

### Exploring Linear Relationships—Stacking Cups and Walking Rates

by Ann Jones and David Staten

#### Activity overview

In this activity, students will explore several examples of linear relationships. They will use a table to organize data that they collect and will make graphs to display that data. Students are prompted to write equations and interpret slope as a rate of change.

#### Concepts

##### NCTM Standards:

In grades 9-12 all students should-

- understand relations and functions and select, convert, flexibly among, and use various representations for them;
- identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- approximate and interpret rates of change from graphical and numerical data;
- understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable;
- for bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools;

##### TN Algebra 1 Standards

##### Course Level Expectations:

CLE 3102.1.4            Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic), to solve problems, to model mathematical ideas, and to communicate solution strategies.

##### Checks for Understanding:

3102.1.13            Change from one representation of a relation to another representation, for example, change from a verbal description to a graph.

3102.1.19            Recognize and practice appropriate use of technology in representations and in problem solving.

3102.3.20            Understand that a linear equation has a constant rate of change called slope and represent slope in various forms.

3102.5.1            Identify patterns or trends in data.

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- 3102.5.8 Examine real-world graphical relationships (including scatter-plots) to determine type of relationships(linear or nonlinear) and any association (positive, negative or none) between the variables of the data set.
- 3102.5.9 Determine an equation for a line that fits real-world linear data; interpret the meaning of the slope and y-intercept in context of the data.
- 3102.5.11 Use an equation that fits the data to make a prediction.

State Performance Indicators:

- SPI 3102.1.1 Interpret patterns found in sequences, tables, and other forms of quantitative information using variables or function notation.
- SPI 3102.1.6 Determine and interpret slope in multiple contexts including rate of change in real-world problems.
- SPI 3102.5.1 Interpret displays of data to answer questions about the data set(s) (e.g., identify pattern, trends, and/or outliers in a data set).
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### Teacher preparation

Before the activity, the teacher should pre-load the activity file Linear\_Relationships.tns on to the student handheld devices. This can be done via Connect-to-Class software, TI-Nspire computer link software, or by using link cables with the handhelds. The teacher should also have several sets of stackable cups on hand for the first activity. Each group will need two sets, each consisting of ten cups, and a ruler to measure the stacks.

### Classroom management tips

1. This activity can be done by the students using the pre-made document file or by them creating various parts of the file as chosen by the teacher.
2. This activity is intended to be student-centered with the students working in small cooperative groups.

### TI-Nspire Applications

- ✓ Notes
  - ✓ Lists & Spreadsheets
  - ✓ Data & Statistics
  - ✓ Calculator
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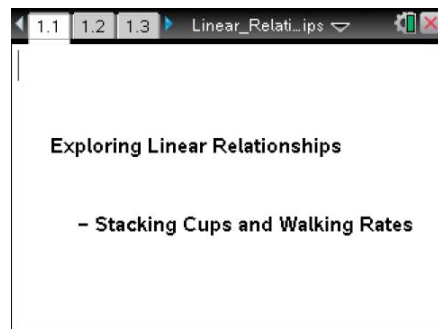
### Step-by-step directions

Steps:

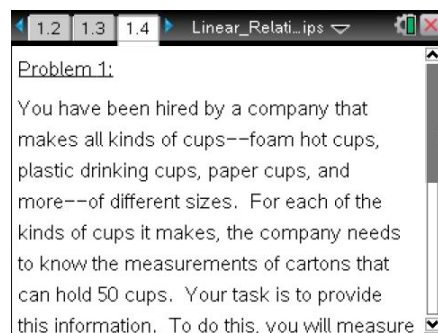
1. From the home screen, choose My Documents and navigate to the appropriate folder containing the .tns file *Linear\_Relationships*. Highlight the file and press **enter**. Choose whether or not to save changes to any previous document.



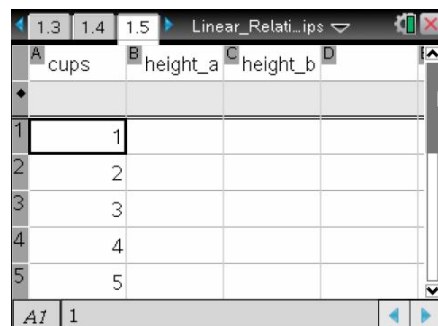
2. Page 1.1 is the title page for the activity. Press **ctrl**➤ and **ctrl**◀ to navigate through the document. Pressing **ctrl**▲ will give you a thumbnail view of all pages in the document. Pages 1.2 and 1.3 list standards addressed in this activity.



