1.

(a) Using the definition of a derivative as $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, show that the derivative of $\frac{2}{3x-5}$ is $\frac{-6}{(3x-5)^2}$. (4 marks)

(b) Using the same function, $\frac{2}{3x-5}$, find $\lim_{h\to 0}\frac{f(x+h)-f(x)}{2h}$. Explain why this limit may be used as a better approximation that the limit used in part (a). (4 marks)

Mark scheme:

(a) $\lim_{h\to 0}\frac{\frac{2}{3(x+h)-5}-\frac{2}{3x-5}}{h}$ (M1)(A1)

$$\lim_{h\to 0}\frac{\frac{2}{3x+3h-5}*\frac{3x-5}{3x-5}-\frac{2}{3x-5}*\frac{3x+3h-5}{3x+3h-5}}{h}$$ (A1)

$$\lim_{h\to 0}\frac{\frac{6x-10-6x-6h+10}{(3x+3h-5)(3x-5)}}{h}$$

$$\lim_{h\to 0}\frac{-6h}{(3x+3h-5)(3x-5)}*\frac{1}{h}$$ (A1)

$$=\frac{-6}{(3x-5)^2}$$ (AG)

(b) $\lim_{h\to 0}\frac{\frac{2}{3(x+h)-5}-\frac{2}{3(x-h)-5}}{2h}$ (A1)

$$\lim_{h\to 0}\frac{\frac{2}{3x+3h-5}*\frac{3x-3h-5}{3x-3h-5}-\frac{2}{3x-5}*\frac{3x+3h-5}{3x+3h-5}}{2h}$$ (A1)

$$\lim_{h\to 0}\frac{\frac{6x-6h-10-6x-6h+10}{(3x+3h-5)(3x-3h-5)}}{2h}$$

$$\lim_{h\to 0}\frac{-12h}{(3x+3h-5)(3x-3h-5)}*\frac{1}{2h}$$

$$\lim_{h\to 0}\frac{-6}{(3x+3h-5)(3x-3h-5)}$$

$$=\frac{-6}{(3x-5)^2}$$ (A1)

When finding the slope of a tangent at a point on a curve, this symmetric differ (R1)