



Investigating the Derivatives of Some Common Functions

IB® EXAM STYLE QUESTION

Topic 5: Calculus

Derivatives

1.

(a) Using the definition of a derivative as $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$, (4 marks)

show that the derivative of $\frac{2}{3x-5}$ is $\frac{-6}{(3x-5)^2}$.

(b) Using the same function, $\frac{2}{3x-5}$, find $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{2h}$. Explain (4 marks)
why this limit may be used as a better approximation than the limit used in part (a).

Mark scheme:

$$(a) \lim_{h \rightarrow 0} \frac{\frac{2}{3(x+h)-5} - \frac{2}{3x-5}}{h} \quad (\text{M1})(\text{A1})$$

$$\lim_{h \rightarrow 0} \frac{\frac{2}{3x+3h-5} * \frac{3x-5}{3x-5} - \frac{2}{3x-5} * \frac{3x+3h-5}{3x+3h-5}}{h} \quad (\text{A1})$$

$$\lim_{h \rightarrow 0} \frac{\frac{6x-10-6x-6h+10}{(3x+3h-5)(3x-5)}}{h}$$

$$\lim_{h \rightarrow 0} \frac{-6h}{(3x+3h-5)(3x-5)} * \frac{1}{h} \quad (\text{A1})$$

$$= \frac{-6}{(3x-5)^2} \quad (\text{AG})$$

$$(b) \lim_{h \rightarrow 0} \frac{\frac{2}{3(x+h)-5} - \frac{2}{3(x-h)-5}}{2h} \quad (\text{A1})$$

$$\lim_{h \rightarrow 0} \frac{\frac{2}{3x+3h-5} * \frac{3x-3h-5}{3x-3h-5} - \frac{2}{3x-5} * \frac{3x+3h-5}{3x+3h-5}}{2h} \quad (\text{A1})$$

$$\lim_{h \rightarrow 0} \frac{\frac{6x-6h-10-6x-6h+10}{(3x+3h-5)(3x-3h-5)}}{2h}$$



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$$\lim_{h \rightarrow 0} \frac{-12h}{(3x + 3h - 5)(3x - 3h - 5)} * \frac{1}{2h}$$

$$\lim_{h \rightarrow 0} \frac{-6}{(3x + 3h - 5)(3x - 3h - 5)}$$

$$= \frac{-6}{(3x - 5)^2} \quad (\text{A1})$$

When finding the slope of a tangent at a point on a curve, this symmetric difference is called the **difference quotient**. (R1)