3. 1.4 introduces the first of two problems in this activity. Students are prompted to complete the table on the next page.



4. Students should complete the spreadsheet one cell at a time with the data they collect as they measure stacks of two different types of cups.



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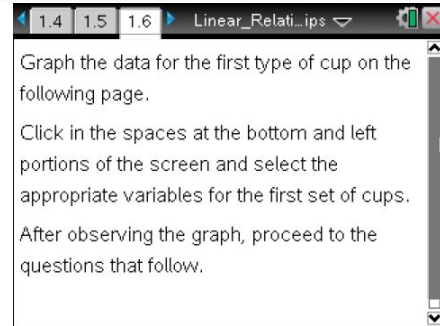
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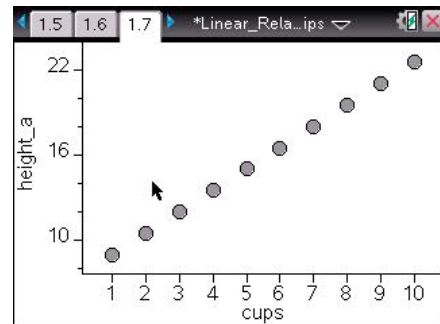
5. Page 1.6 contains directions for graphing the relationship for the first set of data on the Data & Statistics page on 1.7.



6. Click in the areas at the bottom and left portions of the screen to add the variables in the relationship that you wish to graph.



7. Students should select the number of cups for the horizontal variable (x), and the height of the first stack for the vertical variable (y). A sample graph is pictured at right.



8. Pages 1.8 and 1.9 consist of questions that the students must answer. In addition to describing the variables and their relationship, they will also be asked to predict the height for a stack of 50 cups and explain their prediction. To answer the various questions within the document, students should press ▼ until they reach the answer section of the page. A calculator page is provided on page 1.10 for any needed computations.

Question

Describe what variables are being investigated and the relationship between the variables.

Answer

## Exploring Linear Relationships

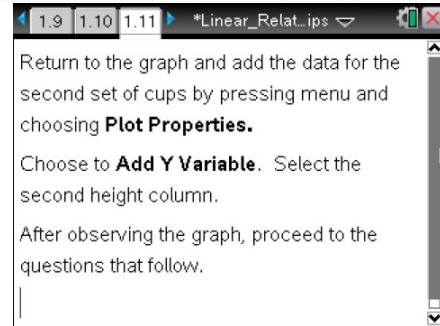
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Grade Level: 8 – 12

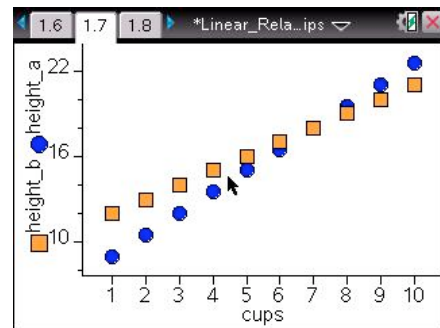
Subject: Mathematics

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9. Page 1.11 provides instructions for graphing the data for the second set of cups.



10. When students add the second set of data to the graph, both relations are displayed. A key is automatically generated on the graph to allow students to distinguish between the data sets for the two different types of cups. A sample graph is pictured at right.



11. Students are prompted to make the same prediction for the second set of cups as they made for the first. They are also prompted to make recommendations for the inside of a carton that would hold 50 of each type of cup and compare their findings for each type on pages 1.12 and 1.13.

**Question**

Predict how tall a stack of 50 cups would be for this second type of cup and explain how you made your prediction.

**Answer**

12. Problem 2 is presented on 2.1. For this problem, students will not be gathering data themselves, but will be given a set of data to work with.

**Problem 2:**

Several students are planning to participate in a walkathon to help raise money for charity. The distance to be walked is 10 kilometers. The students are wondering how long it might take them to walk this distance. Several students decide to do an experiment to determine their walking rates. Here are data for three of the students:

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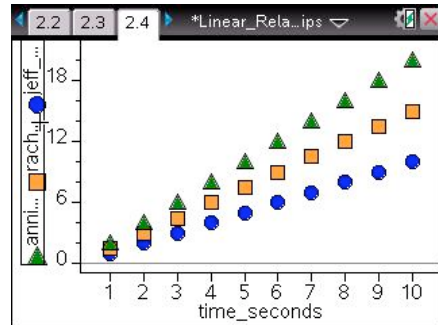
13. The spreadsheet may be completed one cell at a time.

	time_s...	jeff_dis...	rachel...	annie...
1	1			
2	2			
3	3			
4	4			
5	5			

14. Students with spreadsheet skills can use the gray formula cells to complete the spreadsheet.

	time_s...	jeff_dis...	rachel...	annie...
		=a[]	=1.5*a[]	=2*a[]
1	1	1	1.5	2
2	2	2	3	4
3	3	3	4.5	6
4	4	4	6	8
5	5	5	7.5	10

15. Students will answer questions and graph all relationships using the same procedures as in Problem 1. The graph of the three data sets is pictured at right.



