix [A] the 1 × 6 or athlete's degree of difficulty matrix and matrix [B] the 6 × 3 or judges' scoring matrix.







On the home screen, multiply the matrices as shown and use ► to see the complete answer. Record the results.

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3. In the previous question, imagine you are the final performer in the event. You have a comfortable lead over the person in third place, but trail the leader by 47 points. You have three possibilities in your repertoire. One has a degree of difficulty of 4.2 but you only succeed at it 10% of the time. The second has a degree of difficulty of 3.8 and you succeed 65% of the time. Your final choice has a degree of difficulty of 2.6, and you succeed 98% of the time. The three judges rate each performance on a scale of 1 to 6, and your score is the sum of the products of each judge's score times the degree of difficulty. Which of the three routines should you attempt to have the best chance of winning the meet? Explain your reasoning.

Suppose a hypothetical college (perhaps Cal Sci) evaluates applicants on only 5 items. The SAT score is multiplied by 0.01. The GPA (0.0 to 4.0) is multiplied by 6. The student essay is graded out of 100 points and multiplied by 0.1. The teacher recommendation is on a form that rates the student on a scale of 1 to 5 and that score is multiplied by 1. Finally, the number of extracurricular activities is counted (up to a maximum of 10) and that score is multiplied by 0.5. It has come down to two final candidates with these records:

	Candidate X	**Candidate Y**
SAT	1890	2140
GPA	3.8	3.3
Essay	85	90
Recommendation	5	4
Activities	5	6

4. Use matrix multiplication to find the weighted score for each person. Who would be accepted according to these criteria?

5. In the preceding question, suppose you could work to improve either your SAT score or your GPA, but not both. If the effort can add 100 points to your SAT score or 0.2 to your GPA, which is the better choice with regard to your acceptance possibilities under the system given above? Why?

6. What is the ultimate effect of multiplying the SAT score by 0.01 and the GPA by 6?

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

There are many other activities that have variable scoring schemes. In football, a touchdown has a "degree of difficulty" of 6, a field goal has 3, a safety 2, and points after touchdown 2 if by running or passing and 1 if kicked. In basketball, field goals are 2 or 3 depending on length, and a free throw is 1. However, these numbers were not always the ones used. For example, in the early days of football, a field goal was considered more difficult than a touchdown and thus awarded more points. Research the history of these two sports (or any other) and find out how many different scoring systems they have had over the years. Find the rationale behind the changes and tell whether you agree and why. Pick a famous game and recalculate the score, as it would have been in a different era.

Additional Resources

The details of calculating figure skating scores can be found at:
http://www.usfsa.org/content/module4pgs30-34.pdf

For diving, a table for the degree of difficulty and a link to the formula can be found at:
http://www.usadiver.com/dd_table.htm

For the Student

Many schools have a grading system where the grades (A = 4, B = 3, C = 2, D = 1, F = 0) are multiplied by a factor representing the difficulty of the course (e.g., an AP course might be multiplied by 1.5, an honors class by 1.25, etc.). If your school has such a system, speak with the administration to see if you can determine how they arrived at the values. If your school does not, discuss the pros and cons of adopting such a system. If you like the idea, you might write up a proposal for your school or district.

Related Topic

Recently there has been a change in the scoring in figure skating that has led to controversy. The Web site below presents an argument which includes actual scores from real competitions and how they would fare under different systems.
http://cbs.sportsline.com/u/women/skating/jul98/loosemoreb7198.htm

Compare some of the author's concerns to those from voting theory. A good start might be the comparison of methods in the table of voting methods at the Web site below.
http://en.wikipedia.org/wiki/Voting_system

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