### Assessment and Evaluation

- ✓ The teacher can collect student files using Connect-to-Class software or TI-Nspire Computer Link software.
- ✓ Sample Answers to questions in .tns file:

**Q:** Describe what variables are being investigated and the relationship between the variables.

**A:** The variables are the number of cups and the height of the stack. As the number of cups increases, the height also increases at a constant rate.

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- Q:** Predict how tall a stack of 50 cups would be and explain how you made your prediction.
- A:** Answers will vary depending on the measurements of the cups that are used. Explanations may include recognition of a recursive relationship in that the height increases by the same amount with each added cup. Some students may recognize the functional relationship between the two data sets and may define a function rule to make their predictions.
- Q:** For each kind of cup, recommend the inside dimensions of a carton that would hold 50 cups.
- A:** Answers will vary depending on the measurements of the cups that are used. The diameter of the cup should be taken into account when determining the length and width of the carton, while the height will be based on the height of the stack. Measurements for the inside of the carton may be slightly larger to allow some “wobble room”. Some students may design a carton that would hold two stacks of 25, in which case the height of 25 cups must be determined. In this scenario, the width should be double the length to allow room for both sets of cups.
- Q:** Compare the results for the different kinds of cups, noting similarities and differences.
- A:** The heights of both stacks of cups increase at a constant rate with each added cup. The graphs are both linear. Other similarities and differences may be observed depending on the measurements of the cups that are used.
- Q:** Describe the relationship between the time and distance walked, using words.
- A:** The distance Jeff walks increases 1 meter for each second walked.
- Q:** Write an equation for the relationship, using  $d$  to represent distance in meters and  $t$  to represent time in seconds. Describe how the walking rate affects the equation.
- A:**  $d = 1t$  or  $d = t$
- Q:** Write an equation for the relationship, using  $d$  to represent distance in meters and  $t$  to represent time in seconds for Rachel and Annie. Describe how the walking rate affects these equations.
- A:** Rachel's is  $d = 1.5t$  and Annie's is  $d = 2t$ . In each equation, the walking rate is the coefficient of  $t$ , and is the slope of the line.
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- Q.** Using these equations, determine the distance traveled in 1 minute, 30 minutes, and 1 hour for each student. The next tab 2.10 is a calculator screen if needed for computation.
- A.** Jeff: 1 min = 60 m, 30 min = 1800 m, and 1 hr = 3600m  
Rachel: 1 min = 90 m, 30 min = 2700 m, and 1 hr = 5400 m  
Annie: 1 min = 120 m, 30 min = 3600 m, and 1 hr = 7200 m
- Q.** Use this information to estimate how much time it will take each person to complete the walkathon.
- A.** Jeff = 2 hrs 47 mins  
Rachel = 1 hr 51 mins  
Annie = 1 hr 23 mins

### Student TI-Nspire Document

*Linear\_Relationships.tns*

**Exploring Linear Relationships**

**- Stacking Cups and Walking Rates**

**NCTM Standards**

In grades 9–12 all students should–

- understand relations and functions and select, convert, flexibly among, and use various representations for them;
- identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.

**TN Algebra 1 Standards**

Course Level Expectations:

CLE 3102.1.4 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic), to solve problems, to model mathematical ideas, and

**Problem 1:**

You have been hired by a company that makes all kinds of cups--foam hot cups, plastic drinking cups, paper cups, and more--of different sizes. For each of the kinds of cups it makes, the company needs to know the measurements of cartons that can hold 50 cups. Your task is to provide this information. To do this, you will measure

	A	B	C	D
	cups	height_a	height_b	
1	1			
2	2			
3	3			
4	4			
5	5			
A7	1			

Graph the data for the first type of cup on the following page.

Click in the spaces at the bottom and left portions of the screen and select the appropriate variables for the first set of cups.

After observing the graph, proceed to the questions that follow.

Question

Describe what variables are being investigated and the relationship between the variables.

Answer

Question

Predict how tall a stack of 50 cups would be and explain how you made your prediction. The next tab 1.10 is a calculator screen if needed for computation.

Answer



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1.8 1.9 1.10 \*Linear\_Relat\_ips

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1.9 1.10 1.11 \*Linear\_Relat\_ips

Return to the graph and add the data for the second set of cups by pressing menu and choosing **Plot Properties**.

Choose to **Add Y Variable**. Select the second height column.

After observing the graph, proceed to the questions that follow.

1.10 1.11 1.12 \*Linear\_Relat\_ips

**Question**

Predict how tall a stack of 50 cups would be for this second type of cup and explain how you made your prediction.

**Answer**

1.11 1.12 1.13 \*Linear\_Relat\_ips

**Question**

For each kind of cup, recommend the inside dimensions of a carton that would hold 50 cups.

**Answer**

1.12 1.13 1.14 \*Linear\_Relat\_ips

**Question**

Compare the results for the different kinds of cups, noting similarities and differences.

**Answer**

1.13 1.14 2.1 \*Linear\_Relat\_ips

**Problem 2:**

Several students are planning to participate in a walkathon to help raise money for charity. The distance to be walked is 10 kilometers. The students are wondering how long it might take them to walk this distance. Several students decide to do an experiment to determine their walking rates. Here are data for three of the students:

1.14 2.1 2.2 \*Linear\_Relat\_ips

	A	B	C	D
1	1			
2	2			
3	3			
4	4			
5	5			
A1	1			

2.1 2.2 2.3 \*Linear\_Relat\_ips

Graph the data for Jeff on the following page.

Click in the spaces at the bottom and left portions of the screen and select the appropriate variables for Jeff.

After observing the graph, proceed to the questions that follow.

2.2 2.3 2.4 \*Linear\_Relat\_ips

Caption: jeff\_distance

Click to add variable

2.3 2.4 2.5 \*Linear\_Relat\_ips

**Question**

Describe the relationship between the time and distance walked, using words.

**Answer**

2.4 2.5 2.6 \*Linear\_Relat\_ips

**Question**

Write an equation for the relationship, using  $d$  to represent distance in meters and  $t$  to represent time in seconds. Describe how the walking rate affects the equation.

**Answer**

2.5 2.6 2.7 \*Linear\_Relat\_ips

Return to the graph and add the data for the Rachel by pressing menu and choosing **Plot Properties**.

Choose to **Add Y Variable**. Select the Rachel. Repeat this procedure for Annie so that the graph contains the data for all three students.

After observing the graph, proceed to the questions that follow.

2.6 2.7 2.8 \*Linear\_Relat\_ips

**Question**

Write an equation for each relationship, using  $d$  to represent distance in meters and  $t$  to represent time in seconds for Rachel and Annie. Describe how the walking rate affects these equations.

**Answer**

2.7 2.8 2.9 \*Linear\_Relat\_ips

**Question**

Using these equations, determine the distance traveled in 1 minute, 30 minutes, and 1 hour for each student. The next tab 2.10 is a calculator screen if needed for computation.

**Answer**

2.8 2.9 2.10 \*Linear\_Relat\_ips

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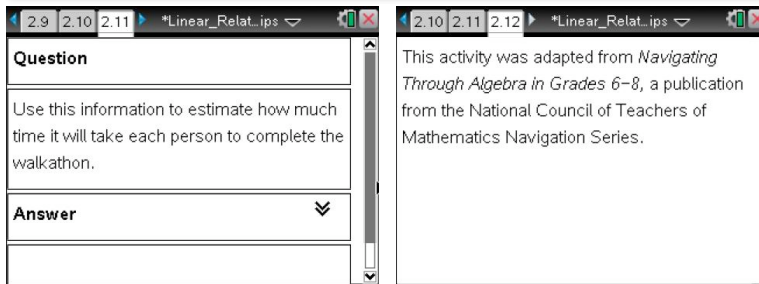
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The screenshot shows a software interface with two windows. The left window has a tab labeled '2.11' and contains a 'Question' section with the text: 'Use this information to estimate how much time it will take each person to complete the walkathon.' Below the question is an 'Answer' section with a dropdown arrow. The right window has a tab labeled '2.12' and contains the text: 'This activity was adapted from *Navigating Through Algebra in Grades 6-8*, a publication from the National Council of Teachers of Mathematics Navigation Series.'

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